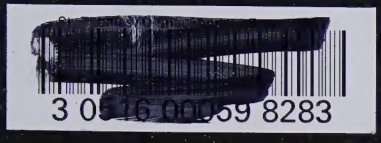
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FOUNDER OF THE *REVUE NÉO-SCOLASTIQUE*

ENLIGHTENED EXPONENT OF THE PHILOSOPHY OF THE SCHOOLS

ZEALOUS ADVOCATE OF ITS ABIDING EXCELLENCE

AND OF ITS HARMONY WITH THE

ESTABLISHED TRUTHS OF MODERN SCIENCE

AS A SMALL TOKEN OF ESTEEM AND VENERATION

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OLD AND NEW:

AN INTRODUCTION TO SCHOLASTIC PHILOSOPHY, MEDIÆVAL AND MODERN.

BY

MAURICE DE WULF,

Doctor of Laws, Doctor of Philosophy and Letters, Professor at the University of Louvain.

TRANSLATED BY

P. COFFEY, PH.D.,

Professor of Philosophy, Maynooth College, Ireland.

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THE SCIENCE OF LOGIC

AN INQUIRY INTO THE PRINCIPLES OF ACCURATE THOUGHT AND SCIENTIFIC METHOD

BY

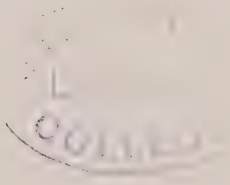
P. COFFEY, PH.D. (LOUVAIN)

PROFESSOR OF LOGIC AND METAPHYSICS, MAYNOOTH COLLEGE, IRELAND

IN TWO VOLUMES

VOL. I.

CONCEPTION, JUDGMENT, AND INFERENCE



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PREFACE

The aim and scope of this treatise are more modest than perhaps its dimensions might suggest. It attempts, in the first place, to present in a simple way the Principles of the Traditional Logic expounded by Aristotle and his Scholastic interpreters; secondly, to show how the philosophical teachings of Aristotle and the Schoolmen contain the true basis for modern methods of scientific investigation, inductive no less than deductive; and finally, to extend, rather than supplement, the traditional body of logical doctrine by applying the latter to some logical problems raised in more recent times. But the treatment throughout is confined mainly to principles, and is meant to be suggestive rather than exhaustive.

Logic has philosophy for its background. The study of logic raises many large questions, leading into various branches of philosophy. Nor can any presentation of the science of logic be truly educative or stimulating which does not involve some definite philosophical standpoint. Without this latter, the more formal portions of logic can be little better than a dry collection of mechanical rules, while the more important applied portions must remain practically unintelligible to the student. But the adoption and application of a definite philosophical standpoint in a work on logic does not render imperative any detailed discussion of the larger and more fundamental questions suggested thereby. Apart from Scholastic authors, who still preserve a clearly marked distinction between the main departments of philosophical in-

340

quiry, modern writers on logic are inclined to discuss many problems which would find a more appropriate place in works on epistemology or ontology. Such problems are but briefly referred to in the present work. But it has been thought advisable to follow the now common practice by dwelling at greater length upon the presuppositions of induction, because the theory of induction has not yet secured from Scholastic writers the amount of attention its growing importance would seem to demand.

The standpoint of the present work is that of Scholasticism as conceived and expounded by those who represent the neo-scholastic movement in modern philosophy.[1] To all such the conviction is common that no recent system of philosophy contains a body of doctrines more in keeping with the established truths of science than are the doctrines of Scholasticism. But to know what the established truths of science are, we must understand the methods of science. Hence the special claims of induction on the attention of the Scholastic logician.

Philosophers who believe in the superiority of the Scholastic system, as compared with other systems now actually in vogue, recognize the need of applying the traditional principles of this system to modern conditions and problems. Many of the latter are new, at least in form ; and much light has been thrown upon them by the intellectual labours of non-scholastic writers. Such useful achievements the modern Scholastic will gratefully recognize and gladly welcome ; while at the same time he remains true to the truth that is in Scholasticism (rejecting whatever he finds to be indefensible in it), and combats the teachings opposed to it.[2] *Vetera novis augere*

[1] For history and bibliography of this school of thought, Professor de Wulf's *Scholasticism Old and New* (Dublin, Gill, 2nd edit., 1910), and Perrier's *Revival of Scholastic Philosophy* (Columbia University Press, 1909), may be consulted with advantage; also the Louvain *Cours de Philosophie* and the Stonyhurst Philosophical Series.

[2] "Nos igitur, dum edicimus, libenti gratoque animo excipiendum esse quidquid sapienter dictum, quidquid utiliter fuerit a quopiam inventum atque excogitatum;

et perficere is a guiding principle which he obeys in all his investigations. This attitude will seek expression in the course of the present treatise. The writer is quite unconscious of having said or intended anything new or original. But neither has he intended to make a mere compilation. It has been his ambition to assimilate and analyse what he has learned from others; and, bearing in mind the requirements of beginners, to set forth the results of his own labours in the manner and order he considers most helpful to those for whom he has written. The main body of the doctrine is, of course, derived from the study of Aristotelean and Scholastic authors; but he wishes here to express his large indebtedness to those recent writers on logic from whose works he has derived much really valuable assistance. This he hopes he has duly acknowledged throughout the work itself. Among the authors he has found most helpful are the following: Mercier (Scholastic), *Logique* (4th edit., Louvain, 1705); Welton, *Manual of Logic* (vol. i., 2nd edit., London, 1904; vol. ii., London, 1901); Keynes, *Formal Logic* (4th edit., London, 1906); Venn, *Empirical Logic* (London, 1889); Joyce (Scholastic), *Principles of Logic* (London, 1908); and Joseph, *An Introduction to Logic* (Oxford, 1906). The latter, a remarkably mature and scholarly work, is largely Aristotelean; and although its positions have not been uniformly accepted in the present work, it has been consulted with uniform profit throughout. The student will be well rewarded for perusing it. All those authors have been allowed to speak for themselves wherever the clearness and brevity of their statements surpassed anything to which the present writer

Vos omnes, Venerabiles Fratres, quam enixe hortamur, ut ad catholicae Fidei tutelam et decus, ad societatis bonum, ad scientiarum omnium incrementum, auream sancti Thomae sapientiam restituatis, et quam latissime propagetis. Sapientiam sancti Thomae dicimus: si quid enim est a doctoribus scholasticis vel nimia subtilitate quaesitum, vel parum considerate traditum, si quid cum exploratis posterioris aevi doctrinis minus cohaerens, vel denique quoquo modo non probabile, id nullo pacto in animo est aetati nostrae ad imitandum proponi."—Encyclical *Aeterni Patris* of Pope Leo XIII. on Christian Philosophy, August, 1879.

could aspire. The references throughout are to the editions just mentioned.

In preparing large numbers of students for the preparatory and degree examinations of the late Royal, and of the present National University of Ireland, the author has felt the need of some one textbook that would meet the requirements of all his pupils. With this object in view, the more advanced portions, together with certain paragraphs that are mainly illustrative, have been printed in these volumes in smaller letterpress, or in footnotes. These may all be omitted on a first reading, when the student is learning the elementary principles and outlines of the science. At this early stage, too, certain chapters may be passed over altogether. The portions thus omitted may be taken up with advantage only in preparation for the degree examination. And it is hoped that even the student who reads an honours course will find a satisfactory programme of studies presented in these pages. Subjoined to each chapter will be found some references to useful readings. The student is also recommended to avail himself of the list of *Questions and Exercises* at the end of each volume.

There is a very close connexion between many of the questions discussed at various stages in a textbook of logic. And it is essential to the proper understanding of those questions by the student that he be made to see their mutual bearings. This the author has endeavoured to secure, not merely by means of the index, but more especially by numerous cross-references (to the numbered sections) in the text itself. Nor, in regard to the more important doctrines, has he thought it advisable to avoid even some partial repetitions. The experienced teacher will admit that at least as an obvious aid to emphasis there is a good deal to be said in favour of such a simple procedure.

For those who aspire to become instructors, educators, teachers, pressmen, statesmen; for those who would lead,

or influence for good, the thought and conduct of their fellow-men in any department of life and action; in a word, for all who aspire to a liberal education: a sound training in the fundamental truths of philosophy—speculative, ethical, and religious—is usually indispensable. And this presupposes such a formation of mind as can be secured only by the study of logic: not indeed of the arid formalisms which sometimes pass for logic, but of the suggestive and illuminating truths which may be gathered from a serious analysis of the thinking processes of the human mind. Such analysis is bound to lead the student into the very midst of purely philosophical inquiries: and at the same time to equip him for carrying these on successfully.

The preparation of the present work has been influenced by those considerations. Possibly it may, on that account, be found to suit the needs of students in other colleges and universities. It has been the author's desire and endeavour to consult for their special wants in the first place. At the same time, he hopes that his treatment of the subject is such as will appeal to all who take a general interest in mental science, whether logical or philosophical.

CONTENTS OF VOLUME I.

INTRODUCTION.

CHAPTER I.

THE MIND AND KNOWLEDGE: PRELIMINARY TRUTHS.

		PAGE
1.	Nature of Man: His Mental Faculties: Senses and Intellect	1
2.	Distinction between Sense Perception and Intellectual Conception: Dependence of Intellectual Thought upon the Sense Faculties	2
3.	Judgment or Interpretation: Inference or Reasoning	5
4.	Relation of Universal Ideas to Individual Things	6
5.	The Problem of Universal Ideas: Its Solutions	8
6.	Some Modern Speculations on Universal Ideas	11

CHAPTER II.

GENERAL VIEW OF THE NATURE AND SCOPE OF LOGIC.

7.	Aim and Object of Logic; Natural and Artificial Logic: The "Ars Artium"; The Science of Logic; Its Material and Formal Object	12
8.	Is Logic a Science or an Art?	14
9.	Mental Process Involved in Knowing Truth: Subject-matter of Logic: Its Divisions	16
10.	Formal Validity or Consistency, and Material Validity or Truth, of Thought: Different Views as to Scope of Logic	19
11.	The "Laws of Thought"	23
12.	The Principle of Identity	23
13.	The Principle of Contradiction	24
14.	The Principle of Excluded Middle	24
15.	Review of the Three Principles	25
16.	The Principle of Sufficient Reason	27
17.	Real, Material, Critical Logic: Criteriology, Epistemology, Theory of Knowledge	28

CHAPTER III.

LOGIC AND KINDRED SCIENCES: DEFINITION AND SOURCES OF LOGIC.

18.	Relations of Logic to Kindred Sciences	30
19.	Logic and Metaphysics	30

PAGE

20. Logic and Psychology 33
21. Logic and Rhetoric 34
22. Logic and Grammar 34
23. Definitions of Logic 37
24. Utility and Necessity of Logic 38
25. Sources and History of Logic 39

PART I.

Concepts and Terms.

CHAPTER I.

Logical Properties and Divisions of Concepts and Terms.

26. Mutual Relations of Term, Thought, and Thing 42
27. Univocal and Equivocal Terms: Analogy and Metaphor 43
28. Universal or General, and Individual or Singular Terms: Basis and Nature of this Division 44
29. Collective and Substantial Terms 46
30. Intension and Extension of Concepts and Terms 48
31. Subjective, Objective, and Conventional Intension: or, Content, Comprehension, and Connotation of Concepts and Terms 48
32. Fixity and Limits of Connotation 50
33. Extension, Denotation, Application of Terms: Realm of Denotation: Universe of Discourse 52
34. Relation between Intension and Extension 55
35. Abstract and Concrete Terms 57
36. Have Abstract Terms Extension or Denotation? Non-Denotative Terms 60
37. Are Abstract Terms and Proper Names Connotative? Non-Connotative Terms 62
38. Opposition in Concepts and Terms: Kinds of Opposition: Material Contradictones 64
39. Formal Contradictiones: Positive and Negative Terms 65
40. Contrary Terms 68
41. Simple Repugnance: Privative Opposition 69
42. Absolute and Relative Concepts and Terms 70

CHAPTER II.

The Predicables.

43. What are the Predicables? Aristotle's fourfold Scheme 72
44. The Five Predicables: How to arrive at them. Essence and Connotation 73
45. The Predicables Defined 76
46. *Genus* and *Species*: Porphyry's Tree: *Species Infimae* 77
47. *Differentia* and *Proprium*: Connexion of *Proprium* with *Essence* . . 81
48. *Accidens*: Separable and Inseparable Accidents 86

CHAPTER III.

Definition.

PAGE

49. Definition gives Distinctness to our Ideas and leads to the Axioms or Principles of the Sciences 89
50. Nature of Definition: *Per Genus et Differentiam* 92
51. Fixity of Definition 95
52. Limits of Definition 96
53. Exemplification or Extensive Definition or Definition by Type . . 97
54. "Nominal" and "Real" Definitions: Verbal Disputes 99
55. Some Nominal Definitions 105
56. Some Substitutes for Definition 106
57. Rules of Definition 108

CHAPTER IV.

Division and Classification.

58. General Character of Logical Division 112
59. Relation to Definition and Kindred Processes 113
60. Formal or Material Aspects of Logical Division: Dichotomy . . . 115
61. Purely Formal Division 117
62. Rules of Logical Division 118
63. Material Division, or Classification 121
64. Its Grounds Determined by its Purpose: This either "General" or "Special": Hence "Natural" and "Artificial" Classifications . 122
65. Classification for Special Purposes 124
66. Classification for General Purposes 125
67. Why Classification for General Purposes is called "Natural" Classification. 128
68. How Classification may be Influenced by Hypothesis 130
69. Scientific Nomenclature and Scientific Terminology 131

CHAPTER V.

The Categories or "Praedicamenta".

70. The Problem of Classifying our Widest Concepts 135
71. Aristotle's "Categories": The Scholastic "Praedicamenta" . . . 136
72. The Categories and the Predicables 138
73. The Aristotelean Categories Enumerated 139
74. The Categories and Language 142
75. The Categories and Reality 145
76. Limitations and Modifications of Aristotle's Scheme 147
77. The Kantian Categories 150

PART II.

JUDGMENTS AND PROPOSITIONS.

CHAPTER I.

NATURE OF JUDGMENT AND PROPOSITIONS.

PAGE

78. Nature of the Mental Process of Judgment: Structure of the Proposition 154
79. Judgment and Truth: Conception: Inference 158
80. The Truth of Judgment is Objective, Universal, and Immutable . . 160
81. Matter and Form of the Judgment: Its "Abstract" Character . . 162
82. Interpretation of Terms and of Propositional Forms: Formulation: "Meaning" and "Implications" of the Proposition 164

CHAPTER II.

KINDS OF JUDGMENTS AND PROPOSITIONS.

83. Problems on Import of Judgment: Basis and Aim of Classification of Judgments 167
84. Judgments Classified according to Relation 169
85. "Necessary" and "Contingent" Judgments with their Synonyms . . 170
86. "*Propositiones per se notae*" and "*Modi dicendi per se*" 171
87. Is the Distinction Objective or Subjective? 174
88. Comparison of the Scholastic with the Kantian View 177
89. Modality in Categorical Judgments 180
90. The Subjective View of Modality 183

CHAPTER III.

QUANTITY AND QUALITY OF CATEGORICAL JUDGMENTS AND PROPOSITIONS.

91. Traditional Fourfold Scheme of Propositions; Distribution of Terms . 186
92. Universal Propositions 188
93. Particular or Indefinite Propositions 194
94. Plurative and Numerical Propositions: Multiple of Quantification: Complex Propositions 195
95. Exponible Propositions 198
96. Indesignate Propositions 200
97. Affirmation and Negation 202
98. Nature of Significant Denial: Its Relation to Affirmation; Its Grounds . 203

CHAPTER IV.

EXTENSION AND INTENSION IN CATEGORICAL JUDGMENTS AND PROPOSITIONS.

99. Possibility of Various Meanings in the Propositions 207
100. Predicative or Attributive Interpretation 207
101. Reverse of Predicative Interpretation 208

PAGE
102. Connotative Interpretation 208
103. Comprehensive Interpretation 209
104. Extensive or Class-Inclusion Interpretation 210
105. Quantification of the Predicate: Hamilton's Eightfold Scheme . . 212
106. Hamilton's Postulate 212
107. General Discussion of the Quantification Doctrine 213
108. Various Alternatives and their Unsatisfactory Results 214
109. Equational Readings of the Logical Proposition 216

CHAPTER V.

Opposition of Categorical Judgments and Propositions.

110. Meaning of Logical "Opposition": The Square of Opposition . . 219
111. Subaltern Opposition 220
112. Contradictory Opposition 221
113. Contrary Opposition 224
114. Subcontrary Opposition 226
115. Opposition in Singular and in Modal Categoricals: Summary of Results 227

CHAPTER VI.

Eductions from Categorical Judgments and Propositions.

116. Immediate Inference: Kinds of Eductions 229
117. Obversion 230
118. Conversion 232
119. Contraposition 241
120. Inversion 243
121. Summary of Results: Table of Eductions 245
122. Material Eductions 245

CHAPTER VII.

Existential Import of Categorical Judgments and Propositions.

123. Reference of Terms and Propositions to a Sphere of Objective Reality: The Possible and the Actual 248
124. Meaning of Inquiry as to "Existential Import" 250
125. Place of the Inquiry in Logic 252
126. Statement of the Question of Existential Import in Categorical Propositions 254
127. Influence of Various Suppositions on Validity of Logical Inferences . 255
128. Existential Import in Ordinary Usage of Language 258
129. Choice of Alternative Interpretations 260
130. Existential Import of Modal Propositions 261
131. Formulation of Existential Propositions 261

CHAPTER VIII.

Conditional and Hypothetical Judgments and Propositions.

PAGE

132. Analysis of the "If" Judgment: Its relation to the Simple or Categorical Judgment 263
133. Two Classes of "If" Judgments, the "Conditional" and the "Hypothetical" 265
134. "Doubt" and "Inference" in the "If" Judgment 267
135. Conditionals and Categoricals 269
136. Opposition of Conditional Propositions 270
137. Eductions from Conditional Propositions 272
138. Hypothetical Propositions: Their Modal Import 273
139. Opposition of Hypothetical Propositions 276
140. Eductions from Hypothetical Propositions 278

CHAPTER IX.

Disjunctive and Alternative Judgments and Propositions.

141. Various Forms of Disjunctives and Alternatives 280
142. Propositions with Alternative Subjects 281
143. Propositions with Alternative Predicates 281
144. The Alternative between two Judgments of Independent Import . . 283
145. Import and Function of Alternate Judgment 284
146. Relation of Alternatives to Hypothetical and Categorical Judgments; Opposition and Education in Alternative Judgments 286

PART III.

Reasoning and Syllogisms.

CHAPTER I.

Nature, Structure and Axioms of the Pure Syllogism.

147. Mediate and Immediate Inferences 292
148. Nature and Structure of the Syllogism: Its Matter and its Form . . 293
149. Kinds of Syllogism 297
150. The Syllogism and the Laws of Thought 298
151. The Syllogistic Axioms of Identity and Diversity 299
152. The Aristotelean Syllogism and the "*Dictum de Omni et Nullo*" . . 300
153. Analysis of the "*Dictum*": Alternative Axioms 301

CHAPTER II.

General Rules or Canons of the Syllogism.

154. Derivation of General Rules or Canons from the *Dictum* 305
155. Examination of the General Rules of Syllogism 307
156. Corollaries from the General Rules 313
157. Simplification and Restatement of the General Rules 315
158. Alternative Statement of the General Rules of Syllogism . . . 317

CHAPTER III.

Figures and Moods of the Syllogism.

PAGE
159. Distinction of Figures 319
160. Distinction of Moods 320
161. Determination of the Special Rules and Legitimate Moods of Each Figure 320
162. The "Original" or "Named" Moods and the "Subaltern" Moods . 324
163. The "Fundamental" and the "Strengthened" Moods of the Syllogism . 326
164. Direct Determination of the Valid Moods in each Figure . . . 327
165. The Valid Moods and the Existential Import of Propositions . . . 331
166. The Syllogism and the Qualification of the Predicate 332
167. Finding Middle Terms and Constructing Formally Valid Syllogisms . 332

CHAPTER IV.

The Doctrine of Reduction: Analysis of the Figures.

168. The Nature and Aim of Reduction: Explanation of the Mnemonic Lines 335
169. Indirect Reduction: Extension of the Doctrine of Reduction . . . 339
170. Characteristics of the First Figure 343
171. Characteristics of the Second Figure 344
172. Characteristics of the Third Figure 346
173. The Fourth Figure and the Indirect Moods of the First Figure . . 350

CHAPTER V.

Hypothetical and Disjunctive Syllogisms.

174. The Pure Hypothetical Syllogism 356
175. The Mixed Hypothetical Syllogism 358
176. "Moods" of the Mixed Hypothetical Syllogism 359
177. Fallacies in the Mixed Hypothetical Syllogism 360
178. The Reasoning in the Mixed Hypothetical Syllogism is Mediate Reasoning 361
179. The Pure Disjunctive Syllogism 362
180. The Mixed Disjunctive (or Mixed Alternative) Syllogism 363
181. Valid and Invalid Moods of the Mixed Disjunctive Syllogism . . . 363
182. Reduction of Hypothetical and Disjunctive Syllogism to Categorical Form 365
183. The Dilemma: Its Various Forms 367
184. Reduction and Validity of the Various Forms of Dilemma . . . 370
185. Other Views of the Dilemma 374

CHAPTER VI.

Abridged and Conjoined Syllogisms.

186. The Modern Enthymeme 376
187. The Polysyllogism 377

PAGE
188. The Sorites 378
189. Figures, Moods, and Special Rules of the Sorites 381
190. The Epicheirema 383

CHAPTER VII.

Nature and Characteristics of Inference.

191. Are there Other Mediate Inferences besides the Syllogism? . . . 385
192. Some Classes of Such Inferences 388
193. The Possibility of Mediate Inference from Particular Judgments . . 392
194. Errors on the Function of the Universal Judgment in the Process of Inference: The "Paradox" of Inference 395
195. The Syllogism and the "Petitio Principii" 401
196. Some Classes of Syllogisms Examined 404
197. The Apprehension of the Universal Judgment and its Application . . 407
198. Logical Grounds and Ultimate Sources of Inferred Conclusions . . 410
199. Summary of Chapter 412

Questions and Exercises 415

General Index 429

INTRODUCTION.

CHAPTER I.

THE MIND AND KNOWLEDGE: PRELIMINARY TRUTHS.

1. NATURE OF MAN: HIS MENTAL FACULTIES: SENSES AND INTELLECT.—Since *Logic* deals with thought and thought is a product of the mind, we cannot better approach our subject than by taking a general glance at the nature of the mind and the way in which it acquires knowledge. There is a special branch of philosophy which investigates *all* our mental activities: it is called *Psychology*. We will here take over from psychology, without any detailed analysis or discussion, those of its conclusions which will help to throw light upon the subject-matter of logic proper. The mutual bearings of logic and psychology will be explained further on (20). It is man himself, who, by his own thought, furnishes the subject-matter of logic. Now man is a *corporeal* being, existing in space and time like all other corporeal or material things, and, like them too, endowed with many mechanical, physical and chemical properties and powers; but he is also *animate* or living, *i.e.* organically constituted in his material structure, and endowed with life in common with the things of the *vegetable* or plant world; and he is *sentient* also, capable of sense perceptions and sense desires, in common with the beings of the *animal* world; finally, he is *rational*, that is to say, possessed of a characteristic aptitude peculiar to himself and entitling him to a place apart in God's visible creation, the faculty of reason or intelligence (46). Such is man's composite nature; and this nature is the *remote* principle or source of all his activities, rational, sentient, vegetative, and non-vital, all alike.

The *proximate* principles or sources of his various activities are called *faculties*. To what faculty do his *acts of thought* belong, and by what features are we to recognize them? Well, even the

very highest and noblest thoughts of man reveal the *compositeness* of his nature. They spring from his reason or intelligence, of course, but no single thought of his is an act of reason or intellect pure and simple. All his intellectual acts are dependent, both in their *origin* and in their actual *exercise*, on the antecedent and concomitant activity of other cognitive faculties of the lower or *sense* order, faculties which man possesses in common with animals, faculties which act only in and through some bodily organ. Of those *faculties of sense knowledge* or *sense cognition*, as they are called, some are known as external senses, others as internal senses. The external senses—of seeing, hearing, smelling, tasting, feeling or touching—are our channels of information about the outer world. The internal senses—of imagination, sense memory and sense consciousness—recall or reproduce in our minds, and modify in many ways, the experiences of our external senses. All those sense faculties, external and internal, subserve and minister to the faculty of thought proper—the reason, intelligence, intellect, understanding, as it is variously called. I cannot *think* of a thing unless some of these senses has already perceived it. Nor can I continue to think of it unless some of them continues to assist me. If I want to recall it to mind I must conjure up some sort of *image* of it: a natural image; or an outline or scheme or formula, such as the mathematician forms in geometry; or an imaginary model or design, such as the artist constructs in his imagination to help him in the conception and execution of his work. All this deserves a little reflection.

2. Distinction between Sense Perception and Intellectual Conception: Dependence of Intellectual Thought upon the Sense Faculties.—The first or simplest exercise of the faculty of thought is called, in logic, *Simple Apprehension* or *Conception*. It is the process by which we form a *concept* or *idea* of any thing or object. To do this we need the assistance of the external senses; each of these seizing and presenting to our reason some sensible quality or other of external things. Here, for example, is a table-bell upon my desk; I look at it and ring it; my eye receives an impression which enables me to see the outline and colour of the bell, my ear an impression which enables me to hear a sound, my fingers the tactile impression which makes me conscious of the shape and resistance of the button pressed, and so on. These are so many distinct *external sensations*. But evidently these various sensible qualities of

colour, outline, sound, resistance, etc., would remain isolated from one another in my mind, did I not possess the power or faculty of *associating* them. Both men and animals possess this power; it is a sense faculty, an *internal* sense; the ancients called it the *Sensus Communis*, modern philosophers call it the *central sense*, or the faculty of *mental association.*

As those sensible impressions are made practically together, it is easy to understand that the sensations produced by them are *associated* with one another. The qualities perceived by the production of those sensations come into our consciousness as forming *one whole;* this whole, the resultant of as many factors as there are qualities perceived, constitutes what we call the *sense object:* the concrete, individual, material thing, existing here and now in the actual conditions and circumstances of time and space in which it is thus perceived by the senses. The cognitive activity of these latter is called *sense perception*, or *sensation*, and the conscious product of this activity is called a *percept.*

Our sensations do not continue indefinitely in consciousness; but on passing out of consciousness they leave behind them traces of themselves, *images* of the sense qualities originally perceived. These images are preserved in the imagination and may be revived, or recalled to consciousness, by sense memory.

Now it is by the exercise of those partly bodily and partly mental activities of external sense perception and imagination that we obtain possession of the *materials* or *data* necessary for *thought* proper. Aided by the sense percept or sense image, our purely mental faculty of thought, our intellect or reason, is able to form a *concept* or *idea* by which we apprehend *what the thing is*, get a *rational* knowledge of it, give it an *intelligible interpretation* or *meaning* and bestow upon it a *name.* In this we surpass the brute creation. Animals have indeed percepts and images of things; but they have not *ideas* or *concepts;* they do not *understand what things are;* they do not *interpret* their sense experiences as we interpret ours and theirs; nor have they *language*, the medium for expressing and communicating *thought.*

It is difficult for the beginner, but it is very essential to accuracy, to distinguish clearly between sensation with its *concrete images*, and intellectual thought with its *abstract ideas;* and to realize that it is intellectual thoughts or ideas or concepts that are expressed in human language, and that it is with these logic deals—not with the products of the sense faculties, which products are only the raw materials of thought (9).

To bring out the contrast between the two orders of mental product, the sensible and the intellectual, let us revert to our illustration of the table-bell. It will enable us to realize that while the object grasped by sense cognition is *concrete*, *individual*, and limited by conditions of time, place, and material existence, the object grasped by intellect, through the idea, is *abstract*, *universal*, and independent of all such changing conditions and limitations. The thing perceived by the senses or reproduced by the imagination is always a definite individual thing apprehended as composed of *this* matter, endowed with *these* properties, existing *here* at *this* particular moment. This thing (the table-bell), which I see with my eyes and touch with my hands, is made of this particular piece of bronze, round in shape, agreeable in tone, resting here and now on this particular corner of this particular desk. All this is perfectly determined. But I can also *think* of a table-bell not of bronze, nor round in shape, nor agreeable to hear, nor resting here and now upon my desk,—of a table-bell which abstracts from all those particularities. No doubt, the table-bell thus thought of, apart from all those particular conditions, will, *if it exist at all*, be made of some metal or other ; it will be of some shape or other ; it will emit some sound or other ; it will be localized some place or other, and exist at some time or other. But, as this table-bell of *abstract thought* may be made of any metal at all, be of any shape, yield any sound, exist anywhere and any time, it will evidently serve to represent to my mind, inadequately, of course, but faithfully as far as it goes, any and every possible and actual table-bell of whatsoever material, shape, sound and whereabouts.

Any object or thing thus considered, apart from all the *particularizing* conditions with which it is really endowed when existing in the actual order of things, is called an *abstract* object ; for to abstract mentally any object is precisely to consider apart—"*separatim considerare*"—that which the thing or object is, while laying aside the particular characteristics which give it this, that, or the other definite and determined individuality. Once an object is thus conceived in the *abstract* by the intellect, it is seen to be capable of being embodied or realized equally and indifferently in an indefinite multitude of individual instances : which is the same as saying that it becomes or is made *universal* by the consideration of the intellect.

These two functions —of *abstracting* and *generalizing* its objects—are the characteristic features of the cognitive activity of human *reason* or *intelligence*.

It is of the greatest importance to distinguish clearly between the *concrete*, *individual* thing, which is the object of mere sense perception or imagination, and the *abstract*, universal object, which is apprehended by *thought* proper. We can *think*, or have ideas, of objects which are not perceptible to our senses : for instance, objects not actually existent but only conceivable, such as *a flying horse ;* or objects which we believe or know to exist, but to be unperceivable because not material, such as *God*, *a pure spirit*, *the human soul*, *truth*, *virtue*, *similarity*. And the things we do perceive by our senses we conceive by our intellects in a manner entirely different from that in which we perceive them. We *perceive* each numerical individual object of a class, as it exists in the concrete, *John*, *James*, *Thomas*, etc. We *conceive* the nature that is embodied or realized in each, and in virtue of which we put them into a common class, *man ;* and we conceive this common *human nature* or *humanity* in the abstract, *i.e.* neglecting or not considering the

different characteristics which particularize it in the individuals, John, James, Thomas, etc. Furthermore, we use this abstract and universal idea, *man*, for the purpose of *interpreting* for ourselves, or giving a *meaning* to, the individual objects John, James, Thomas, etc., which come under the notice of our senses: by thinking to ourselves, "John is a *man*," "James is a *man*," "Thomas is a *man*," etc. *The same is true of all our abstract and universal ideas.* It is by means of these latter that we *interpret* or *know* intellectually the nature and the meaning of the Real World—of Reality itself—as this latter is revealed to us through our senses. This interpretation involves another exercise of thought—the second act of the mind in logic—the *judgment.*

The first act—conception—by which we form abstract and universal concepts of individual sense objects, has many modalities which have secured for it many different names. For example, when the mind considers one object independently of the surrounding objects it is said to pay *attention* to this object. This attention may be brought to bear either upon one single attribute of an object, independently of the other qualities with which that attribute is united; or upon all the attributes which constitute the common, specific or class nature of the object, apart from the characteristics that individualize that nature in the actually existing world: those mental acts are called acts of *abstraction.* Abstraction is the basis of *generalization*, as explained above; moreover it effects a mental process which we call *analysis*, *i.e.* a taking asunder or decomposing of the elements or attributes of a known object. Furthermore, when the mind once again reunites the attributes thus previously isolated, it carries on a work of *synthesis.* But in these activities *judgment* is involved as well as conception (9).

3. JUDGMENT OR INTERPRETATION: INFERENCE OR REASONING.—Not only, therefore, do we form abstract and universal concepts or notions, by means of which we understand more or less fully *what the things are* which come under the notice of our senses. We also *interpret* the individual objects revealed to our senses by *affirming* or *denying* the contents of those abstract and universal ideas about those things. I see an object in the distance. I proceed to *think* to myself about it thus: "That is *something*; it is a *material* thing or being of some sort; it is not a *pillar*, nor a *tree*; it is *moving*; it is an *animal* of some sort; it is a *horse*". All these mental affirmations and denials are *thoughts* of another sort, thoughts by which we *compare* objects we have already conceived, by which we apprehend a *relation of agreement or disagreement* between things already perceived and conceived, and thus get a fuller insight into *what the things are* about which we are thinking. This act of comparison is called *judgment.* By means of it we *interpret* the individual things revealed to our senses —by affirming or denying about these things the objects we have already conceived in the abstract when forming our universal ideas

(*thing* or *being*, *material*, *moving*, *life*, *tree*, *animal*, *horse*, etc.). The act of judgment is thus an act by which we apprehend the identity or non-identity of the objects of two previous apprehensions. It is an *apprehensio complexa* or *complexorum* as opposed to the "simple" apprehension—*apprehensio incomplexa* or *incomplexorum*—by which we conceive an object in the abstract without making any mental affirmation or denial about it. But *conception* and *judgment* are fundamentally the same sort of mental act, an *intellectual intuition* of *what some thing is.*

So, too, is what logic calls the third act of the mind, the act of *reasoning* or *inference.* This is the process by which our reason so compares with one another the ideas and judgments it has already formed that it thereby apprehends new relations between the latter, and thus reaches fresh judgments and additional knowledge or truth about things. Here, too, no less than in judgment, the object apprehended by the intellect is a *relation* of identity or difference between previously conceived objects: and this new apprehension involves, of course, a fuller and better understanding of *what some thing is*—"*quod quid est*".

Conception, judgment, and reasoning are, therefore, fundamentally one and the same type of mental process—the understanding of the nature of a thing. They are all alike acts of the same faculty—the intellect or reason.

4. Relation of Universal Ideas to Individual Things. —Our senses, external and internal, are the channels through which the things that make up the real world come into contact with our minds. All our knowledge is gathered by our *judging* or *interpreting* intellectually the data revealed to our consciousness through the operation of our senses, and by *reasoning* from those data. There is a philosophical aphorism: *Nihil est in intellectu quod non fuerit prius in sensu:* which does not mean, of course, that we can know nothing except the things actually revealed to our senses, *i.e. material* things; but which does mean that whatever we do know, even about suprasensible (or *spiritual*) things, we know by reasoning intellectually from what is revealed to our senses. The abstract and universal ideas themselves, by which we interpret those sense data (*e.g.* the ideas of *thing*, *matter*, *motion*, *life*, *tree*, *horse*, etc.), we get by intellectual conception (or *abstraction*) *from* those data. Precisely the same realities which are apprehended by our senses as concrete, individual, determinate, and incommunicable, are apprehended by our intellect in a state

in which they appear abstract, universal, indeterminate, and common or communicable, *i.e.* realizable equally in an indefinite multitude of individuals. In other words, our senses and our intellect attain to the same realities but in different ways. While our senses apprehend material things in the condition in which these really exist, *i.e.* as concrete, *individual*, separate from one another, changeable and changing in time and space, our intellect grasps a portion—greater or less—of the nature of these things, the portion common to larger or smaller groups of them, the *common* or *class essences*, the *generic* and *specific essences* of these things, and it conceives this common portion in the abstract, *i.e.* in a static, unchanging condition, apart from the influences to which it is submitted in the state in which it is actually found in individual material things. Thus, while my senses perceive, or my imagination pictures, the individual John, my intellect conceives, as embodied in that individual, the various portions of his essence which make him belong to various classes of things, *e.g.* that he is a *corporeal being*, *living*, *sentient*, *rational*.

When, therefore, I interpret any *individual* object of sense experience by attributing to it the object of some *universal* idea —when, for example, I say "*John* is a *man*"—I mean to assert that the object of my universal idea, the entity, essence or nature represented by it (*e.g. human nature*) is embodied in, and constitutes (partially, at least), the individual sense object (*John*). I do not mean to assert that the object of my universal idea exists in the individual sense object *in the same way in which the former is apprehended by my mind.* If I did, my statement that "John is a man" would be false—and so would all statements asserting *universal* attributes about *individual* things. For every object conceived by the intellect through a universal idea (*e.g. man*) is conceived apart from individualizing conditions, as abstract, and hence as universal, *i.e.* common or communicable to, and realizable in, an indefinite multitude of individuals; whereas that same object, as it exists in the individual (*e.g. John*), is concrete and individualized and incommunicable to others, and cannot be attributed to others—John is himself only; nobody else is John: but while John is a *man*, so is James also a *man*: *man* can be attributed to both and to an indefinite multitude. And it can be attributed to them truly, for each assertion means only that the object of my universal idea—*human nature*—is really in each and every one of the individuals, though not in the same way as it is

conceived by my mind. It is multiplied or repeated numerically in each of them; while as conceived in the abstract by my intellect it is one and common and communicable, or indefinitely multipliable.

Again, when I interpret the individual things of sense by means of universal ideas the contents of which I attribute to those things in such judgments as "John is a man," or "John is a living being," or "John is not a spirit," I do not imply that these universal ideas are *adequate* representations of the individual things, or exhaust all that can be known about the latter. I only claim that they are faithful and give me true knowledge so far as they go. They reveal to me the common *generic* and *specific* essences of the realities revealed to my senses, but not the whole *individual essence or nature* of any one of them. I admit that they are inadequate: that no number of abstract ideas about an individual thing will give me a full and complete insight into its reality. But this is an essential limitation of the human mind itself. We are not omniscient.

We have thus accounted for the *origin* of our universal ideas by asserting that they are all *abstracted* by the intellect from the individual data revealed to consciousness by the operation of our senses. We regard as erroneous the view that some or all of them are not thus derived from sense data, but are in some form or other *innate* or inherent in the intellect, independently of, and anterior to, the operation of our senses. The question is purely psychological.

We have accounted also for the *validity* of our universal ideas—in other words, for the trustworthiness of the rôle they fulfil for us in interpreting the realities revealed to us through our senses, and thus giving us an insight into the nature of those realities—by pointing out what we conceive to be the true relation of the universal idea to the singular sense object. This relation we have explained by saying that the object of the universal idea is really embodied in, and constitutes partially the nature, the very reality, of the individual sense object; but that the *feature of universality* which characterizes the object of the universal idea as apprehended by the intellect, does not belong to that object as the latter exists in the individual things of the world: that it is a modality added on to the object by the consideration of the intellect itself. The intellect, therefore, rightly and truly attributes the *object* of its universal idea to the individual thing, but not the *universality* of that object; for the universality is a mental mode—added on by the intellect—hence called in scholastic philosophy "*intentio universalitatis*".

5. The Problem of Universal Ideas: Solutions.—The account just given of the relation of our universal ideas to the individual things about which we affirm or deny the former in

our acts of judgment, is known as the doctrine of MODERATE REALISM. It is summed up in the scholastic formula: *Universale est formaliter in mente, sed fundamentaliter in re.* The object apprehended by means of a universal concept is called a *Universal* because it is something which is conceived as common to many things, something that is or may be attributed to many things in our judgments, something that is conceived as common to all the members of a class. What is this something? The answer of moderate realism is that this something is a *reality* (hence the name, "*Realism*") which is present in the individual things of sense ("*in re*"), helping to constitute the essence or reality of the latter; it is, however, not present in them as *one in all* of them, but as multiplied and numerically distinct in each; in a manner, therefore, which serves as a foundation ("*fundamentaliter*") for the formation of *one* concept that will represent equally well all the realizations of the object in the individuals; nor is it, as it really exists in the individuals, *formally universal*, common, communicable; for whatever exists really is individual and incommunicable: Plato's human nature is his own and cannot be anybody else's; human nature exists as formally and explicitly universal only in our thought; as universal it is only a concept ("*Universale est formaliter in mente*").

The above is not the only answer that has been given to the question: What *are* those objects or entities which we apprehend as universal, common, communicable, in our universal ideas, and what relation have they to the things revealed to us through our senses? The "Problem of the Universals," as it is called, has received other and erroneous solutions. It is of fundamental importance in philosophy; and that is why, notwithstanding its difficulty, we introduce it at this early stage. It is desirable that the student should have the correct orientation on the question from the start. But since the problem is not properly a logical one we merely indicate here the leading solutions it has received from philosophers.

Besides the solution given above—which was first outlined by Aristotle and then developed by the scholastic philosophers of the Middle Ages, especially by St. Thomas Aquinas, and which we regard as the only correct one, there is a view known as EXTREME or EXAGGERATED REALISM. According to this view the universal (*i.e.* what is present to our minds when we form any of those universal concepts or ideas expressed by common or class names, such as man, animal, good, unhappiness, etc.) is not only a reality distinct from the mind, but exists really *as a universal* outside the mind. Plato taught that

these universals exist apart from the world revealed to our senses, and constitute what is in truth the only *real* world, the world revealed to our senses being but a faint shadow of the *real* world (*Platonic* Realism). Other philosophers taught that the universals which we conceive intellectually have their real being as such in the Divine Mind, and that our intellects have a direct intuition of them there (*Ontologistic* Realism). Others again believed that the universals exist actually as such in the individual things of the world revealed to us through our senses ; that they are not multiplied numerically in each individual thing, but that the one common essence (*e.g. humanity*) is numerically the same in all individuals, these being, therefore, manifestations—*apparently* distinct—of what is *really one single reality* (*Empiric* Realism). This view would lead logically to what is known as Monism or Pantheism in philosophy : the doctrine that all existing reality is one single being : that all *distinctions* are only *apparent*, none *real*.

Passing from exaggerated realism we find at the opposite extreme the erroneous doctrine of NOMINALISM. According to this view not only is the universal as such not a reality, but it is not even an idea ; it is a mere *name* (hence the title, *Nominalism*), a mere term. This term (*e.g. man*) has nothing *real* corresponding to it except individuals (John, James, Thomas) ; and it has nothing *mental* corresponding to it except our *perceptions* of actual individuals or our imagination images (some definite, some vague, composite, modified, confused) of individuals formerly perceived. The universal would be simply a common name serving as a label or mark for numerous more or less similar individuals ; but we are supposed to have no concept or idea of that common element which exists in the similar things and is the ground of their similarity. This is a modern doctrine, prevalent for the last few centuries, especially in England from the days of Hobbes, Locke, Hume, etc., and supported later by Bain and Mill. It is based on an erroneous view of the nature of the human mind : the view, namely, that man has no other faculties of knowledge than external and internal sense faculties, that reason or intellect is a sense faculty which acts through a bodily organ, that all our knowledge is reducible to sensations. This system is at variance with an accurate interpretation of the facts of consciousness. It is impossible to deny seriously that, besides sense percepts and imagination images, we have also in our minds, corresponding to the universal terms of our language, other representations—not of a sensible, but of an intellectual order : notions, concepts, ideas—by which we interpret the individual things of sense experience.

Between the error of *Nominalism* which holds that Universals are mere names, and *Moderate Realism* which teaches that they are mental representations of extramental realities, comes another erroneous view known as CONCEPTUALISM, which teaches that Universals are *mere concepts* of the intellect, *mere* mental constructions having no reality outside the mind to correspond to them. This opinion was propounded by some mediaeval philosophers, and in recent times under a new and more erroneous form by the German philosopher, Kant (1724-1804), and his numerous followers. If universals were *mere concepts* of the intellect they would not be validly applicable to the things revealed to us through our senses ; and since it is by applying these concepts to things that we interpret the latter and get all our knowledge

about them, this knowledge would, in the conceptualist view, be fictitious: it would not be a knowledge of real things at all but only of intellectual notions. People generally believe, and rightly, that the various sciences give us genuine knowledge about real things; but science is made up of truths that hold good universally, *i.e.* of truths about *universal* natures, such as the truth that "Water boils at 100° C. at the sea-level"; and if these universal natures are only concepts of the mind the sciences can give us no information about things but only about our own mental notions and the language in which these are expressed.

6. Some Modern Speculations on Universal Ideas.—Moderate Realism takes this as self-evident: that whatever really exists is really an *individual* thing, definite and determined, itself and no other; that it is not *common to others* and cannot be attributed to others;[1] that it is only by being intellectually conceived in the abstract, by becoming an object of intellectual thought, that a thing is stripped of its individuality, loses its incommunicability and becomes *attributable to many*—"*praedicabile de multis, universale in praedicando*".

Plato, however, contended that it is not the individual at all that is *real*, but only the universal; and some modern philosophers, believing that the universal (as such) has as good a claim to be considered real as the individual (as such), and seeing that the universal as such is essentially conceptual, ideal, mental, have concluded that the individual and the universal, or, in other words, the real (of sense) and the ideal (of intellect) are the same. This is the doctrine of the German philosopher, Hegel (1770-1831), a sort of idealistic monism which breaks down all distinction between thought and thing. A similar theory has the support, in England, of Green, Bosanquet, and Bradley, among others. These writers confound the *conceptual* identity of the universal nature, based on similarity of really distinct individuals, with *real* identity. When I say, "John is a man," and then, "James is a man," the nature which I assert to be embodied in, and really identical with, John, I apprehend to be really and numerically distinct from, though similar to, the nature I assert to be embodied in, and identical with, James. The two really distinct natures are so similar, as embodied in the two individuals, that I can represent these natures by *one and the same concept* and describe them by the same name, *human*. This conceptual identity the writers referred to seem to confound with *real* identity. Were the nature I attributed to John really identical with that I attributed to James, I should be entitled to conclude that John and James are really identical—a sufficient *reductio ad absurdum* of this latest speculation on the significance of the Universal.

Mercier *Logique* (Louvain, 1905), c. 1. On the Universals: Joyce, *Principles of Logic* (Longmans, 1908), pp. 132-36. Mercier, *Critériologie Générale* (Louvain, 1906), pp. 337 *sqq*. De Wulf, *History of Medieval Philosophy* (Longmans, 1909), pp. 149 *sqq*., 321 *sqq*., 421 *sqq*. Maher, *Psychology* (6th edit.), pp. 294 *sqq*. Joseph, *Logic*, pp. 20 *sqq*., 49 *sqq*., 55 *sqq*.

[1] Οὐδὲ τὰ καθ' ἕκαστα κατ' ἄλλων, ἀλλ' ἕτερα κατ' ἐκείνων. Non singularia de aliis sed alia de ipsis praedicantur. Aristotle, *Anal. Pr.*, i., 27.

CHAPTER II.

GENERAL VIEW OF THE NATURE AND SCOPE OF LOGIC.

7. AIM AND OBJECT OF LOGIC; NATURAL AND ARTIFICIAL LOGIC; THE "ARS ARTIUM"; THE SCIENCE OF LOGIC: ITS MATERIAL AND FORMAL OBJECT.—The *object* with which logic deals will be determined by the question: What is the *aim* of logic? And about this question there is little or no difference of opinion. The aim of logic is to aid the mind in arriving at a knowledge of the truth. It lays down certain rules which the mind must follow if it is to secure truth and avoid error.

No doubt, every person of ordinary intelligence learns, by the experience of life, to appreciate the proper means of securing the ends he may have in view. Experience teaches us to *regulate* our conduct, that is, to adapt our acts to the ends we pursue. Now the adaptation of an act to an end implies a perception of the relation between act and end. And these relations are formulated in certain practical rules or maxims according to which a prudent man will always regulate his actions. These plain truths, accepted if not acted on by all, are what people usually call by the name of *sound common sense.* They are not the fruit of any deliberate reflection or study, but they imply certain judgments and conclusions empirically formed, one hardly knows how or why, more than half unconsciously perhaps, and yet such that we implicitly rely on them in our estimation of current affairs and of human activity generally. We think and judge and reason and infer according to these spontaneous dictates of our rational nature: they form a sort of *Natural Logic* of which no sane person is entirely destitute.

But this common sense is of itself far from infallible; it really does not carry us very far or very safely in any beyond simple and easy things; even shrewd, clever people often find out *afterwards* that their first spontaneous judgments misled them. In difficult matters they must needs pause and reflect maturely on all

the circumstances of the concrete problem before them, analyse these circumstances and reconstruct and review them in a certain order, to see how best to act in the particular case. Now this kind of reflection results in the *deliberate* and *reasoned formation* of a body of rules for the habitual guidance of the mind in those operations of thought by which all our knowledge is acquired and all our activities directed. And this body of practical rules constitutes what is called *Artificial Logic*.

To have the *art* of doing a certain thing is simply to know and possess and make use of all the requisite means for doing that thing well and properly. "An art," says St. Thomas, "is nothing more than a right conception of the way to do certain things".[1] *Logic*, considered as an *art*, is therefore the collection of those practical rules which should regulate our thinking and reasoning processes. And as it is these latter functions which frame the rules for all departments of external activity—the rules which constitute all the arts—the art of right thinking will underlie and direct all the other arts. For this reason logic is sometimes called the *Ars Artium*, the art of arts.[2] And this is very intelligible; for all conscious mistakes—in any department of activity whatever—arise ultimately from errors of judgment, just as all success in external activity is based upon rectitude or soundness of judgment.

To act well we must first of all judge well. But if we are to judge well—to form sound and true judgments—we must make a careful study of the materials of thought in order to see how they

[1] "Ars nihil aliud est quam recta ratio aliquorum operum faciendorum."—St. Thomas, *Summa Theol.* 1_a 2_{ae}, q. 57, art. 3, c.

[2] "Alia animalia, ait, quodam naturali instinctu ad suos actus aguntur, homo autem rationis judicio in suis operibus dirigitur. Et inde est, quod ad actus humanos faciliter et ordinate perficiendos diversae artes deserviunt. Nihil enim aliud ars esse videtur quam certa ordinatio rationis, qua per determinata media ad debitum finem actus humani perveniunt. Ratio autem non solum dirigere potest inferiorum partium actus, sed etiam actus sui directiva est. Hoc enim est proprium intellectivae partis ut in seipsam reflectatur: nam intellectus intelligit seipsum, et similiter ratio de suo actu ratiocinari potest. Sicut igitur in hoc quod ratio de actu manus ratiocinatur adinventa est ars edificatoria vel fabrilis, per quas homo faciliter et ordinate hujusmodi actus exercere potest; eadem ratione ars quaedam necessaria est, quae sit directiva ipsius actus rationis, per quam scilicet homo in ipso actu rationis ordinate et faciliter et sine errore procedet. Et haec est ars Logica, id est rationalis scientia. Quae non solum rationalis est ex hoc, quod est secundum rationem, quod est omnibus artibus commune; sed etiam ex hoc, quod est circa ipsum actum rationis sicut circa propriam materiam. Et ideo videtur esse ars artium; quia in actu rationis nos dirigit, a quo omnes artes procedunt."—St. Thomas, 1 *Post Anal.*, lect. 1.

ought to be employed in those mental processes of ours which lead us to a knowledge of the truth. Those thought-materials, and thought-processes, and thought-products, are therefore the *object* (or *subject-matter*) with which logic has to deal. They are briefly: our elementary notions, concepts, ideas, in the first place; then, the judgments we form by comparing our concepts with one another; next, the processes of reasoning or inference by which we compare our judgments together for the purpose of arriving at other and more complex judgments; and, finally, those more elaborate mental or rational constructions built up by our reasoning processes and commonly called *sciences*, or *philosophy* itself, as the case may be. The science which studies the proper order and arrangement of all those mental functions which lead us into the possession of truth or knowledge, is the science of *logic*. Thus understood, logic is a *science;* it is even, in a certain sense, the highest in the hierarchy of the sciences—inasmuch as it studies what science is, how a science is constructed, and how all the human sciences should be related and arranged. Our *thoughts* themselves—ideas, judgments, reasoning processes and methods—are its *material object* or *subject-matter*. The special *point of view* from which it studies them—its *formal object*, therefore—is their *adaptability* to the acquiring of accurate and scientific knowledge about all things.[1] Only in so far as they are the means and instruments by which we acquire such knowledge, does logic concern itself with them. To help us to a *knowledge of the truth* by the proper arrangement and utilization of the materials of thought: such is the *end* logic has in view.

8. Is Logic a Science or an Art?—It is both; or rather there is a *Science* of logic—a *practical* science—and an *Art* of logic. This, in brief, we consider to be the most satisfactory answer to a disputed question of secondary importance.[2]

A *scientific* knowledge of any subject-matter is a knowledge of it through its *causes*, and *reasons*, and *principles*, a knowledge of its laws, a systematized, co-ordinated knowledge of it, got by mental application, analysis, demonstration. Science is *specula-*

[1] There can be several distinct sciences about the same subject-matter provided each has its own proper point of view. They are then said to have a *common material object*, but each its own *proper formal object*.

[2] *Cf.* Joannes a S. Thoma, *Cursus Philosophicus, Logica*, p. 2, q. 1; Mill, *Examination of Hamilton's Philosophy*, chap. xx.; Welton, *Manual of Logic*, vol. i., p. 12 (6); Joseph, *Introduction to Logic*, pp. 8, 9.

tive if the knowledge is acquired for its own sake and has no *immediate* application to practical ends, no *immediate* influence on conduct, no *immediate* utility for any ulterior object; it is *practical* if the knowledge is acquired not so much for its own sake as with a view to using it for some ulterior purpose to which it is *immediately* applicable: *Finis speculativae, veritas; finis operativae sive practicae, actio.* Manifestly this distinction is not a fundamental one; for, in so far as it springs, not from the motive entertained in studying the science, but from the nature of the knowledge acquired, it is merely a matter of degree, since *all* true knowledge has, or can have, *some* practical influence on external conduct; and furthermore, it is one and the same mind, one and the same reason, that acquires all science, whether speculative or practical; and, finally, even the most practical knowledge may be acquired for the sake of its own truth, apart altogether from its ulterior value, and will be, under this aspect, speculative.

An *art*, according to the ordinary use of the term, is understood to mean a collection of practical rules or canons or precepts for our guidance in the performance of some work, usually external: *πράσσειν, facere, faire, to make, machen*;—not merely mental: *ποιεῖν, agere, agir, to do, thun.* But it also commonly means *practical skill* derived from experience in the application of those principles or rules. The principles themselves are partly the fruit of *study*—like the truths of a science—and partly of actual *experience* itself. The main division of the arts is that into the *fine arts*—music, painting, sculpture, etc.—and the various *mechanical arts and crafts.*

Now, manifestly, logic is a *science*, for it studies and analyses our mental processes and teaches us a systematized body of truths concerning those processes. It is even *speculative* in character, both in so far as the knowledge yielded by such analysis is desirable for its own sake, and inasmuch as even its practical aim is precisely to secure that very object which all *speculative* science aims at—*knowledge of the truth.* This is St. Thomas's point of view when he writes: "In speculativis alia rationalis scientia est dialectica . . . et alia scientia demonstrativa".[1]

Since, however, the knowledge acquired, the truths brought to light, by logic, are immediately applicable to the exercise of thought; since they are in the nature of canons for securing correct thought, for avoiding and detecting inaccurate reasoning;

[1] *Summa Theol.*, 2ª 2ae, q. 51, art. 2, ad. 3.

since the logician brings them to light from his analysis of thought, not merely for the pleasure of contemplating them, but with a view to using them: it is equally manifest that the science of logic is rather a *practical* than a speculative science. Its *immediate* object being distinctly practical, it must be ranked as a *practical* science.

Finally, is logic not merely a practical science but even an *art?* In the narrower meaning, which would confine the scope of this term to collections of rules for the execution of *external* works, logic would not be an art. But if we extend the term to those rules which direct even internal, mental activity, we may legitimately call it an art—the art of correct thinking, of accurate reasoning. That is to say, the discovery and formulation of those rules or canons—which are no less the outcome of *experience* in thinking than of an analytic *study* of the processes of thought—would be the *practical science* of logic; and the application of those rules, the actual reasoning according to those precepts (whether unconsciously or consciously) would be the *art* of logic.

Every art has some background of theoretical truths or principles behind it; every department of external experience has some counterpart or complement of internal, rational study. The system of practical rules and laws arrived at by the study of our mental processes was called by the Scholastics *Logica Docens*—logic *in the teaching;* the application of those fruits of study for the guidance of those processes, they called *Logica Utens*—logic *in action.*

9. Mental Processes Involved in Knowing Truth: Subject-matter of Logic: its Divisions.—We have said the aim of logic is to lead us to a "knowledge of the truth". When is knowledge true and when is it false? Knowledge can be neither true nor false so long as the mind does not assert or deny anything, but confines itself to the simple contemplation of some object of thought. When, for example, we merely *think* of the sun, or the infallibility of the Pope, or a unicorn, our thoughts are neither true nor false. Ideas as such are neither true nor false, nor is the mental act of apprehension or conception by which we form them, nor are the words of the dictionary which express them. It is only when, by a more complex act, we compare—and identify, or distinguish between—our ideas about objects, that our knowledge assumes the character of truth or error. In other words, it is only when we make a *mental statement* or *pronouncement* (*enunciatio*), an *affirmation* or a *denial* of something about

something—when we announce to ourselves mentally that the sun exists, that the unicorn is a real being, that the Pope is infallible—it is only then we reach knowledge which is *true* or *false*. This mental act of enunciation is called *judgment:* it is the principal act of the mind: in it lies truth or error: Aristotle insists, and rightly, on this fact, and it will come up repeatedly for consideration.

Judgment is, however, not the only act of the mind. The object about which the "statement" just referred to is made, and what is asserted or denied about that object—in other words, the elements into which the judgment can be resolved—are themselves apprehended by acts which are *logically* antecedent to the act of judgment.[1] These elements are called *ideas*, *notions*, or *concepts* (*idea*, *notio*, *conceptus*), and the act in question *simple apprehension* or *conception* (*simplex apprehensio*, *conceptio*).

Furthermore, we unite several judgments together by comparing their elements in a mental process called *reasoning* or *inference*. Reasoning is the process by which we derive one truth from another or others. The deriving of a truth from a single other truth—of the truth that "Some mortals are men" from the truth that "All men are mortal," for example—is called *immediate* inference. When we discover a truth, *e.g.* that A is C—from the juxtaposition of two already known truths, *e.g.* that A is B and that B is C—we are said to reason *mediately;* because, in order to discover whether A is C we have to call in the aid of a *middle* or *intermediate* notion, B, with which to compare the two former successively.

Those three mental acts and their products form the main subject-matter of the science of logic.

The first is *mental apprehension* or *conception;* its product is the *idea* or *concept* or *notion;* and the verbal expression of the latter is the *logical term*. Hence one division of logic will deal with conception, especially as illustrated in the formation of *general ideas*[2] by definition, division, and classification; with

[1] *Chronologically* apprehension is for the most part accompanied by judgment. According as we analyse any complex datum or phenomenon of sense perception, and abstract from it those various aspects which become so many objects of our thought, so many notes or attributes which we predicate about the whole object, we are continually making judgments, mostly semi-conscious and implicit, that "*This is* such or such".

[2] Logic does not deal with what may be called the raw material of thought: sensations, sense impressions, imagination images, etc.;—nor with the mental processes

logical terms, their properties and divisions; with their function as predicates and with the different modes of predicating, *i.e.* with the *prædicamenta* and *prædicabilia*.

The second mental process is *judgment* (*judicium*); its product has the same name—a *judgment;* and the expression of the latter in words is a *proposition* (*propositio*). Hence another section of logic will deal with the nature and import of judgment; with propositions, their divisions and *immediate* implications.

The third mental act is *inference* or *reasoning* (*ratiocinium* or *ratiocinatio*); its product is called an *inference* or an *argument* (*argumentum*); and the verbal expression of the latter is called an *inference* simply in the case of immediate reasoning, an *argument* or *syllogism* in the case of mediate reasoning. Hence a third section of logic dealing with mediate inferences and syllogisms, their various kinds, canons and combinations.

But the mind does not in its actual working isolate those three acts from one another, nor does it rest content with mere spontaneous and fragmentary reasoning about anything and everything that comes before it. It endeavours to introduce *order* into its own acts; to arrange its reasonings according to their respective objects; to group the judgments and arguments it has formulated about any given subject-matter according to their natural dependence on one another. In other words it tries to make its knowledge *scientific*. For it is by thus connecting together all its judgments and reasonings about any object of study, by going around (*discurrere*, *discursus*) the different aspects of it and viewing them together (*comprehendere*, *complecti*) that the mind builds up a *science*. This mental process is not distinct from, but is a combination of, the three acts already described. But the *method* by which these latter should be combined in order to secure scientific knowledge is of the highest importance; and it forms the subject-matter of a distinct section of the science of logic. It is only by thus synthesizing (*συντίθημι*) our reasoning processes and arranging in proper order the judgments which give us fragmentary *explanations*

subservient to conscious, intellectual thought proper: sense perception, imagination, memory and association of sense images, etc. Leaving the investigation of these and other sub-conscious or semi-conscious cognitive processes to psychology, it pursues its analysis of mature reasoning processes and their products only within and not beyond the domain of clearly conscious intellectual thought. At the same time it is inevitable that many matters lying on the debatable borderland of the two sciences will be found in logic—especially in the inductive portion—as well as in psychology.

of things that we can arrive at *scientific* knowledge, and ultimately at that higher unification of the special sciences which we call *philosophy*. Now this method by which we proceed to amass and arrange our knowledge will be found to be twofold—deductive or *synthetic*, and *inductive* or *analytic*—according to the side from which we can best approach the subject-matter of any special investigation. If the subject-matter can be best approached from an intellectually abstract point of view, through the medium of a few great, broad, general truths, as is strikingly the case in the pure mathematical sciences, the *deductive method* will be employed; if on the contrary the subject-matter be presented to us through our senses, in the shape of innumerable individual, concrete facts or phenomena, as is obviously the case in the physical, experimental sciences, then the method used will be the *inductive method*. Hence a fourth section of logic will deal with *Method, Deductive* and *Inductive*.

The various sections just outlined will analyse our thinking processes with a view to the formulation of rules and laws, the observance of which will secure correct thinking and safeguard our rational investigations from error. As the aim of logic is to teach us how to reach *scientific* knowledge with certitude we must next inquire into the nature and requirements of *science*, *proof* and *explanation*. Since, however, it is not always possible to reach *certain* knowledge, we must analyse the nature and conditions of *probability*. And since the snares and pitfalls to which reason is naturally exposed are so numerous and so difficult to avoid, it will add to the practical utility of logic to reconnoitre again the ground already traversed, for the purpose of calling explicit attention to all such dangers. Hence the closing section on Certitude, Probability, Error and Fallacies.

10. Formal Validity or Consistency, and Material Validity or Truth, of Thought: Different Views as to Scope of Logic.—Let us next consider a little more fully what people usually mean by *correct* or *accurate* and *incorrect* or *inaccurate* thinking, by *valid* and *invalid* thought, by *true* and *false* or *erroneous* knowledge. About the meaning of terms *true* and *false* (or *erroneous*) there can be hardly room for ambiguity. Knowledge is commonly understood to be true if it is *in conformity with its object*, with the thing or reality about which it is concerned; to be *false* if at variance with the facts, with the actual state of affairs, with the thing or reality in question. We may let that clear and

simple notion stand without further analysis for the present. The other terms just mentioned have, however, each a wider and a narrower meaning. For, the thoughts that make up our knowledge about anything—at least in so far as they involve *inferences*, which latter are very often latent even in the simplest judgments by which we interpret sense experience—may be perfectly *consistent* with one another and exclude all contradiction and incompatibility with one another, and may nevertheless be all *false*, all *erroneous*, all out of conformity with fact, all out of joint with reality.

This will be made very evident when we come to deal with mediate reasoning. But any simple example will serve to show the possibility of such processes of thought. If, for instance, we take erroneous initial measurements in some problem in mensuration, we may work out our problem in strict accordance with mathematical rules, make all our calculations and inferences correctly, arrive at a result which will be in strict conformity with every step in the process back to the initial data, but which will be nevertheless wrong, *i.e. untrue*, because our starting point was wrong.

Processes and products of this kind are said to be "valid" or "correct" or "accurate" in the narrower sense of being *consistent* with themselves throughout, although not in the *wider* sense of being *true*.

Now there are many logicians who would confine the scope of logic to the securing of mere *consistency* in our thoughts, not to the securing of their *truth*. Distinguishing between the *forms* of our thought and its *content* or matter, they would have logic deal with these forms apart altogether from their relations to the things thought about.[1] Kant, Hamilton, Mansel, and Thomson incline to this view, describing logic as the *science of the formal laws of thought*, *i.e.* of the laws which govern *pure thought* or *thought simply as such*—the laws which regulate the forms of our thinking processes independently of their matter. And as it is almost exclusively in the domain of deductive reasoning that we can thus clearly distinguish between the form and the matter of thought and secure consistency irrespective of truth, these authors unduly confine their attention to deductive reasoning, to the neglect of induction.

[1] Some logicians—Whately, for instance—propose mere *verbal consistency* as the sole aim of logic. They thus confound logic with grammar. Language is the expression of thought, and it is with thought that logic primarily deals.

This view of the Science of Logic is too narrow, and moreover it is based on the erroneous assumption that the forms of thought are completely separable from its matter, and that it is only from the forms of thought that universal and necessary laws of thought can arise. As a matter of fact, the form of any individual thought can never be separated from its matter, although we can distinguish between these elements, and although either can change while the other remains the same.

The *forms* of thought are the natural grooves, so to speak, in which, owing to our actual mental constitution, our thought always runs,—the ways in which we think. The *matter* is the thing or object thought about,—the *content* as opposed to the *form*. When an object of thought is represented in the intellect by an abstract, universal *idea*, the result of an act of simple apprehension or conception, the idea with its properties may be said to be a *form* of our thought. When we analyse that object more fully and proceed to make mental assertions, *judgments*, about it, the *form has changed* from idea to judgment, the matter remaining unchanged. When, further, we proceed to *reason*, to make *inferences* about that object, the form is again changed. Concepts, judgments, inferences : these are forms of thought. And obviously we may change the matter, the form remaining unchanged : *e.g.* from *reasoning* about one subject we may pass to *reasoning* in the same way about an entirely different subject.

"There is a sense in which Logic is undoubtedly formal. By *forms* we mean what is the same in many individuals called *materially* different—the device, for example, on different coins struck from the same die, or the anatomical structure of different vertebrates, or the identical mode in which the law requires the different Colleges of the University to publish their accounts. And all science is formal in the sense that it deals with what is common to different individuals. . . . So the logician studies the forms of thinking, such as that involved in referring a quality to a subject possessing it ; but when he has once grasped the nature of this act of thought, he is quite uninterested in the thousand different occasions on which it is performed during the day ; they differ only materially, as to what quality is referred to what subject ; formally, so far as the notion of a quality existing in a subject is concerned, they are the same ; and the forms that run through all our thinking about different matters are what he studies."

"But those who have insisted most that Logic is a formal science, or the science of the formal laws of thought, have not merely meant that Logic is in this like other sciences, which all deal with what is formal or universal in their subject-matter. They have meant to exclude from Logic any consideration of forms or modes of thinking which are not alike exemplified in thinking about absolutely every subject. . . . But the truth is that we think in different ways about different kinds of subject, and therefore we must, if we wish to study the principles which regulate our thinking, consider to some extent the differences in the matter about which we think. . . . The most general forms of thought exist diversely modified in thinking about different matters ; and they can no more be fully known without attending to the different matters in

which they appear differently, than animal nature can be fully known without attending to the different orders of animal in which it appears differently." [1]

Notwithstanding the fact, therefore, that we can distinguish between the matter and the form of thought, it is obviously impossible to think at all without thinking of *something*, to have the *form* without the *matter* of thought; and hence if we are to understand by a purely formal logic a science of the *pure*, *empty*, "*a priori*" forms of thought, *i.e.* forms devoid of all matter or content and prior to all experience, such a science is impossible, for its object is chimerical and unattainable. It is impossible to separate *thought* entirely either from the things or *objects* to which it has reference or from the *language* which is its expression. Hence to make logic *purely subjective* or *conceptual* or *formal*—excluding all reference to *things* and *words*—is almost as erroneous as to make it conversant with *language* only. Logic deals with both language and things, though only indirectly: the former as expressive of, the latter as represented in, and interpreted by, thought.

But there is a sense in which we may perhaps admit the possibility of a division of logic into a logic of consistency and a logic of truth. Since it is true that we can distinguish between consistency and truth we may distinguish the logical investigations which aim at securing the former from those which aim at securing the latter; and in this sense we should distinguish the investigation of the laws of all formally valid reasoning—*i.e.* of reasoning that is consistent, or true hypothetically—from practically all the remaining sections of logical doctrine—about conception, judgment, classification, induction, scientific explanation and proof: all of which have in view not merely the subjective consistency but the objective truth of our knowledge.

Were we, however, to confine logic to an investigation of the necessary and universal laws which secure mere consistency in thought, we should bear in mind two things: firstly, that we must make provision in some philosophical science for the adequate treatment of the means and tests for securing *truth*, over and above the laws that secure *consistency* (17); secondly, that even these latter laws are not *exclusively* subjective or formal. This latter point deserves attention.

That there are such universal and necessary laws of thought is beyond all dispute. That their necessity and universality arise exclusively from the nature of the thinking subject—and not at all from the nature of the object thought about—is a mistake. For, although the things about which we think may differ very much from one another, yet they have something in common: they are all *things*, *realities*. The laws revealed by logic as underlying the consistency of thought derive their necessary and universal

[1] JOSEPH, *Introduction to Logic*, pp. 4-6; *cf. ibid.*, pp. 339 *sqq.*

character no less from this common constitution of the *object* of thought than from the mental constitution of the thinking *subject*. The laws of thought are not *purely formal* in this sense of being *totally and absolutely independent* of the nature of its *matter* or *content*. They are, however, formal, or non-material, in the sense that they do not vary even when the particular subject-matter of our thought does vary.

Hence it is that these processes and products of thought may be represented by symbols when they are being analysed for the purpose of illustrating logical laws and principles. The advantage thus gained by using brief symbols—usually the letters of the alphabet—instead of concrete examples, consists not merely in a saving of time but also in an increased facility in fixing our attention on the formal validity of those processes, while the matter of concrete examples would be likely to mislead or at least to distract us.

11. THE "LAWS OF THOUGHT".—Of the laws brought to light by logical analysis as underlying all our thinking processes, a few are so fundamental that it will be useful to follow the now common practice of setting them forth explicitly at this introductory stage. They have been variously described as *First Principles of Thought; Regulative Principles of Thought; Formal, A Priori, Laws of Thought; Postulates of Knowledge.* How far those titles are appropriate or misleading will appear from an examination of each of the principles in question. All logicians enumerate at least three: the *Principle of Identity*, the *Principle of Contradiction*, and the *Principle of Excluded Middle;* to which some add a fourth: the *Principle of Sufficient Reason.* They are formulated each in a variety of ways.

12. THE PRINCIPLE OF IDENTITY is simply the self-evident truth that *Everything is identical with itself; Everything is its own nature.* It is involved in every judgment—more directly in every affirmative judgment—and demands that throughout every thought-process the objects represented by our concepts and expressed by our terms remain identical with themselves. It thus expresses the unambiguity of the judgment and the immutable character of truth. It does not give us any positive information about a thing, beyond what we possess by thinking of the thing. But we cannot think definitely about anything without mentally marking it off from all that is not itself. Hence the principle is not a bare tautology, capable of being expressed by the statement that *A is A*. Such a reiterated reference to one

object of thought, *A*, does not get us beyond the *unity* expressed by the first reference to *A*. But *identity is more than unity*. We cannot conceive *identity* unless we conceive *diversity*: and what the principle really expresses is *identity amid diversity*. It finds its real application in the proposition *A is B*, no less than in the tautology *A is A*—in the statement that "Snow is white," no less than in the formula "Snow is snow". What it really demands, therefore, is this: that an object of thought, when subjected to mental analysis and regarded from *different* points of view and under *different* aspects, be considered and understood to remain objectively identical with itself throughout. *Everything is its own nature: Everything is what it is: Whatever is, is: Once true always true: Truth is at all times true: Truth must be ever in conformity with itself*—*Δεῖ γὰρ πᾶν τὸ ἀληθὲς αὐτὸ ἑαυτῷ ὁμολογούμενον εἶναι πάντῃ* (ARISTOTLE, *Anal. Pr.*, i., 321).

13. THE PRINCIPLE OF CONTRADICTION may be stated thus: *The same thing cannot be and not be at the same time and under the same respect: Idem non potest simul esse et non esse secundum idem: A cannot both be B and not be B: The same attribute cannot be at the same time affirmed and denied of the same subject: Ἀδύνατον ὀντινοῦν ταὐτὸν ὑπολαμβάνειν εἶναι καὶ μὴ εἶναι* (ARISTOTLE, *Metaph.*, iv., 3; *cf. idem.*, iii., 4). *Contradictory judgments cannot both be true; of two such one must be false.*

This principle is likewise involved in every judgment we make —more directly in every negative judgment. With the Principle of Identity—of which it is the correlative—it expresses the nature of affirmation and denial; and both principles underlie all immediate inferences from our judgments. It must obviously be understood to refer to one and the same object of thought in exactly the same circumstances, not to different parts or aspects of the same thing nor to the same thing at different times. It would indeed be more accurate to say that the principle abstracts altogether from time, and simply states that contradictory attributes cannot be asserted of any single object of thought; when, however, reference is made to time, the principle must be understood as referring to the impossibility of contradictory attributes inhering in the same subject at one and the same time.

14. THE PRINCIPLE OF EXCLUDED MIDDLE is variously expressed by saying that *Everything must either be or not be: A either is or is not B: Any attribute must be either affirmed or denied of any given subject: One of any pair of contradictories must*

be true; both cannot be false together: Between affirmation and denial there is no middle course; Ἀντιφάσεως οὐδὲν μεταξὺ ἀνὰ μέσον (ARISTOTLE, *Metaph.*, iii., 7; *cf. Phys.*, v., 5; *Anal. Post.*, i., 2).

The remarks just made about the Principle of Contradiction apply equally to the Principle of Excluded Middle. The two principles are closely related. The latter forbids us to think that both of two contradictory attributes can be simultaneously absent from a given subject; the former forbids us to think that both can be together present; neither tells us which must be present or which absent. Both principles taken together bring out the distinction between affirmation and denial and make us realize that every affirmation involves a denial and *vice versa*, and that we cannot understand the force of either of the latter without grasping the force of the other.

Some authors have raised difficulties about the universal applicability of the latter two principles. But those difficulties are due, in part at least, to a confusion between *contradiction* and *contrariety* (38-45). Of course between contraries—which are the most widely divergent attributes in a given sphere—there are numerous intermediate alternatives; but there is no such alternative between affirmation and negation. The paper on which I write need not necessarily be either *white* or *black* but it must evidently either *be white* or *not be white*. A thing need not necessarily be either *greater* or *less* than another thing—for "greater" and "less" are not contradictories—but it must either *be greater* or *not be greater* than that other thing: and if the latter, it will be either equal to or less than that thing.[1]

Another sort of difficulty is raised by asking such an admittedly absurd question as this: Must honesty either be green or not be green? For, if the principle is universal, one or other alternative answer must be held to be true rather than the other. This point will recur for discussion when we come to deal with negative terms and certain forms of predication and of inference.

15. REVIEW OF THE THREE PRINCIPLES.—The three principles so far dealt with are absolutely *primary* and *self-evident*, springing directly from our very notion of being or reality.

They are *necessary* principles of thought in this sense that no one can *consciously* and *deliberately* think in a way that would *violate* them. Of course whenever people reason fallaciously or

[1] *Cf.* KEYNES, *Formal Logic*, 4th edit., pp. 457-463.

erroneously they do *de facto* violate those laws—owing to carelessness of thought, ambiguities of language, or some of the many causes of error—but no one does or can *consciously* violate them by thinking a contradiction.

They are *laws* not merely in the scientific sense of being uniformities—embodied and repeated in every conscious thought of every rational being—but in the deeper sense of being constraining principles *productive* of such uniformities: as the law of the land, though in a different way, by its binding force produces uniformity of conduct in the citizens.

They are *a priori* laws or principles in the sense that they are not mere generalizations arrived at by experience—like the physical laws of falling bodies, for example—but reveal themselves as directly operative in our very first conscious thoughts about things; not, however, in the sense of their being innate endowments possessed by the mind antecedently to all thought and experience.

They are *formal* in the sense that their validity is absolutely universal and entirely independent of the particular subject-matter on which our thought may be exercised; not, however, in the sense that their inviolable necessity and universal applicability result *exclusively* from the constitution of the mind, which is the *subject* of thought, and not at all from the nature and constitution of the reality which is the *object* of thought.

Conceptualist logicians are wont to assume that the necessity and universality of those laws of thought are grounded exclusively in the subjective factor of thought—the mind. But such an assumption is both unnecessary and unwarranted in logic—besides being erroneous. Whether these first principles of *Thought* are not also first principles of *Being*, of *Reality;* whether their necessity is not ultimately grounded in the *matter* of thought as well as in its *form*, in the *object* no less than in the *subject;* and whether therefore they do not apply to all being as well as to all thought—it is the province of metaphysics, rather than of logic, to decide. Nowhere yet, at all events, has any valid reason been advanced why we should doubt the soundness of man's spontaneous convictions that the necessary truth of those self-evident first principles is rooted in the nature of things no less than in the nature of thought (10). They not merely assure us that we *cannot think* that a thing can be other than itself, or that we *cannot conceive* a thing being and not being at the same time and in the same respect, or that we are *forced to think* that a thing must either possess a certain attribute or not possess it: they assure us that *the things themselves are so, as we think them*, and that it is not merely a matter of *how we must think about things*, but also a matter of *how things really are.*

Of course they refer *primarily* to our thoughts, *i.e.* our judgments about things. It is just because they are seen to be true of *all* conceivable things

that they "do not profess to give any material knowledge, and their validity is in no way dependent on material conditions,"[1] *i.e.* on any special conditions or changes of the particular subject-matter we may be thinking about. "The three laws," writes Dr. Keynes,[2] "may be expressed by these formulæ: *I affirm what I affirm, and deny what I deny; If I make any affirmation I thereby deny its contradictory; If I make any denial, I thereby affirm its contradictory*".

"It follows that we cannot make any progress in material knowledge except in subordination to these laws. But at the same time they do not directly advance our knowledge of things. They are distinctly laws relating to judgments, and not directly to the things about which we judge."

The close relation of these two—the subjective and objective, the formal and material—aspects in the principles in question, will be still more evident in a fourth principle: the Principle of Sufficient Reason. In this principle the material or real side—the reference to reality—is so prominent, and so overshadows the reference to thought, that the advisability of ranking the latter with the three foregoing principles in an introduction to logic has been sometimes questioned. We will merely state it here, deferring a fuller treatment of it to Induction.

16. The Principle of Sufficient Reason, as formulated by Leibniz (*Monadolagie*, §§ 31-39), states that "no fact can be found to be real, no proposition true, without a sufficient reason why it is in this way rather than in another," or, "whatever exists, or is true, must have a sufficient reason why the thing or proposition should be as it is and not otherwise". The principle contains very explicitly a double reference: firstly, to our thoughts, judgments, reasoning processes; secondly, and more fundamentally, to the reality itself with which such mental processes are concerned. In its first or more formal aspect—under which it says that every judgment must have a reason—it is at the bottom of all our reasoning processes, giving explicit expression to the necessity with which consequent follows from antecedent, conclusion from premisses. But since every judgment cannot have for its reason an anterior judgment, since we must start from judgments which do not depend on others, we are entitled to ask what sufficient reason have we for assenting to these: to which the only answer is that the *reality* which is the object of those self-evident judgments *compels* or *constrains* us to judge as we do about it. This "constraining power" which is described by some modern philosophers as "the characteristic of the real,"[3] scholastic philosophers have always called "objective evidence". For the

[1] Keynes, *op. cit.*, p. 463. [2] *ibid.*
[3] Welton, *Manual of Logic*, vol. i., Introd., p. 1.

ultimate grounds of our judgments, therefore, the Principle of Sufficient Reason carries us beyond the subjective connexions of thought with thought, and into the reality itself which our thoughts have for object. It is more than formal, more than a law of thought: it is material or real, a law of being, of reality.

17. REAL, MATERIAL, CRITICAL LOGIC: CRITERIOLOGY, EPISTEMOLOGY, THEORY OF KNOWLEDGE.—Finally, to recur to the point mentioned above (10): if we were to concern ourselves in logic merely with the consistency of thought, we should assign to some branch or other of philosophical inquiry the whole question of the validity of thought in the larger sense of its *truth*. Beyond all questions of the inner dependence of thought on thought in our reasoning processes there is the question of the relation of thought to things. If we assent to each step in a reasoning process because of the previous step, why do we assent to the first step? If the *formal* canons of inference guarantee the truth of a conclusion not absolutely but only hypothetically, *i.e.* on the assumption that the premisses are true, what guarantee have we for the truth of the latter? What are the *tests* or *criteria* of the truth of our knowledge, and which of them is fundamental? What, in ultimate analysis, are the *motives* of our *certitude* about truths which we consider to be evident? Can we, on reflection, find adequate reasonable grounds for our spontaneous assents and beliefs? Or does a critical examination of our intellectual assents lead to universal doubt and scepticism? What is the nature of the "evidence" to which we appeal in justification of our beliefs and convictions? What is the nature of the mental act of judgment and what precisely is it that constitutes the *truth* we claim for it? Do the abstract and universal concepts which we use as predicates, and often as subjects, in our judgments, represent anything given to us in the data of our sense experience? And are these sense phenomena themselves exclusively mental creations—pure products of sense consciousness—or do they represent an extramental reality? Can an analysis of our processes of sense perception and of the origin and growth of intellectual knowledge throw any light upon the validity of the latter? Are there any rules or canons to guide us in the methods by which we are to observe the phenomena of mind and matter, to form our general concepts about things, to classify things, to *acquire* universal or scientific knowledge about things, and to *prove*, *establish*, *justify*, this knowledge?

Obviously these are all questions of fundamental importance. They have been discussed at all times by philosophers; but there has always been much difference of opinion and practice as to the proper place for treating them. The earlier scholastic philosophers usually discussed them immediately after the questions of *Formal* logic and described them as forming *Material* or *Real* or *Applied* logic, or again as *Logica Critica*, in opposition to *Logica Dialectica*, which was the title they gave to the logic of formally valid inference. Modern scholastics are inclined to state and expound briefly the practical rules and canons which constitute *Method*—both inductive and deductive—in *Logic*, and to leave the fuller discussion of all the great underlying principles of knowledge to a special treatise which they call *Criteriology* or *Epistemology*, or *Theory of Knowledge*, and which they claim to be a special branch or department of psychology, or of metaphysics, rather than a section of logic.[1] It is quite true that the treatment of many of these questions must be largely psychological and ontological. Nor is it possible or desirable to draw a sharp line of demarcation between these sciences.

There is a tendency among modern writers on logic to discuss in logic itself all the presuppositions of the science: all the deeper questions about truth, certitude, and knowledge to which an analysis of thought gives rise. But these had better be left as far as possible to some department of metaphysics, which analyses the principles of all the other sciences—most appropriately to that special department which investigates the validity of human knowledge and the ultimate grounds for human certitude. In the present treatise the questions that have an *immediate* bearing on the truth, as well as those concerning the consistency, of our thoughts, will be brought under notice; the presuppositions of induction will be dealt with. But the larger questions concerning the ultimate criteria of truth and the ultimate motives of certitude are left to epistemology.

WELTON, *Logic*, i., pp. 1-40. JOYCE, *Logic*, ch. i. and note. KEYNES, *Formal Logic*, Introd. and Appendix B. JOSEPH, *Logic*, ch. i. MERCIER, *Critériologie Générale*, Introd.

[1] *Cf.* MERCIER, *Critériologie Générale*, 5me edit., Introd., pp. ii. *sqq.*

CHAPTER III.

LOGIC AND KINDRED SCIENCES. DEFINITION AND SOURCES OF LOGIC

18. Relations of Logic to Kindred Sciences.—What has been said in the preceding chapter on the nature and scope of logic will be made clearer by a brief comparison of this science with certain other more or less nearly related sciences.

Logic is sometimes called the "science of sciences," because, although it does not deal with the special methods and rules of procedure peculiar to any particular science, it brings to light general laws and canons to which reason must conform in all; and because, furthermore, its scope embraces the principles that underlie the classification of all the other sciences and the unification—as far as this is possible—of all human knowledge.

19. Logic and Metaphysics.—The mind has all *Being*, all *Reality* as the object of its knowledge. *Metaphysics* considers being in its most abstract state, being in general, apart from all its specific and individual realizations. And hence, having such a simple, abstract, universal object, it helps us to comprehend the more complex objects of the special sciences. Just as mathematics helps the study of physics, so, *a fortiori*, does metaphysics aid us in the study of all the sciences; for it gives their principles and axioms to all the sciences, and guarantees the validity of all their initial assumptions—including those of logic itself.

But since *being* is the object of our *knowledge*, while logic aims at knowing *the process by which all being is known*, it is manifest that logic also has a sort of indirect interest in all being. It has, therefore, the same material object as metaphysics; not, however, the same formal object. For while metaphysics studies the common nature of those things that are treated in detail by the special (physical and mathematical) sciences, *i.e.* while it studies *real being* as it is in itself (*Ens Reale*) and the *real attributes* of real being, logic studies that

same being *as it is in the mind*, not as it is in itself, not as clothed in its real attributes, but as *subjected to the process of knowledge*, in the modes and with the attributes bestowed on it by the human reason (*Ens Rationis*). Metaphysics is thus the most abstract and universal science of *Reality;* logic the most abstract and universal science of our *Knowledge* of reality.

Whatever is real, *i.e.* whatever exists or can exist, can be an object of human thought. When the human mind tries to understand the nature of things, it can, by an effort of abstraction, consider successively their changing sensible properties, their stable mathematical quantity, their inner essence or nature and metaphysical attributes. All these various aspects are characteristics which *belong to the real being* under consideration. But the *abstract character* of the object as considered by the mind, its *universality*, and the *various other modes and attributes* which inevitably affect it from the fact of its becoming an object of intellectual thought—these are not real in the sense of being realizable outside the mind: they have no reality other than what they have in and from the mind which gives rise to them, and they are therefore called *attributes of reason* as opposed to *real attributes*. When concrete and individual things become objects of intellectual thought they become abstract and universal, and in this state they give rise to mutual relations such as could not exist between concrete individual things: for example, one mental object becomes *predicate* of another which serves as subject, *i.e.* one becomes *predicable*, *affirmable* or *deniable* of the other in the mental act of judgment; comparisons in intension and in extension (30) between these mental objects give rise to relations of identity and diversity, relations established in the mental processes of definition, division, classification, judgment, inference, etc.; and the matter of all these different intellectual operations, from beginning to end, is *being*, not as it is in itself, independently of thought, but *as it is moulded and elaborated by thought.* In a word, the *formal object* or special view-point of logic in studying being is not being itself as such, but being as conceived by the mind—the *Ens Rationis;* while the *formal* as well as the material object of metaphysics is real being, simply as such—the *Ens Reale.*

When the mind concentrates its attention upon things, its abstract ideas of their various phases and aspects have been called by logicians *primae intentiones mentis*—first or direct mental views of things—and these aspects

themselves *objecta primae intentionis mentis:* these constitute the object of *metaphysics*. But when the mind proceeds to reflect on those first abstract objects of its direct thought, to compare them with one another and thus to establish mental relations between them, these reflex acts are called *secundae intentiones mentis*—second or reflex views of the objects, not now as simply existing in themselves, but as directly known or thought of by the mind—and these mental or logical relations thus established between the *objecta primae intentionis* are themselves called *objecta secundae intentionis*. With these latter *logic* is concerned.[1]

When the reality is brought by a direct abstract thought—a *prima intentio*—into the domain of intelligence, and there acquires that peculiar mode of presence given by knowledge—a presence which the ancients called *intentionalis*,—it can become the object of reflex thought, can be endowed with numerous *purely mental or rational* attributes—*objecta secundae intentionis*, and thus falls within the scope of logic.

The relations thus established by the mind between its own objective concepts by reflection upon the latter, *do not* and *cannot* exist or occur in any sphere of reality, actual or possible, about which the mind can think, other than the sphere of its own thought about such actual or possible reality. They are products of the mind's thinking processes, nothing more. They are psychologically real, as subjective or mental products; but the only *objective* reality they have is that which the mind itself gives them by thinking them: as it does in logic, where it distinguishes clearly between them—as its own creations, devoid of all other actual or possible reality—and the realities which it does not create but only apprehends by its direct thought, as actual or possible.

How logic actually deals with these *entia rationis*, these compartments, comparisons, and relations, which it establishes in the contemplation of things, we have already discussed, in dealing with the consistency and truth of thought. It examines the various forms of thought in detail: concept, judgment, class, *genus*, *species*, subject, predicate, inference, syllogism, antecedent, consequent, etc. It examines them, however, not in their empty, abstract form, but in their applications to reality; it applies to man, for example, the concept of *species*, and to animal that of *genus*, and compares these two real objects of thought—man and animal: *objecta primae intentionis*—in the light of the logical relations of *genus* and *species* which it establishes between them. Here, obviously, we reach the point of contact between logic and metaphysics. *Each* science alike contributes to our knowledge of *all* being: metaphysics by seizing on those deepest and widest of its attributes which must pervade and elucidate the assumptions of all the other sciences, logic by analysing the processes

[1] "Sciendum est," says St. Thomas, "quod alia ratione est de communibus Logica et Philosophia Prima. Philosophia enim Prima est de communibus, quia ejus consideratio est circa ipsas *res* communes, scilicet circa ens et partes et passiones entis. Et quia circa omnia quae in rebus sunt, habet negotiari ratio, Logica autem est de operationibus rationis, Logica etiam erit de his, quae communia sunt omnibus, id est de *intentionibus* rationis, quae ad omnes res se habent. Non autem ita quod Logica sit de ipsis rebus communibus, sicut de subjectis. Considerat enim Logica, sicut subjecta, *syllogismum*, *enunciationem*, *praedicatum* aut aliquid hujusmodi."—*In Post. Anal.*, i., sect. 2.

whereby we introduce reality into the sphere of mind and the peculiar mental modes and forms which affect reality on its being thus made the object of our knowledge.

20. LOGIC AND PSYCHOLOGY.—Psychology is the (speculative) philosophical science which investigates the ultimate causes and principles of *life* in general, and more especially of *conscious* and *mental* life. Embracing all mental activities, it has a wider *material* object than logic (3, 4). And even where the two sciences partially coincide—in the investigation of processes of intellectual thought and reasoning—their *points of view* are quite different. Psychology studies our thought processes as natural phenomena, as natural activities of the living being, with a view to finding out their *genetic* principles, and the laws that regulate their production and development. It aims at finding out how men *do* think, not how they *ought* to think ; at discovering the *natural* laws of the *actual* association and interaction of our thoughts, not at laying down *normative* or *regulative* laws to which *ideally* accurate thought must conform. The psychologist confines his attention to the *subjective* or mental conditions of the thought-*process ;* he is not concerned with the *objective reference* of the thought-*product*—with its accuracy, its truth, its validity.

The distinction between the two sciences is, therefore, sufficiently clear. Yet it is practically impossible to keep them entirely separate. Some psychological knowledge of the nature and functions of the human mind is indispensable for a proper logical analysis of our fully conscious thought processes. Then, too, this latter analysis is sure to carry us back to purely psychological inquiries concerning the origin of intellectual knowledge and the sentient activities which subserve its acquirement. Hence the many psychological discussions which arise in regard to conception, generalization, judgment, induction, etc. Nor, again, is it possible in psychology to treat satisfactorily of the origin and growth of knowledge, without raising questions about its *object*, its *validity*, and the *tests* of its *truth*. The psychological analysis and explanation of our beliefs is hardly separable from the question whether or how far they are logically justifiable.

Those "critical" questions regarding the objective side of human knowledge—questions as to the *criteria* or tests of its objective truth, arising out of psychology—are nowadays usually treated apart (in Criteriology, *cf.* 17); except in so far as logic claims to derive from an analysis of them certain rules or canons

of method by which the mind may be guided not merely to subjective consistency but also to objective truth.

21. LOGIC AND RHETORIC.—These have kindred aims, that of logic being to *convince* by appealing to the *intellect*, that of rhetoric to *persuade* by appealing rather to the *emotions*. Both sciences touch psychology, the former in the intellectual, the latter in the emotional domain. But rhetoric is more intimately connected with the study of *Language* and *Literature* than logic is.

22. LOGIC AND GRAMMAR: *Thought and Language: Words, Syncategorematic and Categorematic: Parts of Speech: Names and Terms, Single-worded and Many-worded.* — Besides the grammar of each particular language there is a science of *Universal* or *General Grammar* which investigates the laws to which all languages must conform. It deals with the mutual relations of those parts of speech that are essential to all rational language, and investigates their connexions with the thoughts they express. Having language as its subject-matter, this *general* grammar is closely connected with logic, whose subject-matter is the thought itself: for logic deals also, in a secondary way, with language, which is inseparable from thought. Language may be defined as *a system of articulate sounds produced by the organs of speech and used as instruments of thought and as signs for the communication of thought.* This is a definition of the *oral* or *spoken* language of man. It does not include the cries, barks, bleatings and various other more or less inarticulate calls (*voces*) which animals instinctively use as signs to manifest and communicate their conscious states. These are in a wider sense called the "language" of animals. Nor does the definition include the "language" of gesture; nor *ideographic* writing which stands immediately and directly for objects of thought without the intermediary aid of spoken language (hieroglyphics, for instance); nor, properly speaking, does it include the systems of signs used by deaf-mutes. It does, however, indirectly include the ordinary or *phonographic written* language, inasmuch as the written words of the latter are visual symbols of the spoken words themselves; and also the systems of raised print invented for the blind to read by the sense of touch.

Language, whether oral or written, is accordingly a system of signs used for the *communicating* and *recording* of thought. And this is undoubtedly the leading function of language: communicating our thoughts *orally*, and storing up a record of them by com-

binations of *written* (or printed) signs (or by phonographic records in more recent times). The other function fulfilled by language, that of supplying a natural *aid* to thought, or *instrument* of thought, is, however, one of great and distinct importance.

A *sign* or *mark* of a thing is anything which arouses a knowledge of that thing in a being capable of knowledge. (A sign is either *formal*—when it reminds us of the thing by virtue of its *likeness* to the thing, as in the case of all *images*,—or *instrumental*—when its connexion with the thing is other than a connexion of resemblance. Such other connexion may be either *natural*, in which case we have a *natural* instrumental sign, as smoke is of fire; or it may be established by *common agreement*, in which case we have an *arbitrary*, *artificial*, *conventional* instrumental sign, as, for example, in the case of military and naval signals. The various auditory signs (*voces*) which constitute the so-called "language" of animals are *natural* signs. The articulate sounds called *words* (*vocabula*) which constitute human language, are *largely*, though *not at all exclusively*, *arbitrary* signs. Not exclusively; for human languages are in a true sense spontaneous, natural growths, even though the great mass of their details may be settled by convention.

Leaving to its proper place in psychology the whole question of the origin of language, its natural connexion with thought, and the possibility or impossibility of thinking without words or language of any sort, we will here simply emphasize the fact that all our thinking processes are much more dependent on words and much more intimately assisted by language than we might, without reflection, be inclined to imagine: (*a*) In our *analysis of sense experience* into its component elements we are enormously aided by our power of giving *names* to each of these elements. (*b*) In the formation and retention of the *abstract, general concept* —including, as it may, quite a large group of attributes—the *term* by which we express the concept serves powerfully to hold the contents of the latter together. (*c*) And terms not only tend to make our concepts definite but also to fix and concentrate those processes of *judgment and reasoning* by which we establish relations between our concepts. Hence it is that language is regarded as a *practically indispensable instrument* to even moderately developed thought.

Logic, therefore, must needs deal with language. Yet, even in so far as it does, it differs from grammar. The latter deals *solely with the expression of thought in speech*, with the mutual relations of the words which make up language and which are called *parts of speech*. Logic deals with language only as an instrument and vehicle of thought, and therefore analyses it only up to the

point at which it expresses the simplest conscious element of intellectual thought—the *concept*. Just as the logical analysis of thought does not proceed farther back than the *concept*, so neither does the logical analysis of language go beyond the *term* or *name*.

From the point of view of logic, therefore, words fall not into nine parts of speech but into two great classes: into words which, of and by themselves, express a concept, a notion, an idea; and words which do not: that is to say, into words that are logical terms or names, and words that are not such. The former are called *Categorematic Words;* the latter, *Syncategorematic Words* (from σύν, and κατηγορέω, *I predicate*). The former can stand by themselves as subjects, or at least as predicates, of logical propositions, the latter only in connexion with some categorematic word, without which they remain meaningless. All such non-significant words are, by themselves, outside the scope of logic; all significant words logic places in one class—calling them "names" or "terms"—whatever grammatical parts of speech they may happen to be. The parts of speech which are significant or categorematic, and which can therefore be subjects or predicates of a logical judgment, are primarily the substantive and the adjective. Pronouns in the nominative case, standing for nouns, are also categorematic. Participles and possessive and demonstrative adjectives and the possessive case of nouns and pronouns, are logically equivalent to adjectives, and, therefore, also categorematic. All verbs are changed in logic to participles united with the verb *to be*—which is the only verb recognized by logic, being the *copula* or connecting link between subject and predicate in all statements when reduced to logical form. Adverbs, being mere modes or qualifications of adjectives and participles, have no meaning of themselves and apart from what they qualify, and are therefore syncategorematic. So likewise are prepositions, conjunctions, and interjections (74).

Nor is the grammatical analysis of the *sentence* into substantive (or pronoun) and verb the same as the logical analysis of the *proposition* into subject and predicate (78, 96). Not all grammatical sentences are logical propositions, but only those which make a statement, by the indicative mood of the verb used. On the other hand, grammar recognizes no form of words as a sentence unless it contains, explicitly or equivalently, a noun and a verb; whereas logic takes account of even monosyllabic exclamations, such as "Fire!": but claims the liberty to restate their meaning fully and explicitly in the propositional form of subject, copula,

and predicate: "These premises are on fire". Again, we fix the logical subject of the sentence, not so much by looking to the form of words in which the latter is expressed, as to the underlying thought, and finding out what the main thing is about which we are thinking: this we make the logical subject; what we think about it, the logical predicate. Frequently the logical and the grammatical form of a statement do not coincide: the latter has to be recast in order to obtain the former. The logical subject is always grammatically resolvable into a substantive. The adjective with its equivalents is logically a predicate, and it is only by an ellipsis it can stand as logical subject: "The virtuous (*people*) are happy". Of course any part of speech, *when made to stand for itself* by what is known as *suppositio materialis*, can be a logical subject: *e.g.* "*Seldom* is a word of two syllables"; "*alas* is an interjection"; "*and* is the English of *et*," etc.

Finally, it will be observed that a name or term, the subject or predicate of a proposition, is perhaps more frequently expressed by two or more words than by a single word. A *Name*, therefore, might be defined as *a word, or combination of words, serving as a sign or mark to raise up in our own minds and in the minds of others an idea of some object of thought.* When a name is used as subject (*S*) or predicate (*P*) in a logical proposition it is called a *term* (from *Terminus*, a *limit* or *boundary:* the two ideas compared in a judgment forming the extremes of the comparison). A *Logical Term* is usually defined as *the verbal expression of an idea*, or *the result of the analysis of a logical proposition into subject and predicate.* When the verbal expression is considered in itself, apart from any proposition, it is called a *name;* as forming subject or predicate of a proposition, it is called a *term.*[1]

The fact that the term or name may be either a single word, as "student," or a combination of words, as "student of philosophy," gives rise to the division of terms or names into *Single-worded* (*Terminus Incomplexus*) and *Many-worded* (*Terminus Complexus*). The latter are combinations of words some of which are themselves simple terms and others syncategorematic words, which latter fall in this indirect manner under the consideration of logic.

23. DEFINITIONS OF LOGIC.—From what has been said so

[1] The word "term" is sometimes used by logicians as synonymous with the *object* about which we think, the object of our thought, what scholastic logicians called the *conceptus objectivus* (*cf.* JOSEPH, *op. cit.*, pp. 14-18).

far concerning its nature and scope, its subject-matter and divisions, its formal object and its relations to certain other sciences, we are now in a position to attempt a definition of logic. Many definitions have been proposed—good, bad, and indifferent. But a discussion of their relative merits and demerits would, at the present stage of our inquiry, be neither profitable nor desirable. Logic has been variously defined as the *Art of Right Thinking;* the *Art and Science of Right Reasoning;* the *Art of Attaining Truth;* the *Science of the Formal Laws of Thought;* the *Science of the Principles which Regulate Valid Thought;* the *Practical Science which Directs the Operations of the Mind towards Truth;* etc. On the whole, we are inclined to prefer some definition on the lines of the last just given. We would, therefore, prefer to define the science of logic somewhat more explicitly as: *the Practical*[1] *Science which Directs our Mental Operations in the Discovery and Proof of Truth.*

24. UTILITY AND NECESSITY OF LOGIC.—It would be a mistake to imagine that, above and beyond what is called the *Natural Logic* of sound common sense, the study of the *Science of Logic* is absolutely necessary for right reasoning. Men reasoned rightly before Aristotle ever formulated a canon of logic. It was, in fact, by an analysis of such reasonings that he discovered those canons: they could never have been discovered otherwise. Here as elsewhere the art came before the science; theory followed practice. A man may reason rightly without knowing a single rule of the syllogism; or, conversely, he may know all the details of logic and be an indifferent guide to truth—just as a first-rate geometrician may be a failure as an engineer. But still, just as his knowledge of geometry will enable the geometrician to detect the defects in a piece of engineering, so too will an explicit knowledge of the canons of reasoning enable us to discover more readily where the fallacy of a misleading argument lies. Without professing to guard us infallibly from error, logic familiarizes us with the rules and canons to which right reasoning processes must conform, and with the hidden fallacies and pitfalls to which such processes are commonly exposed.[2] Hence one obvious benefit derivable from a careful

[1] *i.e.* including the theoretical groundwork ("*Logica Docens*") on which the actual exercise or art of correct thinking ("*Logica Utens*") is based (8).

[2] "A man who is very ready at integration begins to hesitate and flounder when he is asked such a simple question as the following: 'If all triangles are plane

study of logic: a facility in detecting error in reasoning processes, and a consequent likelihood of avoiding such errors, and of thinking and reasoning about difficult matters with clearness and consistency—a capacity much rarer, even among educated people, than is commonly suspected.

But there is another—and perhaps greater—utility in the study of logic: the advantage of the admirable *mental discipline* which the study of the science indirectly and unconsciously involves. It is by this mental training rather than by the explicit, positive knowledge of its technical rules, that logic gives us the power and the habit of thinking clearly. Probably more than any other science, a careful study of logic trains and develops the reasoning powers, and not merely the power of thinking *consistently*, but the power of discovering and proving *truth*.

Yet another and no small advantage of logic is that it gives us some insight into the nature and powers of our own minds, and suggests to us in one way or another perhaps most of the great problems with which it is the function of philosophy to deal: hence its value as an introduction to philosophy.

Of course the exclusive study of the more formal, deductive, abstract side of our reasoning processes might easily result in an abnormal and one-sided mental development. And it may be admitted that an occasional undue accentuation of such studies is perhaps partially accountable for the reaction which has manifested itself in an affected distaste and disregard for logic in some centres of education and intellectual culture. The English mind has never been remarkable for any deep respect for logical consistency. It has an undeniable respect for concrete facts, and protests against the abstract—as if the abstract were unreal. Cardinal Newman's attitude, for instance, towards the traditional, deductive, abstract logic, was unsympathetic. Yet, in the main, his *Grammar of Assent*, so far from discouraging the study of logic, will prove an invaluable aid to the latter if only by acting as a wholesome corrective against the possible danger of over-emphasizing the rôle of the pure reasoning faculty and under-estimating the place and the importance of those more delicate and complex mental processes by which we arrive at practical conclusions concerning the concrete facts and phenomena of real life.

25. Sources and History of Logic.[1]—Most of the questions studied nowadays in logic were dealt with by Aristotle in several

figures, what information, if any, does this proposition give us concerning things which are not triangles?' As to untrained thinkers, they seldom discriminate between the most widely distinct assertions. De Morgan has remarked, in more than one place, that a beginner, when asked what follows from 'Every A is B,' answers 'Every B is A *of course*'" (Jevons, *Studies in Deductive Logic*, Pref. ix, x).

[1] *Cf.* Veitch, *Institutes of Logic*, Part i., ch. ii. and iii.; Prantl, *Geschichte der Logik*.

distinct treatises: The Book on the Categories, *Ἀριστοτέλους κατηγορίαι, Aristotelis Liber de Praedicamentis ;* the Book on Judgment, *βιβλίον περὶ ἑρμηνείας, Liber de Interpretatione ;* the (two books of) Prior Analytics, *ἀναλυτικὰ πρότερα, Analytica Priora (libri duo) ;* the (two books of) Posterior Analytics, *ἀναλυτικὰ ὕστερα, Analytica Posteriora (libri duo) ;* the (eight books of) Topics, *τοπικὰ, Topicorum Libri Octo ;* and the Sophistical Arguments, *περὶ σοφιστικῶν ἐλεγχων, De Sophisticis Elenchis.* These separate tracts were all grouped together under the common title of (Aristotle's) *Organon, ὀργανον,* a name first given them by Diogenes of Laërte.

The Book on the *Categories* treats of *Simple Apprehension* and *Concepts.* The *De Interpretatione* deals with *Judgment,* affirmation and enunciation, denial, subject and predicate. The two books of *Prior Analytics* deal with the *formal* side of *Inference,* the canons of the *Syllogism ;* the *Posterior Analytics,* the *Topics,* and the *Sophistical Arguments* investigate the *material* side of the reasoning process, the syllogism as *Demonstrative,* as *Probable,* and as *Erroneous,* respectively.[1]

In Aristotle's theory of logic, *Demonstration,* as the ideally perfect means of reaching *Science,* is his supreme concern. His view of logic is, therefore, not the narrower but the wider view. He paid more attention, however, to the application of the syllogisms to the *necessary* matter of metaphysics and mathematics than to the *contingent* matter of physical phenomena and the concrete facts of social life. His theory, therefore, as developed in after times, especially by the scholastic philosophers of the Middle Ages, tended towards a predominantly deductive and formal treatment of our thought processes.

The advances made in the physical sciences in the seventeenth and subsequent centuries led men to concentrate their attention more carefully on the mental processes by which we gradually bring to light—from isolated observation and experience of individual facts—a knowledge of general truths. Hence the prominence universally accorded to *Induction* in the numerous logical treatises which saw the light during the course of the last century. Nor have the results of the analysis of those processes which lead to the discovery and establishment of the general truths of the positive sciences been yet moulded into any one definite or generally accepted theory of induction.

[1] *Cf.* Joseph, *op. cit.*, pp. 348, 357-363.

Naturally, too, the excessive development of the purely formal side of Aristotle's treatment of logical processes led to a diminution of the great esteem in which the *Organon* had been traditionally held. But the soundness of his logical theory as a whole has stood the test of centuries. His title of Founder of Logic has never been disputed. A careful and impartial study of the *Organon* in our own time is convincing many that a great deal of fruitful and suggestive doctrine may still be learned from the Stagirite.[1]

See readings for preceding chapter. Also VEITCH, *op. cit.* JOSEPH, *op. cit.*, p. 34, c. xvii., pp. 348-9 ; c. xviii., pp. 361 *sqq*. MELLONE, *Introductory Text-Book of Logic*, ch. i.

[1] The works of MELLONE and JOSEPH are remarkable for the great amount of valuable matter they owe to a close first-hand acquaintance with Aristotle's logical treatises.

PART I.

CONCEPTS AND TERMS.

CHAPTER I.

LOGICAL PROPERTIES AND DIVISIONS OF CONCEPTS AND TERMS.

26. MUTUAL RELATIONS OF TERM, THOUGHT, AND THING. —As logic deals primarily with *thought* and secondarily with *language* it will make no difference whether we speak, with some authors, mainly of *Terms*, *Propositions*, and *Syllogisms*, or, with others, mainly of *Concepts*, *Judgments*, and *Reasonings*,— provided always we bear in mind what has been said about the function and scope of logic: that thought is its chief concern. As a matter of fact we purpose to employ sometimes one, sometimes the other set of terms, according as the treatment of the subject may demand. Nor can we expect entirely to avoid certain references to the nature of the *things* themselves, the realities, the objects about which thought is concerned.

Again, since *judgment* is the chief act of the mind—the act in which truth or error is contained—logic will treat of concepts and terms only in so far as these are materials of, and enter into, judgments and propositions. Every concept *stands for*, or *refers to* some object, but it is only the judgment that *predicates* or *announces* one object of thought about another. These two thought-objects, compared in judgment, are known as *concepts* or *objective concepts*. They are connected by a mental bond expressed orally by the verb *to be*, which is called the *copula;* and they are themselves called the *subject* and *predicate* of the judgment. The concept will therefore be treated as a thought-unit capable of fulfilling the function of subject or predicate of a logical judgment: "*notio subjicibilis vel praedicabilis in enunciatione*". So, too, the *Name* will be treated only in so far as it stands as *Term* in a logical

proposition. It will be well for the student to bear this in mind even when the investigation of certain divisions and properties of concepts and terms may seem to regard these latter solely as they are in themselves and without any direct reference to the judgment or proposition.

As we shall be constantly referring to *terms*, *concepts*, and *things*, it will be well to have a clear idea from the outset regarding the exact relations between these three. The term represents and refers to *the thing* rather than *the concept*. It does not, however, express the thing as it is in itself, and apart from all relation to thought, but the thing *as known through and by the concept*. In a word, the reference of the term is to the *known object*, or to the *object as known*. "Voces," St. Thomas rightly teaches, "referuntur ad res significandas mediante conceptione intellectus."[1]

When, for example, we pronounce the word *sun* we do not signify or convey to others our idea of the sun—our mental state—but the thing itself, the object, the sun; and when we say that the sun gives light and heat we manifestly assert the attributes, light-giving and heat-giving, to belong to the sun itself, not to *our idea* of the sun.

At the same time the term or name, *sun*, can scarcely be said to denote the thing or object exactly and necessarily as it exists in nature; because people thought for ages that that thing or object which they denoted by the name *sun*, was a moving body revolving around our planet: which was evidently not true of the sun as it really was and is, but only of the sun as it was thought to be before the discoveries of Galileo and Copernicus made people aware of their error. The term, therefore, represents the object not necessarily as it is in nature but rather as it is known or thought of by those who use the term.[2]

If, therefore, Mill was right in accusing Hobbes of teaching the erroneous doctrine that names stand for ideas, he left himself somewhat open to the charge of erring in the opposite extreme by laying undue emphasis on the antithesis that "names are names of things, not of our ideas".[3]

27. UNIVOCAL AND EQUIVOCAL TERMS: ANALOGY AND METAPHOR.—An univocal term is one which serves as a name for one class of things; an equivocal term, one which stands for two

[1] *Summa Theol.*, 1ª, q. 13, a. 1.

[2] This point will recur in our treatment of the connotation and denotation of terms, and of the existential import of propositions.

[3] *Cf.* KEYNES, *Formal Logic*, 4th edit., p. 9.

or more entirely distinct classes of things and has therefore two or more distinct senses: as *vice*, a bad habit, and *vice*, a mechanical instrument. All languages contain many such terms, and they are all essentially ambiguous and a fertile source of confusion of thought. The meaning of such terms must be determined from the context; and when the logical characteristics of such a term, apart from any context, are asked for, it is to all intents and purposes equivalent to two or more logical terms according to the number of distinct meanings it may have, and each of these must be dealt with separately. Hence the present division comes naturally first in our treatment of the logical divisions of concepts and terms.[1] It concerns language rather than thought; and the same is true of analogy and metaphor. When we apply a term to an object in an *analogical*, *transferred*, *metaphorical* sense, it is because of some resemblance, or other relation, of the object in question to the object which the term properly and primarily denotes; as when we say that certain medicine, or climate, or food, or complexion, is *healthy*, because of the connexion of each of these with the living being of which health is properly and primarily asserted; or when we speak of a *smiling* meadow; or of the *foot* of a *mountain*, of a *man*, or of a *page*. In all such instances it will be seen that the idea underlying the name and expressed by it is not *quite* the same in each case: the term is not applicable *univocally* or in *exactly* the same sense to the objects it is made to denote—nor yet can the underlying idea be said to be *totally different* from case to case: nor is the term therefore applied *equivocally* —but the idea is in fact partially the same and partially different. An idea applied in this way, with the term that expresses it, is usually described as *analogical*. Of course every universal idea or term may conceivably be used in this way as well as univocally: the distinction is one which affects the *predication* that takes place in judgment rather than the concept (or term) itself. Further light will be thrown upon this point by the treatment of connotation and denotation of terms.

28. UNIVERSAL OR GENERAL, AND INDIVIDUAL OR SINGULAR TERMS: BASIS AND NATURE OF THIS DIVISION.—Terms are divided into *Singular* and *General* according as they can apply to one or more than one object of thought.

A *Singular* or *Individual Term* is *one which can be applied in*

[1] The division into single-worded and many-worded terms (22) is of minor importance from the point of view of logic.

the same sense to only one definite, individual thing. Singular terms are of two kinds: (1) *Significant Singular Terms*, and (2) *Non-significant Singular Terms*, or *Proper Names.* A *General* or *Universal* or *Common Term*, or *Class Name*, is *one which can be applied in the same sense to each of an indefinite number of individual things:* as man, horse, book, Centaur, Emperor of Ireland, triangle.

The general term or class name is the verbal equivalent of the *Universal Concept*—the concept which signifies "something common to many": *unum commune pluribus* (4). It is, therefore, applied to each and every member of a class because it signifies some attribute or attributes which they possess in common and in virtue of which the common name is given to them. There need be no *actually existing* object to which the name applies; or there may be only one such; but provided there may be *conceived* an indefinite multitude to which the name *can* apply, the latter is general; whereas the singular term is not even *potentially* applicable to more than the one individual. A good test for deciding whether a term is general or singular is to see whether it will take "*all*" or "*some*" before it; for if it is a class name, statements may be made about *all* or *some* members of the class.

All general terms may be changed into *Significant Singular Terms* by prefixing some individualizing word or phrase by which the term will be made to identify or point out some one individual thing:[1] book—*this* book; king—*the present* king; man—*the first* man; universe—*the material* universe. Sometimes the definite article alone will suffice, in a known context, to make a term individual, as "Let us walk in *the* garden"; sometimes it leaves the term general, as "*The* horse is a beast of burden". It is in fact *mainly* by using general terms with such limiting epithets that we refer to individual objects. It is the only way we can refer to the vast majority of individual objects, which have no proper names; and even the very limited classes of objects which receive proper names—persons and places: some domestic animals: more rarely, inanimate objects, such as the principal stars—are often referred to by such individualized general terms instead of their proper names. These individual terms are described as *significant* because they signify not merely the attributes signified by the corresponding general term but the individualizing attributes of the individual thing in addition.

[1] *Cf.* VENN, *Empirical Logic*, pp. 161 *sqq.*

Proper Names, on the other hand, are *non-significant :* a proper name may be defined as *a term used conventionally to serve as a mark or sign to indicate an individual person, place, thing, or event, without implying any attribute of the latter.* It is, therefore, a mere arbitrary verbal sign, a mere label for the identification of an individual object, and not intended to convey any information about that object. From the accidental fact that many individuals have, or may have, the same proper name, we must not therefore infer that it ought to be a *general* term, for such individuals are not understood to have got the name *in virtue of any attribute they possess in common ;* on the contrary, every occasion on which such a name is fixed upon an individual the reference is understood to be to that individual alone (*cf. infra*, 37).

There are two cases in which proper names are apparently general terms. The first is where a statement is made about the class of persons who happen to agree in this point of having some proper name in common, as "Some Patricks are not Irishmen". In such examples the name is not really used as a proper name, but as indicating an attribute (the extrinsic, accidental attribute of having the same name) possessed in common by the class referred to. The second case is where the proper name of some notable individual is applied to others who possess the characteristics of that individual in a marked degree, as when we say of a person that he is a Socrates, a Napoleon, an Ignatius. Here, evidently, the name is not used as a proper name, but as signifying certain attributes and applying to all individuals who may be found to possess those attributes.

'29. COLLECTIVE AND SUBSTANTIAL TERMS.—A *Collective Term* is the name of a group of similar units, as army, library, nation. The origin of such names is due to the need for some special term to express (*a*) a number of *separate* units (*b*) collected into one whole or group because of *some similarity* of the units. The object expressed by a *non-collective* or *unitary* term—such as *paper*, may be composed of a number of similar elements, but these are *not thought of as separate ;* and on the other hand if there be *no similarity* between the members of a group—as, for example, a group composed of a man, a boat, the House of Commons, and a month—it is neither needful nor possible to get a special name for the group.

Collective terms may be either general, as *library*, or singular as *Maynooth College Library*, according as they come under one or other of the definitions of such terms given above (28). When singular, they fall for the most part into the class of *significant* individual terms: only a comparatively small number of collective

terms are proper names; and these are geographical, as *The Pyrenees, The Archipelago.*

Of course it is only an exceedingly small number of all actually existing groups or collections of things that have got special collective names; and if, in the absence of such a collective name, we want to make a statement about some such group *as a whole*, we must have recourse to the word "*all*" in the sense of "*all together*". For example, "All the angles of a triangle are equal to two right angles," "All these books will fill the shelf": here we have the *collective use* of the ordinary non-collective or unitary general term; while we have the *distributive use* in the statements: "All the angles of a triangle are less than two right angles," "All these books could be read in a week". Sometimes a *collective* term is used *distributively*, as in the example: "The meeting dispersed". Oftentimes it is not easy to decide whether the statement is intended to be made of the whole of a group or of the individual units; but the context usually enables us to decide: "The people filled the hall"; "The people were excited". Arguing or assuming that what is true of many things distributively is true of them collectively, or *vice versa*, is the fallacy known as the "*sensus compositus*" or "*sensus divisus*"—"*composition*" or "*division*".

The names of materials such as gold, wood, water, air, salt, have been called "*substantial terms*". We may ask, are such terms *general* or *singular*, and do we usually employ them *collectively* or *distributively?*[1] When they are used as predicates—which is not often—"This is gold; that is salt," etc.—their use is evidently distributive: they refer not to the one single collective heap of all of that substance (gold or salt) in existence, but to particular portions of it. When we employ them as subjects, the same is generally true: we rarely if ever have in mind the *one single collection* of all the existing material in question, but rather *some* or *any portion* of it: "Some water is unfit for drinking"; "Water is composed of oxygen and hydrogen" (*i.e.* any and every particle or portion of water); "Oil is lighter than water" (*i.e.* any definite quantity or volume of oil, compared with an equal volume of water). The terms are here *general* and *distributive*, referring to the portions as units. Moreover we can speak of different *kinds* of water, etc.—which would show that such

[1] *Cf.* WELTON, *Logic*, i., p. 51 (2nd edit.); VENN, *Empirical Logic*, pp. 170, 171; KEYNES, *Formal Logic*, p. 12 (4th edit.).

terms are used as *general* and *distributive*. Still they do not take the indefinite article—we do not speak of *a* water—because, although distributive and general in the sense of referring to portions of the whole material, still each portion so referred to is thought of as a collection of homogeneous, uniform, and indefinitely divisible parts. It is just because of this divisibility that we have no unitary term for the possible ultimate units of such substances, that the ordinary names of such materials are collective in form, that when we want to refer explicitly to some definite portion of them we have to use such expressions as "a *piece* of salt," "some *kinds* of water," etc.; whereas when the individuals of a group come clearly before the mind as distinct units it is for them individually we use the principal (unitary) term, *e.g. birds*, while if we wish to refer to the group we must have recourse to a secondary and derivative (collective) term, *e.g.* the *bird family*.[1]

30. INTENSION AND EXTENSION OF CONCEPTS AND TERMS. —There are two kinds of signification or meaning involved in *most* of our ideas and terms (36). A concept or term *applies to* or *stands for* ("*supponit*," "*suppositio*"[2]) an object or class of objects, and it *implies* certain attributes which those objects possess in common. The relation of the term to the objects indicated by it has been partially examined in the preceding section. There also we saw that a general term is applicable to an indefinite number of objects—actual or possible—*because it implies some attribute* (or group of attributes) which they are understood to possess in common. This implication of attributes we must now examine more in detail, comparing it with the application of the term to the things possessing the attributes. The latter aspect or reference (the *applicational*) we propose to call the *Extension*, the former (or *implicational*) the *Intension*, of the term or concept.[3]

31. SUBJECTIVE, OBJECTIVE, AND CONVENTIONAL INTENSION; OR, CONTENT, COMPREHENSION, AND CONNOTATION, OF CONCEPTS AND TERMS.—These expressions, which are loosely used as synonyms, we shall find it convenient to employ in distinct, though allied, senses. For, if we examine the connexion there is between a concept or term and the attributes brought to

[1] VENN, *loc. cit.*

[2] For history of the term "*suppositio*," see JOSEPH, *Logic*, pp. 14, 140.

[3] *Cf.* KEYNES, *Formal Logic*, ch. ii., whose treatment is largely followed in this section. Other synonyms for *Application* are *Extent*, *Sphere*, *Breadth*, *Scope*; for *Implication*, *Intent*, *Depth*, *Force*.

mind by it, we shall find room for three distinct ways of understanding this connexion.

First, there is the sum-total of attributes brought up before the mind of any individual by the presence in his mind of the concept expressed by the term in question: the whole *content* of his mental state, whatever way this content may be directly or indirectly implied, or suggested by psychological association of ideas with the mention of the term. This quantity is essentially variable from individual to individual. To no two individuals does the mention of a term or the presence of an object of any class, bring up exactly the same *subjective*, mental state. This variable group of attributes forms the *Subjective Intension* or *Content* of the notion or term. It is of little logical importance, because logic deals not with what is subjective and variable in thought, but with what is objective and fixed. In its most scientific form, the *content* or *subjective intension* of a concept would be the group of attributes *generally known* at any given time, by people acquainted with the class, to be actually common to all its members.

Secondly, there is the sum-total of all the attributes *de facto* common to the objects referred to, whether these attributes be known or not. The mental concept that would embrace or *comprehend* all these may be regarded as the ideal at which our knowledge of things ought to aim. Here the standpoint is *objective;* and this *Objective Intension* of the concept or term we will call its *Comprehension.* Beyond the content of each individual's notion of the objects in question—assuming that content in each case to be correct as far as it goes—the comprehension would usually include other yet unknown attributes: unless we could venture—as we cannot—to say of any one class of object, that our knowledge of it is absolutely perfect and complete.

Finally, between the subjective and objective groups of attributes we can conceive an intermediate group containing *those qualities and those only which are regarded as essential to the objects, in the sense that the name would not be given to any object found wanting in any one of those qualities.* Such attributes are regarded as constituting the *definition* of the objects in question. No individual would be regarded as a member of that class of objects without all of those attributes. The fixing of this group is undoubtedly guided by reference to the objects themselves, to their *generally known* attributes; but it is to a certain extent *conventional*, though not of course arbitrary. And the fixing of

such groups to determine the application of general terms is obviously an absolutely necessary condition for the utility of language as a means of intercourse, and for securing uniformity of thought behind uniformity of language. This group of attributes, conventionally determined by competent authorities in each department of knowledge—not so much by any conscious act or compact as by tacit agreement in usage—forms the *Conventional Intension* or *Connotation*[1] of the concept or term.[2]

32. FIXITY AND LIMITS OF CONNOTATION.—It is, as we have said, a necessary assumption for the accurate communication of thought by language that terms should be understood to convey the same meaning[3] to the minds of all. And indeed language is a special help to thought precisely because of the unquestionably powerful influence exerted by the term in making *fixed* and *definite* at least some elements of the living, palpitating, vaguely outlined and ever-varying content of the concept in the individual thinker's mind. This is the aim in fixing connotation. Logic imperatively demands that, if truth is to be secured, the connotation of the terms employed in any process of inference or proof remain unchanged throughout. But this is the most it can hope to secure. An ideal language would have absolute fixity of connotation. But the language we use is a plastic medium: it is subject to gradual processes of *generalization* and *specialization* in the application of its general terms: these may have not quite the same connotation in technical use and in everyday discourse:[4] moreover, in new departments of knowledge the connotation of new terms must be at first vague and elastic; and the progress of science in a particular department is bound to react on the received connotation of its terms and more or less to alter this latter. For all these reasons connotation is not absolutely rigid, but more or

[1] For historical note on the use of the terms "connotation," "connotative," see JOSEPH, *op. cit.*, pp. 140-42.

[2] "For anyone who is given the meaning of a name but knows nothing of the objects denoted by the name, subjective intension coincides with connotation. Were the ideal of knowledge to be reached, subjective intension would coincide with comprehension."—KEYNES, *op. cit.*, p. 26, note.

[3] When we speak of the *meaning* or *signification* of a term or concept, without further qualification, the reference is as a rule to the *implicational* meaning, to the *attributes connoted* by the term; for these form the fixed standard of meaning: and it is this that secures identity of *denotation* for the term in the minds of all.

[4] Of course where a term has some special signification or meaniug attached to it in some science, clearly different from the commonly accepted sense, it becomes *equivocal*, or equivalent to two terms: *e.g.* "accident" as opposed to "substance" in philosophy, and in the usual sense of "He met with an *accident*".

less variable. This variation, however, unlike that which attaches to subjective intension, is only accidental. And arising as it does from inherent and inevitable defects of clearness in language and thought, it need not prevent us from demanding that as far as possible the connotation of our concepts and terms be kept unchanged throughout our reasoning processes.

We may next inquire on what principles the attributes that form the connotation of a term are selected. What are to be the limits of the group? There has been a great deal of controversy and not a little confusion as to whether connotation should include *all* the attributes, known and unknown, actually common to all the things denoted by the term—or all known to *each individual mind* to be common—or all known *by men generally* to be common—or only some of the latter, namely, those fixed upon by competent authorities and understood to be the test of classification in the sense that no individual thing will be recognized as a member of the class in question without possessing all of them. But from the distinctions we have indicated between content, connotation and comprehension, it will be seen that the last alternative just mentioned gives the *connotation.* The attributes included in this are, of course, those most *directly implied* by the name, those understood to be most *fundamental* and important in the *nature* of that class of objects; not those that are only *indirectly* implied, in the sense that they are *properties* necessarily connected with, and derivable from, the more fundamental ones directly implied; nor those that may be known by some people or by all people to be actually common to all members of the class, though having no apparent connexion with the fundamental attributes, and which may be merely *suggested* by the mention of the name through some subjective association of ideas.

Thus, for example, we take the connotation of *equilateral triangle* to be "plane, rectilinear figure with three equal sides"; we do not include the property of being "equiangular," nor any other of the innumerable properties brought to light by the geometrical study of such triangles. We take the connotation of *man* to be "rational animal"; we do not include the attributes of "cooking his food" or "using tools," though these are properties that follow from his rationality; nor the attribute of "two-legged," though this is *de facto* naturally common to all men. Similarly, though all known "ruminants" are "cloven-hoofed," we do not regard this attribute as essential to the ruminant, nor include it in the connotation of ruminant; nor the attribute of "Australian" in the connotation of "kangaroo". And although the notion of "wearing a soutane" is inseparably associated in our mind with

the notion of "Maynooth student," yet we do not include that attribute in the connotation of the latter term.

We have described the general term as applicable to an indefinite number of things *in the same sense*, *i.e.* in virtue of some attribute or attributes which make up the *sense* or *implicational meaning* or *connotation* of the term, and which those objects possess in common. It is clear, therefore, that the term or notion *stands for* or *denotes* or *applies to* those objects *in virtue of its connotation*: in other words, that, *logically*, *the connotation precedes and determines the extension or denotation* of the term or notion. As a matter of fact, however, we find both aspects of meaning already determined for us in the current usage of the language we speak, so far as the very vast majority of our general terms are concerned. But if we ask ourselves whether, in the actual progress of knowledge in any department—in the definitions, divisions, and classifications of things, and in the selection and use of appropriate nomenclature and terminology which such progress always and necessarily involves—connotation is fixed antecedently to denotation, we shall find that both processes usually go hand in hand and react on each other: that usually we first select a small number of objects (denotation) presenting some striking feature in common (connotation), and fix on this latter as the test for including in the newly formed class, or excluding from it, subsequently discovered objects: that further knowledge of this and kindred classes may induce us to include in this class some objects that will involve a narrowing of the originally fixed connotation of our classname, or to exclude from it, by increasing the connotation, some objects originally included in it.

The *applicational* side of the concept and term can now conveniently receive a more detailed notice.

33. Extension, Denotation, Application of Terms: Realm of Denotation: Universe of Discourse.—By the *Extension* or *Denotation* of a concept or term is understood *the number of individual objects for which it actually stands and to which it actually applies in the same sense*, *i.e.* with the same connotation. For example, the number of real beings to whom the term *man* is correctly applicable—all living human beings, past, present, and future—form the extension or denotation of the term *man*; all the beings believed really to exist in the past, present, or future, to whom the term *angel* is correctly applicable, form the extension or denotation of that term; all the dragons ever spoken

or written of or alluded to by those who believed in their real, physical existence, or by any others who ever discussed such accounts, form the extension or denotation of the term *dragon;* all the different fairies that ever have been or ever will be described or referred to by writers of fairy-tales or others—whether these believe or not in the real existence of such entities—form the extension or denotation of the term *fairy;* all the entities ever referred to or yet to be referred to under the name of ghosts, and to which the term would be understood to be accurately applied, form the extension or denotation of the term *ghost;* all the objects to which the term "highest mountain in Asia" can be accurately applied (obviously only *one*) form the extension or denotation of that term; all the figures or outlines of material things, past, present, or future, to which the term "circle" may be correctly applied, form the extension or denotation of the term *circle.*

These various examples have been purposely chosen from different spheres or realms of the objects of our thought in order to emphasize a first important fact in connection with extension or denotation: that it *must always be sought in its appropriate realm or sphere.* This may be—and of course most frequently is—the sphere of *material* real things, existing in time and space, as in the example of *man;* or it may be a sphere which some believe to be real though not material, and which other unbelievers in its reality consider to be an unreal sphere created by the belief of the former class, as in the examples of *angel* or *spirit;* or it may be a realm once believed by some to be real, though now universally admitted to be only imaginary, and thus created by that belief, as in the instance of the *dragon;* or it may be any one of the purely fictitious worlds of romance, poetry, heraldry, fairyland, created by the inventive imagination of man, as in the example of *fairy.* It must, however, be a realm which is not only present to, but also independent of, the individual thinker's actual thought, and to which an appeal can be made to verify his judgments about the things therein (*cf.* 80, 123).

Furthermore, *actual* membership of a class belonging to any such realm is something different from mere logically *possible* membership of that class. In other words, those various spheres of objects are all distinct from the sphere of the *purely possible*, of that which is merely conceivable by any individual thinker's mind; they are *actual* spheres, not merely *thinkable* spheres. The *condition* for actual membership of a class in one or other of those spheres may be either visible, material existence; or real, though

immaterial existence ; or existence in a sphere created by the imagination of a novelist, poet, or storyteller ; but it is always something over and above the mere logical existence of the object before the minds of those who think of it.

What the *appropriate realm* is, in which the objects denoted by a term are to be sought, either the connotation of the term itself, or the context in which it is used, will tell us. The connotation usually includes some attribute which serves to indicate or reveal the realm within which the objects are to be sought, to which the term is correctly applicable: *man*, the sphere of material reality ; *dragon*, the sphere of writings and traditions referring to fanciful beliefs ; *lion rampant*, the domain of heraldry, etc. When the connotation leaves this ambiguous, the context in which the term is used in rational discourse will generally reveal the appropriate realm and remove all ambiguity.

But there is in the next place this very important consideration, that in ordinary rational intercourse, people, when employing concepts and terms as subjects of their judgments and propositions, do not usually have in mind the whole realm of objects to which those concepts and terms are correctly applicable, but only a more or less limited portion of this realm. And such limitation is usually tacit, not expressed. Using general terms, we are constantly taking it for granted that people will understand their application as limited by time, place, and other such circumstances. When we say, "Everybody is talking about it," we usually mean not everybody in the world, but all within certain more or less narrow limits of space. If we make such statements as that "Europeans are short-lived," we shall be understood to refer not to all European living things but to European *human beings* only.

Now this *more or less limited sphere, within which people tacitly understand their statements to apply*, is called the UNIVERSE OF DISCOURSE. It may be only a portion of the whole realm of things within which the term would find correct application, or it may be identical with this realm. Since its extent is not determined by the connotation of the term, it is not easy for logic to deal with it ; yet logic must demand, in the interests of truth—as in the case of connotation—that these conventional limitations should not vary throughout a given reasoning process.

As in regard to intension, so now in regard to extension, we shall find it convenient to retain for the latter word the wider or generic meaning of

applicational reference simply; to call the *objective extension* of a term—*i.e.* all the actual individual things to which the term is *correctly applicable*[1]—by the name of *denotation;* and to describe its *conventional extension*—*i.e.* all the objects to which the term is actually applied by a given speaker in a given context within a possibly restricted universe of discourse—as forming the *application*[2] of the term (corresponding roughly to what ancient logicians called *Suppositio*[3]).

The multitude of *purely, intrinsically, logically possible* or *conceivable* objects in any realm is simply *indefinite.* And although there is an intelligible sense in which we may conceive a proportionate variation in the indefinite multitudes making up such purely possible classes—the indefinite multitude of possible men being less than the indefinite multitude of possible animals, or than the indefinite multitude of possible human hands, and these ten times less than the indefinite multitude of possible human fingers, and so on,—yet we do not see the utility, in logic, of giving any special title, as Dr. Keynes does (*op. cit.*, p. 30), whether *extension* or *denotation*, to the range of such purely or intrinsically possible objects of thought. These titles should be reserved for things supposed to exist actually in some—real or imaginary—realm, at some—real or imaginary—time, and to be, at the time of predication, at least *adequately capable* of existing in their proper realm in the sense that if they either actually have not existed or do not exist, they at least *will* (and not merely *may*) exist in it. The word "exist" here evidently implies mere membership of a class in any realm, real or imaginary.

34. Relation Between Intension and Extension.—Since intension refers to attributes and extension to the objects possessing those attributes, it is obvious that, generally speaking, according as the former is increased the latter is diminished and *vice versa* in any related series of general concepts; that, for instance, in the series of concepts expressed by the terms, figure, plane figure, rectilinear plane figure, quadrilateral, parallelogram, rectangle, square, the intension gradually increases while the extension decreases; and that in the series, man, animal, living thing, material thing, thing, the intension progressively decreases while

[1] Dr. Venn (*op. cit.*, p. 178) limits *denotation* to the objects actually existing at the point of time at which the term is used: so that the names of extinct animals would have *now* no denotation; while that of the term *man*, for example, would change with every human birth and death. We prefer not to make such a limitation.

[2] Thus, what the term *denotes*—its *denotation*—depends on, and is the correlative of, its *connotation*, and is *objective* in the sense that it is determined by this latter and cannot be limited by any further subjective or conventional arrangement made or implied by the speaker; while the *application* of the term—the collection of objects to which the speaker intends actually to *apply* it within a restricted universe of discourse, is *conventional* in the sense of being expressly or tacitly determined by the speaker. This usage differs from that of Dr. Keynes (*op. cit.*, p. 30), who gives the title of "subjective extension" to "the whole range of objects real or imaginary to which the name can be directly applied, the only limitation being that of logical conceivability."

[3] *Cf.* Joseph, *op. cit.*, p. 14.

the extension increases. As a general rule, therefore, the less a name implies the wider the group of things to which it applies.

But this inverse variation is, of course, not to be taken in the mathematical sense of numerical variation in the proportion of two quantities. For, in the first place, we cannot measure mathematically the intension of a concept, though we may speak of restricting or enlarging it. In the second place, even did we try to enumerate exactly—as so many distinct and individual entities—what we are pleased to call the "attributes" that constitute intension, we should not necessarily in every such case interfere with the denotation by the addition or subtraction of an attribute or attributes to or from the connotation. For instance, we do not lessen the denotation of "man" by increasing the connotation to "*mortal* man," nor do we increase the denotation of "*cloven-hoofed* ruminant" by dropping from the connotation the attribute "cloven-hoofed"; for these attributes are common to all the members of the classes in question. And in the third place, even when the addition or subtraction of attributes which are not common to the whole class does change the denotation of the original concept, this change is much greater in the case of some attributes than in the case of others. For instance, by adding to "man" the attribute "white," we diminish the denotation considerably; but by adding instead the attribute "red-haired," we diminish it much more.

What is true therefore is this, that *if connotation be increased or diminished, denotation will either remain unaltered or will change in the opposite direction.*

Similarly, if the denotation of a given class be arbitrarily increased by annexing to it another class whose members do not possess all the attributes essential for membership of the first class, the connotation of the concept of the larger class thus formed will be necessarily smaller—the number of common attributes fewer—than in the concept of the original class. If to the class of horses we add the class of apple-trees, the concept of the new class (living organisms), of which horses and apple-trees are alike members, will be much poorer in connotation than that of either of the original classes: for, to say an object is a living organism is to convey much less information about it than to say that it is a horse, or an apple-tree, as the case may be.

Of course the relation between a class and its sub-classes (a *genus* and its *species*) is not exactly the same as that between a class and the sum-total of

individuals which constitute it. The former relation is between universal concepts only: it brings out the contrast between the unity and the variety in a general notion. The latter relation is between a general notion and the individual instances of which it is verified. Mr. Joseph rightly draws attention to the distinction between the two kinds of relation. But when he says of the latter relation that "what is meant by the common term predicated of them all [the individuals] remains the same,"[1] he means to imply that this is not so when we predicate a generic notion about a specific notion. But it is so: when I say "the horse is an animal" and "man is an animal," the predicate "animal" has the same implication in both propositions, no less than it has when I say "Bucephalus is an animal" and "Socrates is an animal".

Describing the relation of a class to its individuals as *denotation* and that of a class to its sub-classes as *extension*, Mr. Joseph contends further that the doctrine of any sort of inverse relation between *extension* and *intension* is unsound:[2] that the wider term is not the poorer in meaning but rather the richer, inasmuch as it brings before the mind the rich choice of possible alternatives involved in the known variety of sub-classes latent in the unity of the higher class itself. But this wealth of *indefinite* meaning constitutes what we have called the *subjective intension* or *content* suggested to the individual by the use of the term in a given judgment or context, not the *definite* meaning which we have called *connotation*. Extension is not connected inversely with the former (31), but it is with the latter.

35. ABSTRACT AND CONCRETE TERMS.—The next question that suggests itself is whether *all* our concepts and terms have intension or connotation and whether all have extension or denotation. We shall find it convenient, however, to refer first to another division of terms: the division into *Abstract* and *Concrete*. We have been speaking hitherto about "things" (or "objects") and "attributes" in connexion with intension and extension. We must now see what is the meaning of each of these words; for a *concrete term* is usually defined as *the name of a thing;* an *abstract term* as *the name of an attribute considered alone or apart from any thing*.

It will be seen presently that this division is primarily applicable to terms or names rather than to concepts, that it is more grammatical than logical, and that it is of importance in logic only indirectly—by the light it throws on the intension and extension of concepts and on the nature of the mental act of judgment. Here we have first to observe that an abstract term—*generosity*, *humanity*, for example—is not so called because the idea it expresses was arrived at by a process of mental abstraction. For we have seen already (4, 5) that all universal ideas, and therefore all general terms—concrete no less than abstract—are the outcome of

[1] *op. cit.*, p. 139. [2] *op. cit.*, pp. 123 *sqq.*

that process. The reason of the designation is that when the mind forms the concept expressed by the abstract term, it holds its object, *i.e.* the attribute or group of attributes it is considering, *apart from the individual thing or things* which were revealed to it in sensation, and in which it found those attributes in the first instance; whereas when it expresses its concept by means of a concrete term—*generous*, *human*, *man*—it is considering the attributes which make up the intension of that concept, not apart from, but as embodied in, the existing individual *things* revealed to it either directly or indirectly by sense experience. From this it follows that all adjectives are concrete; for they are not the names of attributes *considered apart* from things (*yellowness*), but rather of attributes *as existing in things* ("gold is *yellow*, *i.e.* a yellow *thing*").

By protracted analysis of our ideas of things "we may obtain higher and higher *abstractions*, each of which may be considered, *by comparison with those from which it was derived*, as being *abstract;* and, in turn, *when compared with those derived from it* may be considered *concrete*. 'Party spirit' might be reckoned an abstract quality of a political party;—which is itself by no means so concrete an entity as one of the persons composing that party. The *virulence* of that party spirit may again be reckoned as an attribute derived from the spirit itself, and so on. The fact is that hardly any object, *as objects are regarded by us*, can be selected, which is not to some extent a product of our powers of abstraction, and the more or less of this faculty called into play in any particular case hardly warrants us in labelling the instances respectively with such distinct designations."[1] In fact so far as our *concepts* are concerned, these are *all* abstract in the sense that they are formed by repeated efforts of abstraction: even the concepts which we express by individual significant names, or which we associate with proper names (28), are groups or syntheses of *abstract attributes*, and are therefore *potentially* universal, though *actually* restricted to a single individual.[2] But we may certainly think of all such attributes *either* apart, in themselves, *or* as existing in, and forming or constituting, some individual existing being or thing;[3] and we may have different terms to indicate the same

[1] VENN, *op. cit.*, p. 190 (italics ours).

[2] See VENN, *op. cit.*, pp. 163, 167, 175, for the sense in which intellectual concepts can be said to be singular.

[3] *i.e.* the οὐσία πρώτη of Aristotle, the *Substantia Prima* of the Scholastics—the individual, or first underlying subject of all *real* attributes and of all *logical* predicates.

object of thought, to express the same concept, in each of these two distinct states.

So long as we confine ourselves to material things and their qualities there is no great difficulty in distinguishing between the "thing" and the "attribute": as *strong* things—*strength; men—humanity; democrat—democracy; envious—envy; square—squareness.* And such terms usually go thus in pairs: the concrete being as a rule the first and more easily understood,[1] and bringing before the mind prominently the individual units characterized by a certain quality; while the latter, when thought of apart from those individual things, is or may be designated by an abstract term. Every general term therefore which directly denotes a class of things, may conceivably have a corresponding abstract term for their common quality considered in itself and apart from those things; though the names of such pairs of terms in any language is in reality comparatively small.

But can we maintain that the distinction between abstract and concrete terms is an absolute and exhaustive one: that as soon as a term is given us we can at once place it in one or other of the two classes, or that the same object of thought cannot be regarded now as a thing possessing attributes and again as itself an attribute of some other thing? May we not say of a man that "he is *brave*" and immediately go on to say that "his *bravery* is physical rather than moral," thus making the (abstract) quality of bravery a subject about which we predicate the further attributes of "physical" or "moral"? We do undoubtedly think and reason about abstract qualities of things just as we do about the concrete things themselves: witness our common use of confessedly abstract terms (all of which are grammatical *substantives*) as subjects about which we affirm or deny attributes.[2] We think of them after the analogy of the things in which we find them: we not only make them subjects about which we predicate further qualities—as when we say that "laziness is demoralizing," or that "honesty is the best policy"; but we even constitute in our thoughts a world of abstract attributes which we classify and generalize after the manner of concrete objects—speaking of different *kinds* or *varieties* of *courage*, of *beauty*, of *colour*, etc.

Hence it has been suggested that we ought to regard as concrete whatever term we use as the name of a *subject of attributes* and as abstract whatever term we use as the name of an *attribute of subjects*.[3] The same

[1] Not necessarily always: some abstracts are easier to grasp than their corresponding concretes. *Cf.* VENN, *op. cit.*, p. 190.

[2] We may call attention here to the existence of a natural tendency to substitute the abstract for the concrete term: to speak of the "Deity" or "Divinity" for "God," a "reality" for a "thing," a "relation" for a "relative," a "nationality" for a "nation," etc. These then sometimes lose their abstract character and the need is felt of coining such double abstracts as "relationship," etc.

[3] KEYNES, *op. cit.*, p. 16.

term might thus be concrete in one context and abstract in another, according to the terms with which it is compared, and the distinction would thus become relative, not absolute: it would be a distinction between the abstract and the concrete *use* of terms. This way of understanding the distinction would attach a simple, intelligible sense to "thing" and "attribute" in logic. It would enable us to classify as abstract or concrete, according to the context, such terms as signify events, processes, states, changes, rather than "things" in the more restricted meaning of material, individual objects: such terms as *equation*, *logic*, *the weather*, *an after-thought*, *action*, *a parliamentary election*, *space*, etc. It would, however, coincide with the use of terms as subjects and predicates of logical propositions. Undoubtedly, logic deals with concepts and terms not so much for their own sake as on account of their place in the judgment and proposition. But when we reflect that the subjects of all our judgments are used *concretely* in the sense just indicated (being used as *subjects of attributes*) *no matter how evidently abstract be the form of the term;* and that the predicates of all our judgments are used *abstractly* in the sense indicated above (being used as *attributes of subjects*) *notwithstanding the evidently concrete form of most of them*—of all adjectives, for example—we cannot help thinking that this way of understanding the distinction does too much violence to the usually accepted sense of the terms "abstract" and "concrete"; besides being superfluous owing to its coinciding with the use of terms as subjects and predicates.

We prefer, therefore, simply to regard as abstract those terms which are *the names of attributes considered alone or apart from the things* from which they are derived—*whether those attributes are regarded as the subjects of further attributes or not.* On the other hand, the *names of things or of classes of things or of attributes thought of as existing in things* (and therefore expressed by adjectives) will be concrete. Although this principle may not be easy to apply in all cases it is at least simple and intelligible, and it is based on the sound Aristotelean division of all objects of thought into *substances* and *accidents* (71). Not that it merely reproduces this latter distinction; for it recognizes as concrete not merely the names of "first" or "individual" substances—the *substantiae primae*—but also of common natures —*substantiae secundae*—as well as of all accidents considered as inhering in the individual substances. Were we to define an abstract term as the name of an attribute *in whatever state the latter be thought of*, we should classify adjectives as abstract.[1]

Owing to the fact that the adjective is not properly the name of an attribute, nor properly the name of a substance or subject, it has been placed by some logicians neither among abstract nor among concrete terms, but in a class apart, and called an "*attributive*" term. But this separation of adjectives from other terms "corresponds to no further distinction in thought"[2]: it regards not the thought-object or thought-term itself but the attribution of it to another thought-object in predication or judgment.

36. HAVE ABSTRACT TERMS EXTENSION OR DENOTATION? NON-DENOTATIVE TERMS.—We may now inquire whether the

[1] *Cf.* JOSEPH, *op. cit.*, p. 26. [2] *ibid.*, pp. 25, 26.

division of concepts and terms into general and singular (28) is exhaustive, or whether perhaps there may not be certain classes of terms which have no extension at all, *i.e.* which do not denote *things* at all (30). *Extension* in general we have understood to mean the reference of our concepts and terms to things. If, therefore, we have *concepts* which represent as their objects *attributes apart from all reference to things*—and we undoubtedly have such concepts, purely abstract and *potentially* applicable to things, though not *actually applied* to things by the mind,—and if we have abstract *terms* which express the attributes conceived in this way, then consistency would demand that we deny such concepts and terms all *extension* and recognize for them *only one kind of meaning, viz. implicational* or *intensive* meaning. It would seem that we cannot with propriety speak of the extension or denotation of such concepts and terms at all—any more than we can speak with propriety of the intension or connotation of a certain other class of terms to which we shall presently refer, and which have only the *applicational* or *extensive* kind of meaning.

Hence we must recognize a division of terms into those which have extension and those which have not: into *Denotative Terms* and *Non-denotative Terms.* And if we compare this division with that given above (28)—into general and singular terms—we can now see that the latter division is not an exhaustive division, since it can have reference only to terms that have extension, *i.e.* to *denotative* terms, not to non-denotative ones.

When logicians discuss the question whether abstract terms are singular or general, they do not imply any reference of these latter to *things:* they are thinking exclusively of a sphere or system of abstract attributes (35), calling those attributes *general* which they can analyse into different kinds or varieties, and those others *singular* which they cannot further analyse. So, *colour*, *virtue*, etc., would be *general*, while perhaps *squareness*, *yellowness*, *equality*, might be instanced as singular. It will be noticed, however, that this distinction into singular and general, as applied to abstract terms, has not the same meaning as it had above (28): there it concerned exclusively the reference of our concepts and terms to individual *things;* here it refers to the relations of abstract attributes to one another. And, furthermore, *denotation* is the reference not of a wider class to its *sub-classes* or *varieties*, but to all the *individual members* of that wider class. For this reason also the use of the term *denotation* in the present connexion is inappropriate.[1]

We see no sufficient reason to deny that concepts of *purely possible* or *logically conceivable* objects of thought—such as the objects thought of in pure mathematics—are *general* (as also the terms we may invent to express

[1] *Cf.* Joseph, *op. cit.*, p. 135.

them), though we have not given their indefinite applicability the name of extension or denotation (33).[1]

37. ARE ABSTRACT TERMS AND PROPER NAMES CONNOTATIVE? NON-CONNOTATIVE TERMS.—Dealing next with the reference of our concepts and terms to attributes we may ask: Have all concepts and terms intension or connotation? There are two classes of terms about which there may be room for doubt: (*a*) abstract terms, and (*b*) proper names.

(*a*) Seeing that the direct and in fact the *only* reference of abstract concepts and terms is to attributes, we should have no hesitation in saying that they *have* content, connotation, comprehension—that they have, in fact, no other sort of meaning but *implicational* meaning.

We would say, therefore, that these have *implication* or *intension*, but not *application* or *extension*. Furthermore, if we use the word *denotation* for reference (of our concepts and terms) to *things*, and *connotation* for reference to *attributes*, we must say that abstract terms *have connotation* and *have not denotation*. Although, however, the expression is appropriate as regards denial of denotation for abstract ter ms do not *denote*, or *note down*, *point out*, or *stand for*, *things*—it is awkward as regards connotation—for, etymologically, to *connote* means to *note* something *together with* or *in addition to* something else *denoted;* and there is no "something else denoted" in the case of the abstract term. We doubt, however, if it improves matters to say that abstract terms are non-connotative because they *denote* the attributes connoted by the concrete terms, and have themselves nothing left to connote.[2] This usage endows the same word, *denotation*, with a twofold reference—to attributes and to things.

Other logicians, distinguishing, as above (36), between general and singular abstract terms, hold that the latter are non-connotative, the former connotative. Such abstract terms as *shape*, *colour*, *virtue*, etc., being general, would be likewise necessarily connotative—each *denoting* the different discernible *kinds* of shape, colour, virtue, respectively, and *connoting* the common attribute (or group) by the possession of which these kinds belong to the same genus or class. The distinctions which arise in our thoughts about things are thus carried by analogy into our thoughts about attributes.

(*b*) We have assumed in what we have just been saying that a *Connotative Term* is *one which implies attributes* whether it denotes a subject or not,[3] and that a *Denotative Term* is *one which denotes or points out a thing or things* whether it connotes attributes or not. We have seen that in this sense abstract terms are

[1] *Cf.* VENN, *op. cit.*, p. 178. [2] JEVONS, *Principles of Science*, chap. ii., § 3.
[3] The words "in addition to denoting a subject" would convey the more commonly accepted definition.

non-denotative. Are there any terms which are non-connotative —whose sole function is to point out or stand for things, without any reference to attributes? All concrete general terms and all significant individual terms not only denote subjects but also imply attributes and are therefore connotative. About *proper names*, however, there has been much dispute—owing to a confusion of the three distinct varieties of intension referred to above (31).

Every term which points out a thing to us and enables us to identify it, must convey to our minds, by some psychological process of suggestion or association of ideas, one or more attributes: and must, therefore, have *subjective* intension, or *content.* This, therefore, is true even of the proper name. The latter undoubtedly *suggests* all the attributes associated with the habitual application of it to the individual that bears it.[1]

Furthermore, since nothing is or can be entirely destitute of attributes, and since their sum-total, known and unknown, forms, in each case, the *comprehension* of the thing denoted by the name, it follows that even proper names have *comprehension.*

But since a proper name is not given to an individual *by reason of any attributes* possessed by that individual, since its application to the individual possessing it is not determined by selecting any of those attributes and fixing on them as implied by it, but rather by pointing out the individual possessing the name, it follows necessarily that such names must be regarded as *non-connotative.*[2]

Of course it is often difficult to draw the line between what a proper name *suggests* and what it might perhaps be claimed to *imply.* The difficulty is not so great in the case of such proper names as "Lake of Lucerne," "Major Jones," etc.—the first part of each of these being manifestly connotative, *i.e.* applied to the individual in question because of the possession of a certain attribute; the second part being non-connotative. Nor does any real difficulty arise from the fact that most if not all proper names were *originally* applied to individuals by reason of certain attributes possessed by those individuals. The names were *then* of course significant or connotative. "Maynooth" meant "The Plain of Nuad". The first "John Smith" was probably so called because he plied the trade of smith. "Wednesday"

[1] All the known individualizing attributes or characteristics are regarded by Mr. Joseph as *meant* or implied by the proper name, which he therefore holds to be connotative (*op. cit.*, p. 138).

[2] In a certain sense proper names and designations are not terms at all—inasmuch as their mental equivalent is not properly an intellectual concept (which is always theoretically abstract and universal) but rather a mental combination of the abstract concept with a percept or imagination image of an individual.—*Cf.* JOSEPH, *op. cit.*, p. 67.

originally meant the day dedicated to the worship of Woden. The first "Godfrey" may have been so called because he was considered to possess in a marked degree the "peace of God". And so on. But we must not confound the *etymology* or *history* of a name with its connotation. No matter what may have been the origin of the name or the reason for which it was assigned to an individual in the first instance, as soon as it came to be regarded merely as a proper name, and to be used and applied as such, its application became independent of its original meaning. In other words, its connotation was dropped or lost: it is now no longer applied to an individual on account of any attributes he may or may not possess; although it may and invariably does suggest attributes when so applied.

In favour of the view that proper names are connotative it might be argued that the proper name "John Henry Cardinal Newman," for instance, not merely suggests but implies or connotes (*a*) that the bearer held a high office in the Catholic Church; (*b*) that he was an organic-living-human-being of the male sex; (*c*) that his father's name was Newman, (*d*) and probably that he had ancestors named John and Henry. We may admit the first point (*a*). But though we may feel quite certain about the second (*b*), the name does not *signify* or *imply* it. The name might, not without impropriety, but yet without any necessary deception, be given to a racehorse,—as *Victoria Nyanza*, for instance, is the name of a lake. Nor, *a fortiori*, is identity of name in (*c*) father, or (*d*) ancestors, necessarily implied in the use of such a name.[1]

The application of a purely proper name being determined by pointing out the individual, it follows that such a name is unintelligible—*i.e.* has no sort of meaning—for any one who hears it without having pointed out or otherwise revealed to him the individual—person, place, or thing—to which the name is applied.

38. Opposition in Concepts and Terms: Kinds of Opposition: Material Contradictories.—Terms which *imply attributes that cannot co-exist in the same subject* are called *Incompatible Terms*. We may distinguish various degrees of incompatibility: (*a*) contradiction, (*b*) contrariety, (*c*) simple repugnance.

Two terms may be so opposed as to be *mutually exclusive in their connotation and collectively exhaustive in their denotation.* Neither will be applicable to any object to which the other is applicable, and both between them will exhaust the whole common sphere of their denotation. This is called *contradictory opposition*, and such terms are called *contradictories.*

This opposition may be discovered in two ways: either from a knowledge of the connotation and denotation of the terms in question, *i.e.* of the *matter* to which they refer, or by mere inspection of the *form* of the terms. Hence two kinds of contradictories: *material* and *formal.* Material contradiction can be discovered only by an examination of facts. The two terms themselves do

[1] *Cf.* Keynes, *op. cit.*, pp. 45-47.

not reveal it in their form. Both have positive connotations and stand on equal and independent footings. They always possess a certain amount of connotation in common—that portion, namely, which indicates their common sphere of denotation.[1] In addition to this, each includes an attribute (or group of attributes) peculiar to itself and such that every individual denoted by the terms must possess one or other of these attributes (or groups) while no individual can possess both.

A few examples will illustrate this. "Male" and "female" are such a pair of material contradictories, possessing in common the connotation which fixes their sphere of denotation as that of "living organisms with differentiation of sex"; "matter" and "spirit" possessing the common connotation of "substance"; "substance" and "accident," possessing the common connotation of "being"; "Irish" and "foreign," for the sphere of "material products"; "Irish" and "alien," for the sphere of "human beings". We may observe that as the common portion of the connotation increases and the sphere of application grows narrower, the possible numbers of such material contradictories within this narrower sphere increase. For, it is easier to find a pair of positive attributes which divide a smaller class, than a pair which divide a larger class, into two mutually exclusive sub-classes.

39. FORMAL CONTRADICTORIES: POSITIVE AND NEGATIVE TERMS.—While a *positive term* may be defined as one *which implies the presence of an attribute or group of attributes*, a *negative term* is *one which merely implies the absence of* (all or some of) *the attributes connoted by the positive term, while it implies the presence of no attributes whatsoever.* It is formed from the positive term by prefixing *not* or *non* to the latter; it is called the formal contradictory of the latter. The opposition between a positive and a negative term—*man* and *not-man*, *holy* and *not-holy*, etc.—is called *formal* contradiction because it can be learned from the *mere form* of each pair of such terms, independently of their meaning altogether.

Now, while both members of a pair of *material* contradictories, *e.g.* "male" and "female," have *positive* connotation, one member of a pair of *formal* contradictories—the purely negative term—has no positive connotation at all. And as to denotation, while the sphere exhausted by a pair of material

[1] The context often comes to the aid of their common connotation in determining the common *Universe of Discourse* within which they are meant to be actually applied (33).

contradictories is always more or less limited, that exhausted by a pair of formal contradictories is supposed to be coextensive with all being: "non-man" is supposed to denote *all things* other than men. Whether we ever *de facto* use such purely negative terms and concepts in our actual thought and speech may be seriously doubted. It has even been contended by some logicians that they *cannot* figure as genuine terms in our thought: that they stand for no *reality* but only for a mere creation of formal logic, an *ens rationis:* that they are a perverted way of expressing the logical relation established by the mind between subject and predicate in the negative judgment. We certainly cannot deny that it is only in connexion with *judgment*, with *predication*, with affirmation and denial, that the division of terms into positive and negative can become intelligible at all: contradiction is primarily between judgments, not between terms.[1] The very use of the word "negative" reminds us of negation, and negation is judgment. Taken in itself, apart from judgment, what can the idea or the term "not-white" imply or denote? Not certainly the absence from the mind of the idea "white": on the contrary it involves the presence of the latter in the mind. "Not-white" is not, therefore, the pure negation of the idea "white". Nor again can its object be everything that is absent from the mind thinking of white; for certainly an idea cannot stand for what is merely *absent* from the mind. But perhaps it has for object "everything that *is not white*," whether present to the mind in any more explicit way or not? That is to say, when the mind thinks of "white things" or of "things possessing the attribute of whiteness," perhaps it can do so only by mentally distinguishing them from "things that are not white": so that every positive mental concept would divide the realm of its denotation into two mutually exclusive departments or classes, a class of which the concept can, and a class of which it cannot, be predicated? And would not the term "not-white" come in this indirect way to denote all the members of the latter class—to stand for "whatever is not white"?[2] If this be so, the mind has reached the concept "not-white" only through a process of implicit judgment: a process of conceiving objects which have a certain attribute, *denying* it of all other objects, and holding these latter together only by this purely negative bond of denial. But to arrive at such an indefinite denotation for "not-white," it is not necessary, as Sigwart seems to imply,[3] to have gone through an indefinite series of judgments. From the perception of even one single "not-white" thing, we can abstract the general concept: "whatever is not white".

Purely negative terms, thus understood to include in their denotation everything thinkable except the things denoted by the corresponding positive term, have been described as *infinite* or *limiting* terms, and the propositions in which they are predicates, *infinite* or *limiting* propositions.

It is not true to say that such terms are utterly meaningless and inconceivable, expressive of no real concept, on the ground that it is impossible to

[1] Two contradictory terms looked at in themselves, apart from their relation to judgment, have been called by Dr. Keynes *complementary* terms, as exhausting the sphere of denotation between them (*op. cit.*, p. 162).

[2] "While it may be said that *A* and *not-A* involve *intensively* only one concept, they are *extensively* mutually exclusive,"—Keynes, *op. cit.*, p. 58.

[3] *Logic*, p. 135, quoted by Keynes, *loc. cit.*

hold together in one mental synthesis, in one concept, such a chaotic jumble of not-white "entities as virtue, a dream, time, a soliloquy, New Guinea, the Seven Ages of Man".[1] It is not impossible to do so. Let the things be as varied and vague and chaotic *to the imagination* as you please, the negative bond of the absence of a certain attribute, "whiteness," is a *perfectly intelligible* bond, and denotes a group of things about which we can think and speak intelligibly. What is to be freely admitted of course is this, that in ordinary thought we very rarely if ever think of such a group: we almost invariably limit the application of such formally negative terms to the universe of discourse indicated by the next highest class of things, the *proximate genus:* by not-white things we usually mean "not-white *coloured* things," by "non-voters" not all things nor even all men with no right to vote, but "all the *inhabitants of a certain town or district or country*" who have not that right. And so on. But such limitation, though always assumed in ordinary thought, is not necessary for intelligibility. It has been argued that such propositions as "sound is not-white," "virtue is not-blue," etc., are meaningless and absurd. It would be more correct, perhaps, to say that they are so obviously true as to be wholly superfluous—seeing that sound and virtue are not coloured things at all. They are not; and therefore it is *a fortiori* true—though needless—to say they are neither white nor blue: "As a rule, it is needless to exclude explicitly from a species what does not even belong to some higher genus. But the fact of exclusion remains."[2]

On another ground, however, we should rightly deny that such purely negative notions and terms as "non-white" are genuine, or deserving of the name of concepts and terms: for this reason, namely, that they have no positive *intension* distinct from that of the positive term, that their only intelligible use and function is in *judgment*, in *predication*, and that the form of negation belongs to the proposition and not to the term. We regard such propositions as "virtue is not-blue" as perfectly identical with "virtue is not blue". That is to say, the terms are the same; in our thought it is invariably the *copula* or *form* of the proposition, and not the predicate, that is affected by the negative. Lotze rightly contends (*Logic*, §§ 40, 72) that "everything which it is wished to secure by the affirmative predicate *non-Q* is secured by the intelligible negation of *Q*". But we see no reason to refuse recognition to the form "A is *non-Q*". He rejects the latter form and consistently denies the lawfulness of *obversion* (117)—the process of passing from, *e.g.*, "spirit *is not* matter" to "spirit *is* not-matter". He admits, however, the *universal* validity of the *principle of excluded middle* (14): an admission which implies that a predicate (such as *blue*) can be intelligibly denied, or its corresponding negative (non-blue) intelligently affirmed, even of things that do not belong to the class (*coloured* things) immediately above those predicates. But if this be so, negative terms used in predication may have an intelligible meaning without being limited to the universe of their next highest genus, or in fact limited at all.

When we deny an attribute of a subject, the reason of the denial is often to be found in the subject itself (97, 98)—in the fact that there are in that subject other attributes incompatible with the one denied of it. For these latter we have often no special name, contenting ourselves with a negative

[1] KEYNES, *op. cit.*, p. 59. [2] *ibid.*, p. 60.

term based upon the denial of the positive attribute. "*In*human," "*dis*agreeable," "sense*less*," "*un*fortunate," "*dis*respectful" are examples. Their denotation is always tacitly confined to some limited universe of discourse: "senseless," for instance, being understood in practice to apply to those things to which "sensible" could be appropriately applied, and not to all things whatsoever —such as distances, week-days, railways, etc.—not endowed with any capacity for "sense". In rare cases—*e.g.* "unequal," "uneven"—the attributes implied by the negative terms are found in all the things of which the positive attributes are denied, so that they remain genuine contradictories of the positive terms, exhausting with the latter the whole sphere of their common denotation. But in most cases they imply more than what is required for the mere absence of the attributes connoted by the positive terms, both thereby failing to exhaust between them the common sphere of denotation. While remaining mutually exclusive they cease to be collectively exhaustive, and hence such pairs of terms are not true contradictories, but are either *contrary* or *simply repugnant*. Between "happy" and "unhappy," for instance, there is the intermediate state of indifference which is the mere negation or absence of the state of positive happiness.

40. CONTRARY TERMS.—Two concepts or terms are said to be *contrary* when they *connote the greatest conceivable degree of difference between the things denoted by them in the sphere within which they are actually applied.* "First" and "last" (in the universe of any order or series), "black" and "white" (in that of colour), "wise" and "foolish" (in that of conduct), "happy" and "miserable," "pleasant" and "painful," are examples of such pairs of contraries. The distinction is obviously a material one—based exclusively on knowledge of facts. Moreover, it is not an exhaustive division of terms: not all concepts or terms have contraries, but only those occupying an extreme position in any given universe, and over against which, at the opposite extreme, there are other and correlative terms. Then, furthermore, between these contrary extremes we may conceive an indefinite multitude of *intermediate* states or positions, all more or less *incompatible* with either extreme, though not contrary to it: unless we use the word "contrary," as some authors do,[1] in the wide sense in which it would include any degree of incompatibility. Hence, too, it will be seen that two contraries are not collectively exhaustive. Between two contraries (in the strict sense) there is always a mean. Therefore, although both cannot be true together of the same thing, they may be false together of the same thing (unlike contradictories), while the truth lies somewhere between them. Though the same

[1] *Cf.* KEYNES, *op. cit.*, p. 63.

thing cannot be in the same respect simultaneously white and black (contraries cannot be true together), it may be neither white nor black (but blue, *i.e.* contraries may be false together), though it must either be white or not be white (contradictories cannot be either true together or false together: *cf.* 112, 113).

41. SIMPLE REPUGNANCE: PRIVATIVE OPPOSITION.—Concepts and terms are described as *simply repugnant*, when, *without being collectively exhaustive like contradictories, or in extreme opposition like contraries, they nevertheless imply attributes which are mutually exclusive*, and cannot, therefore, be simultaneously affirmed of the same object. Thus "red," "blue," "green," are incompatible in this sense; so also "made of wood," "of delf," "of iron," etc. Such objects of thought are also called *disparate*.

This division, likewise, is purely material and of little logical importance. It is sometimes not easy to draw the line between simple repugnance and contrariety. What are called *privative terms* will illustrate this. We may define these as *terms which connote the absence of an attribute in a subject capable of possessing it*, whether this subject might be *specially expected* to have that attribute or not. For example, "lame," "blind," "deaf," "dumb," etc., imply the *privation* of an attribute naturally expected in the subjects of which they are usually predicated. Unhappy, senseless, unfortunate, disagreeable—and all that class of terms referred to above as apparent contradictories—are also *privative* inasmuch as they imply the absence of some attribute which the subject is *capable* of possessing. But these have all come to possess a positive element of connotation, which destroys their formally negative character and makes them positively incompatible with—and sometimes even directly contrary to—the original positive term. Thus "unhappy" connotes the absence of some element essential to happiness, the presence of some elements making the subject capable of happiness, and the presence of some positive element or elements incompatible with the absent one, and producing the positive state of unhappiness. If these latter be multiplied or intensified to the extreme limit conceivable, they make a person not merely *unhappy*, but even the *contrary* of happy, that is to say, *miserable*.

The whole doctrine of logical opposition has reference only to connotative or significant concepts and terms; it can be applied in no intelligible sense to proper names.

42. Absolute and Relative Concepts and Terms.—A *Relative Concept* or *Term* is described as *one which, over and above the class of object it denotes, implies in its very signification a reference to some other class of object* (called *correlative*), so that without such reference the meaning of the concept or term in question could not be understood.

Of course there is in reality not a single thing of which we can have an idea or for which we can have a name, that does not stand in manifold relations to other things—associated somehow or other in our minds, whenever we think of it, with other things. The nature of things and the nature of our minds alike demand this. No actual thing is absolutely isolated and separated from everything else. No object can be thought of except as implying a relation of distinction from all other objects. And no thought can apprehend its object out of all relation to all other objects. But these facts are not sufficient to make the concept or term relative in the sense just defined. Thus the mention of the words "king" and "man" immediately *associates* the two objects in thought, and *suggests* the facts that kings are men and rule men. Nevertheless, we have not here a pair of relatives, or correlatives; for neither concept *implies a reference to the other in its meaning;* each can be understood without the other.

Not so, however, in the case of "king" and "subject": each of these *connotes* a reference to the other, nor can either be understood without reference to the other. So, for example, "friend—friend," "partner—partner," "equal—equal," "near—near," "husband—wife," "father—son," "logical subject—logical predicate". All these, therefore, are relative terms. It will be noted that they always go in pairs, that sometimes both correlatives have the same name, sometimes each a distinct name.

The relation implied in relative terms always arises from some fact or series of facts connoted by both terms alike. These facts constitute what is called the *Fundamentum Relationis*—the basis or foundation of the relation. The series of acts or facts constituting friendship, partnership, paternity and sonship, citizenship and kingship, etc., are the respective foundations of those relations. Whenever the correlatives have different names, so too, as a rule, has the *fundamentum relationis*, according to the side from which we regard it—*e.g.* paternity, sonship.

An Absolute Concept or *Term* is *one which does not imply in*

its connotation or meaning a reference to anything else. For example, man, book, Monday, honesty, Ireland, are absolute terms. In their *subjective intension* or *content* they may *suggest*, by association of ideas, other objects of thought; in their *objective intension* or *comprehension*, too, they may involve innumerable relations with other things; but so long as no such reference is included in their *conventional intension* or *connotation* they are to be regarded as logically *absolute* terms.

VENN, *Empirical Logic*, pp. 160 *sqq.*, 163, 167, 175, 190 *sqq.* JOSEPH, *Logic*, chap. ii., vi. WELTON, *Logic*, I., Bk. I., chap. i. KEYNES, *Formal Logic*, Pt. I. JOYCE, *Logic*, chap. ii.

CHAPTER II.

THE PREDICABLES.

43. WHAT ARE THE PREDICABLES? ARISTOTLE'S FOURFOLD SCHEME.—From the preceding chapter it will be gathered that intension and extension are the two most important characteristics of our ideas of things. It will be remembered also that it is by comparing these concepts with one another in the mental act of judgment that we obtain and formulate consciously our knowledge of whatever we know. It is possible, therefore, to compare any two terms—the subject and predicate of any proposition—with each other both as to their intension and as to their extension. The results of such comparison will give us an enumeration of all the possible *kinds of predication*, or *ways of predicating* one concept or term about another. These relations are called *Praedicabilia*, κατηγορήματα, from *praedicare*, κατηγορεῖν, to predicate or assert. The *Predicables*, therefore, may be defined as a classification of the *relations of predicate to subject in a logical judgment or proposition.* They are called by the Scholastics *quinque modi praedicandi*—the five ways of predicating—and also *quinque voces*—"the five words"—because each of the five relations has got a special name of its own; and, being characteristically logical entities, they have always held an important place in the scholastic treatment of logic. That they are "logical entities," *entia rationis*, will be evident when we reflect that they are not names given to the objects of our direct concepts, but to *relations which the mind*, by reflecting on its direct concepts and comparing these with one another, *itself creates* or establishes between the latter. In other words, they are the names of *secundae intentiones mentis* not of *primae intentiones mentis* (19).[1]

Before explaining the traditional fivefold scheme, first elaborated by Porphyry[2] in his *Isagoge*, or *Introduction to the Categories of Aristotle*,

[1] *Cf.* CLARKE, *Logic*, pp. 165 *sqq.*

[2] A Neo-Platonic philosopher who flourished A.D. 233-304.

we may refer to the fourfold classification implied by Aristotle—who did not explicitly treat of the predicables. It may be summed up as follows:[1]—

(1) When *P* (the predicate) agrees both in denotation and in connotation with *S* (the subject), thus giving all the attributes which fully explain the nature of the subject, its relation to the latter is called *definition* (ὅρος), *e.g.* man is a *rational animal.*

(2) When *P* agrees in denotation with *S* while differing wholly from it in connotation, thus giving some attribute that necessarily follows from the connotation of *S*, the relation of *P* to *S* is described as *property*, *proprium* (ἴδιον); *e.g.* men *use tools.*[2]

(3) When *P* differs in denotation from *S* but partially agrees with it in connotation by giving that portion of the connotation of *S* which the latter has in common with some other things, then the relation of *P* to *S* is called *genus* (γένος); *e.g.* man is an *animal.*[3]

(4) Finally when *P* differs in denotation from *S*, and also in connotation by giving some attribute that may or may not be found in *S*, then the relation of *P* to *S* is called *accidens* (συμβεβήκος); *e.g.* some men are *red-haired.*

The analysis of this four-fold division will be included in our exposition of the traditional scheme of *Porphyry's Five Predicables.* It is briefly summed up thus by Mr. Joseph:[4] "where the predicate of a judgment is commensurate with the subject, there it is either the definition or a property of it: where it is not commensurate, there it is either part of the definition, *i.e.* genus or differentia, or an accident".

44. The Five Predicables: How to Arrive at them. Essence and Connotation.—The following simple consideration will enable us to see that there are five and only five possible relations between the intension of the predicate and the intension of the subject of a logical proposition. The predicate of any affirmative proposition is an attribute (or group of attributes) which (*a*) either belongs to the nature or essence of the subject as connoted by the class name of the latter, or (*b*) does not belong to it. In the former case it will either give us (1) *the whole* nature or essence of the subject, or (2) the part of that nature

[1] Welton, *Logic*, i., p. 78.

[2] If *P* partly agreed in connotation with *S* by giving that part of the connotation of *S* which differentiated *S* from other things that also agreed partially in connotation with *S*, the relation of *P* to *S* would be called *differentia* (διαφορά); *e.g. man is rational.* Porphyry's scheme differs from the present one in making distinct and explicit mention of this predicable.

[3] When *P* gives the portion of the connotation of *S* which differentiates the latter from other things, *e.g.* its *differentia*, this latter will sometimes not have the same denotation as *S*. For example, angels are rational as well as men. Only when the *differentia* is such that it can belong to *S* alone, is it identical in denotation with the latter; and it is only in such cases that we have the ideally perfect *differentia* from the point of view of scientific knowledge. *Cf.* 47, 58, 62; also Joseph, *cp. cit.*, p. 116.

[4] *op. cit.*, p. 59.

which the subject has *in common* with other classes of things (the *pars determinabilis naturae*), or (3) the part which *differentiates* the subject from all other classes of things (the *pars determinans naturae*). In the latter case the predicate must give us (4) either something which, though not regarded as essential to the subject or a part of its nature, is nevertheless *necessarily* connected with that nature; or (5) finally, it will give us something not necessarily, but only contingently, accidentally and *de facto* conjoined or coincident with that nature. There is no other possible alternative. Hence, in every logical judgment which affirms a relation between two concepts, subject and predicate:

the predicate must be either a { 1. *species* (εἶδος) / 2. *genus* (γένος) / 3. *differentia* (διαφορά) / 4. *proprium* (ἴδιον) or / 5. *accidens* (συμβεβηκός) } of the subject.

It will be noted that the results are here reached by examining relations of *intension*, which we know already to be more fundamental in our concepts than their extension.

Comparing our ideas from the point of view of their *extension*, we find (*a*) that the predicate of a proposition may give exactly a whole class coextensive with the subject-class, either (1) in virtue of its giving the whole connotation of the latter (*species*), or (2) the differentiating portion of it (*differentia*), or (3) a *property* necessarily connected with its connotation (*proprium*); or (*b*) that the predicate may give a wider class to which the subject belongs, either (4) by giving the common portion of the connotation of the latter (*genus*), or (5) some other attribute not necessarily connected with the connotation of the subject, but which the latter happens to possess, and which makes it a part or subclass of this wider class (*accidens*).

It must not be forgotten that those various *objective concepts* (or *objects of our concepts*), which we distinguish *mentally* from one another, and between which we apprehend the various relations of *genus*, *differentia*, *species*, *proprium*, and *accidens*, are not so many *really* and *numerically* distinct entities composing any really existing individual thing, as stones compose a heap, or as words are in a book, or as the various organs are in the human body: the individual is *one* existing reality: the mind, by the process of *abstraction* (2), analyses, or breaks up, or distinguishes

in its thought, various aspects of that single reality, and sets up between these aspects the relations we are discussing in the present chapter.[1]

Lest any ambiguity should arise from our use of the traditional philosophical terms, *essence*[2] and *nature*,[3] we may state here that by *essence* we mean simply what we regard as the fundamental constituent of the thing: the attributes we would assign to explain the thing if asked *what the thing is:* what the Scholastics forcibly if not very elegantly entitled the *quidditas* or *whatness* of the thing: the attributes whose presence guides and determines us in fixing the connotation of the name we give to that class of things, and the absence of all or any of which from an individual instance would prevent us from putting it into that class or giving it that name: the attributes we regard as directly implied by that name. In substituting the modern logical word "connotation" or "conventional intension" for the word "essence"[4] in this whole context—regarding predication and definition—we are in no way committing ourselves to the nominalistic view that these processes deal merely with the use of language and not with the nature of things.[5] For we have already seen that the fixing of connotation must always be checked and guided by constant objective reference to the nature of the things named. And furthermore, the Scholastics distinguished clearly between the *individual* essence—the essentia *atoma*[6]—and the *specific* or *generic* essence—the essentia *specifica* or *generica;* the former being the sum-total of reality in the individual thing, the sum of all the attributes, common and proper, possessed by the individual, which would constitute the *objective intension* or *comprehension* of the *individual term;* the latter being merely the sum or synthesis of the attributes which the individual possesses *in common* with other individuals belonging to the same (lowest) class or *kind* (*species*) and possessing the same class name. The latter essence alone they regarded as the *proper object of*

[1] *Cf.* JOSEPH, *op. cit.*, pp. 68, 69.

[2] *Cf.* CLARKE, *Logic*, pp. 170 *sqq.*; MERCIER, *Logique*, pp. 100 *sqq.*

[3] The *nature* is simply the *essence* considered as the root-principle of all the activities and characteristics by which the thing reveals itself to us.

[4] *Cf.* VENN, *Empirical Logic*, p. 271.

[5] See below, 46.

[6] Also called *substantia prima vel individua*—the *first* essence or substance—the underlying subject of all predicates in the mental or logical order, and of all attributes in the real or ontological order (45). We must never lose sight of the fact that it is upon the validity of the relation between the universal concept and the individual reality, as expressed in the *singular* judgment, that the validity of the whole superstructure of our knowledge rests (4-6). Yet, of the individual as such we can have no adequate intellectual concept properly speaking; for *all* concepts are abstract and universal—at least potentially or theoretically universal. Now, the predicables are properly a classification of the possible relations *of universal concepts to one another.* Hence when the doctrine of the predicables is applied to judgments with *singular* subjects, it applies to these latter only in so far as we succeed in forming intellectual concepts of these singulars. Mr. Joseph contends (*op. cit.*, pp. 93-96) that Porphyry abandoned Aristotle's point of view by conceiving the predicables as relations of *universal* predicates to *singular* subjects; and that the change has led to much confusion.

intellectual knowledge; such a group of attributes constituting the *species* or *specific nature*, or *definition*, of the thing. Though their point of view in determining the *specific essence* was objective, it was subject to the same conventions as the fixing of connotation, for they too distinguished between attributes which they regarded as *constituting* the essence, and those other attributes called *properties*, which, although flowing necessarily from the concept of the essence, they nevertheless did not include in the latter. It is somewhat misleading, therefore, to say that "the application [of the word *essence*] has varied through the whole range from objective to subjective, *i.e.* from a necessity imposed upon us by the laws of nature to a necessity arising from conventions imposed upon us by the usages of language".[1] That such necessity does not spring from the forms of thought—or language—alone, the mediaeval Scholastics did undoubtedly hold (15); that it results entirely from conventions of language, very few, we think, would at any time be found to maintain.[2] "The essence of any individual object," so far from being "entirely determined by the name through which we regard it,"[3] *entirely determines* the name we apply to it. Why do we give a thing a certain *class name?* Is it not because our *concept* of the thing includes the attributes we desire to signify by that name? And why does our concept of the thing include such attributes? Is it not because the *thing* has revealed itself to us as possessing those attributes? Because those attributes reveal to us *what the thing is?* As thought precedes words, so do essences precede and determine thought. Were our knowledge of *what things are*—in other words of the *essence* or *nature* of things—merely a knowledge of the meaning of the names we impose upon them, would not all scientific knowledge thereby seem to be reduced simply to a knowledge of the meaning of names? Would it not seem to be a knowledge of "*nominal* essences" rather than of "*real* essences"? Many English philosophers, from the days of Locke (1632-1704), are accused of having thus underrated, if not degraded, the power of the human mind. But it may well be doubted if they seriously *meant* that knowledge is *merely* of names: for, obviously, we fix and determine the meaning of names by *an appeal to things;* and a knowledge, therefore, of the former essentially involves a knowledge of the latter.

The process of determining accurately the constituent factors of the real essences of things, is, of course, a difficult process. It is the work of the various sciences. We shall recur to it when dealing with *Definition* (50-53) and *Classification* (63-69). The fact that we see in things resemblances and affinities which enable us to classify them, suggests that they are individual embodiments or realizations of *specific* schemes or types, and these again of wider or *generic* types. These differentiations of the abstract and universal objects of our knowledge furnish us with data for our definitions *per genus et differentiam.*

45. THE PREDICABLES DEFINED.—The predicable SPECIES may, therefore, be defined as *the relation borne to the subject of a logical judgment by a predicate which gives the whole connotation of that subject* (the *tota essentia specifica*).

[1] VENN, *op. cit.*, p. 269. [2] *Cf.* JOSEPH, *op. cit.*, pp. 77-92.
[3] VENN, *op. cit.*, p. 272, footnote.

The predicable GENUS is *the relation borne to the subject by a predicate which gives that portion of the connotation which the subject possesses in common with other things* (the *pars determinabilis essentiae*).

Regarded from the point of view of their denotation, it will be observed that *genus* and *species* are correlative, the former being always a wider class within which the latter is included. Hence, from this point of view they are often defined respectively as "a wider class made up of narrower classes called *species*," and "a narrower class included in a wider class called the *genus*". But the "inclusion" here is not *physical* inclusion as of coins in a purse, nor mere membership of a miscellaneous collection, as De Wulf's *History of Medieval Philosophy* among my books, but *mental* inclusion of one objective concept (*species*) in another (*genus*), because the former realizes or embodies in a definite way the mental type which constitutes the latter.

The predicable DIFFERENTIA may be defined as *the relation borne to the subject by a predicate which gives that portion of the connotation—the attribute (or group of attributes)—which distinguishes the subject from other groups of things within the same genus* (the *pars determinans essentiae*: the attribute by which the generic type is *determined* or *definitely realized* in the species).

The predicable PROPRIUM may be defined as *the relation borne to the subject by a predicate which gives some attribute (or group of attributes) that follows necessarily from the connotation of the subject.*

The predicable ACCIDENS may be defined as *the relation borne to the subject by a predicate which gives an attribute (or group of attributes) that has no necessary connexion with the connotation of the subject.*

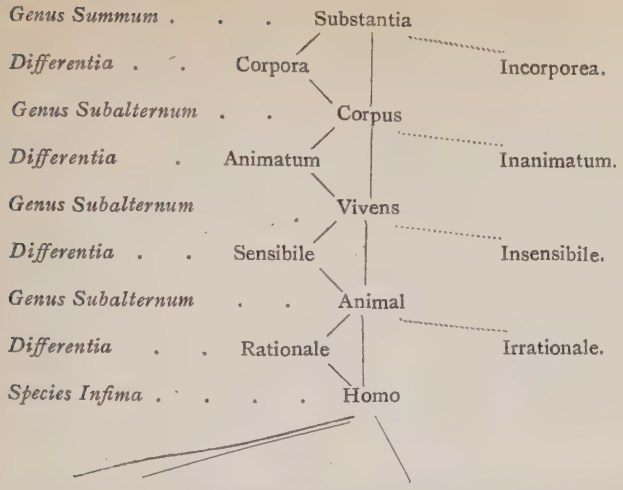
46. GENUS AND SPECIES: PORPHYRY'S TREE: SPECIES INFIMAE.—*Genus and Species:* It is evident that if we are asked to classify any given class name or general term in one or other of the five predicables we cannot do so *except in relation to some other term* with which we compare it. And the same term can belong to different predicables according to the nature of its relation to the other term with which it is compared. "Animal," for instance, is a *genus* compared with the two *species* "man" and "beast," while it is itself a *species* (co-ordinate with "plant") under the *genus* "living organism". We saw above (34) how, in an ascending series of widening classes, the extension increases while the intension decreases. In all such series of subordinate classes each is a *species* of those above it and a *genus* in regard to those below it.

The largest and widest class in any scheme of classification is called the *summum genus* (γένος γενικώτατον) in regard to that order or department of things ;[1] and in a classification of *all* knowable things it would be a *summum genus* not relatively to any particular department, but in an *absolute* sense.[2] At the other end of the scale is the lowest species, below which there are no sub-classes but only individuals. This is called the *infima species* (εἶδος εἰδικώτατον). The direct series of classes between any *infima species* and its *summum genus* is called a predicamental line—*linea praedicamentalis*. As every *genus* must have at least two *species* under it, it is plain that every *summum genus* must give rise to numerous lines or trees of division (called *predicamental* lines) branching off progressively towards its various *infime species*. In every such line the class next above any given class is called the proximate genus (*genus proximum*) of the latter ; that next below it its proximate species (*species proxima*). A *subaltern* genus—*genus subalternum* (γένος συνάλληλον)—is one which has other *genera* (at least one) both above and below it. Similarly, a subaltern species—*species subalterna*—is one which has at least one other *species* above and below it. In every predicamental line the *genera* which lead down progressively from the *summum genus* to any given class, are said to be the *cognate genera* of that class. Thus, "substance," "matter," "organism," "animal," would be the *cognate genera* of "man". The co-ordinate classes into which any genus is divided are called *cognate species* of that genus. Thus, "equilateral," "isosceles" and "scalene," are cognate species of the genus "triangle".

Porphyry's Tree : Among all the classifications of the objects of human knowledge the one which has most interest for us, and which is in many ways the most important, is that in which we ourselves, individual human beings, appear as members. The predicamental line commencing with "substance"—one of Aristotle's *summa genera*,—and ending with "man" as *infima species*, was first drawn up by Porphyry, and has been used ever since to illustrate the relations, explained above, between *genera*, *species*, and *individuals*. It is referred to generally as the *tree of Porphyry*, or *Porphyry's tree*, less frequently as the *Ramean tree* (from Ramus, a sixteenth century logician).

[1] "Man" for the science of anthropology ; "animal" for zoology ; "plant" for botany ; "living organism" for biology.

[2] We shall deal with the tenfold scheme of *Prædimenta* or *Categories* attempted by Aristotle, and other similar schemes in this sense, in Chapter V.



Socrates, Plato, et alia singularia individua seu supposita (Personae).

The members of this predicamental line—from *substantia*, through *corpus*, *vivens*, *animal*, to *homo*, are got by modifying each successively by the respective *differentiae specificae* given in the left-hand column. The *differentiae* in the right-hand column also realize certain other classes which would terminate finally in various *species infimae*, each of these latter having under it no further sub-classes, but only individuals.

Species Infimae.—Of course, there is no reason why we should not go on dividing and subdividing the class "man" into smaller and smaller groups, whenever such subdivision may be found to serve any useful purpose: for instance, into Europeans and non-Europeans, the former into Irishmen and non-Irishmen, the former again into County Dublin Irishmen and those from other counties, the former again into Maynooth (Co. Dublin Irish) men and those (Co. Dublin Irishmen) not from Maynooth.

But it was not customary with the ancient and mediaeval logicians to describe these latter sub-classes as *species*, or their differentiating attributes as *specific differences*. Fixing their attention on the great, broad lines of demarcation traced out in Nature itself between different classes of things—especially in the animate world,—and influenced, no doubt, by the belief that these separate classes had their origin in separate creative acts and were *distinct and unalterable natural kinds* of things, they called these classes *species*, regarded the class name of the *species* as connoting the *whole specific nature* of the individuals within that class whether the latter were subdivided or not, and pronounced all further subdivisions to be *artificial* rather than *natural*, and the attributes on which these were based *accidental*, or not necessarily involved in the *specific nature* of the individuals—as opposed to differences that were *specific*, or *essential* to the *specific* nature of the individuals.

We shall see in a later chapter (70) that there are ample grounds for drawing a distinction between *natural* and *artificial kinds* or *classifications* of things, but that it is the duty of the various other sciences, rather than of logic, to say in particular cases what attributes are to be regarded as essential for the specific nature of any individual thing, and, consequently, for the connotation of its ordinary or lowest class name,—the name one would give in answer to the question: *What is that thing?*

But we must be careful to distinguish the logical use of the terms *genus* and *species* from their use in the biological sciences. In these latter, *species* means a group of individuals supposed to have descended from common ancestors and to be indefinitely fertile in breeding among themselves; *genus* means the next highest group; while under the *species* come the smaller classes called *varieties*, and above the *genera* several wider classes called *tribes*, *orders*, *divisions*, *kingdoms etc.* Logic calls all classes alike *species* and *genera*, provided they fulfil the conditions indicated above. This, however, must be borne in mind: that logical divisions, and, therefore, logical *genera* and *species*, ought to aim at following the *natural* divisions of things established in the natural (biological) and physical sciences (70).

We can therefore maintain that our knowledge of things—of the essences of things—is real knowledge, and not merely a knowledge of the meaning of names (44), without denying the titles *genus* and *species*—in the sense in which these terms are used by modern logicians—to subdivisions of what earlier logicians regarded as *species infimae*. That men and monkeys are different *kinds* or *species* of things zoologically and psychologically; that they differ in *kind*,[1] and not merely in the *degree* in which certain attributes or qualities may be developed in either class; that, on the other hand, monkeys differ from dogs by a *smaller* difference than that which separates either class from men; and that terriers differ from greyhounds by a *still smaller* difference than either class does from monkeys: all these things we believe to be true; but to contend that their truth depends in any way upon the recognition of the class "man" as a lowest *logical species*, or upon the recognition of the "existence of an absolute *infima species*,"[2] is to confound two quite distinct meanings of the same technical term—the biological and the logical meanings. We fail to see how the subdivision of the traditional *species infima* by modern logicians necessarily involves a nominalistic or conceptualistic solution of "the whole question of the formation of universals" or jeopardizes "the absolute character of truth".[3] If some modern logicians have fallen into the errors just referred to, it is certainly not because they have emphasized the sufficiently obvious fact that, whether we are concerned with divisions above or below the traditional *species infimae*, the attribute which forms the basis of any step in the process of division, and which is *essential* to the sub-class or species—constituting, as it does, the *differentia specifica* of the latter—is always and necessarily a *separable accident* of the class that is being divided (the genus), and cannot, therefore, be *essential* to the latter.

[1] They differ, not merely *specifically* ("*in specie*") but *generically* ("*toto genere suo*") in this case, for they have not even the same *genus proximum*: the *genus proximum* of monkey being "brute beast" or "*animal irrationale*," which is a co-ordinate species with "man" under the higher genus, "animal".

[2] CLARKE, *Logic*, p. 185. [3] *ibid.*

In his endeavour to establish a distinction between the character of the *species infima* and that of its sub-classes, Father Clarke[1] apparently fails to distinguish between the full (explicit) contents of the whole *individual* "nature," and the ever-diminishing (explicit) contents of the *specific* "nature" and the *generic* "nature" as represented to us by each of the ascending series of general class names and concepts in and through which we can think of that individual. When he says of such attributes as "*white* and *coloured*, *virtuous* and *vicious*, *heathen* and *Christian*, *European*, *Asiatic*, *American*, *African*, and *Australasian*," in reference to "man," that "every one of them might be conceived to be reversed without the *man*, so to speak, losing his identity," he seems to forget that "rationality" and "irrationality" might be likewise reversed without "animal" losing any part of what makes us think of it as animal. So too, if a man "is an *European*, he will not have his nature changed if we suppose him born in Asia";—that is, his *human* nature: but his *Europeanism* will be changed. "If he is a negro we can think of him as remaining in all respects the same, though his skin should be white." Not quite; he would remain a *man*, but would he remain a *negro?* "But if we take any of the attributes which belong *to man as such*, it is quite different. Take away from man the faculty of *sensation* and he is a different being at once . . . it [sensation] cannot be separated even in thought without destroying his nature." Of course it cannot be taken away (no more than *rationality*) without destroying his *human* nature, for it forms a part of his *human* nature, and if it be taken away "he is a different being at once"; but its removal in thought does not prevent his being still thought of as a *living organism:* just as on the mental removal of "European origin" or "dark-coloured skin" the individual *would cease to be thought of as an European or a negro, though he would still be thought of as a man.* So far from being "quite different," the cases are quite similar as regards the relations of the attributes to the logical classes in question; nor is there any need to distinguish logically between them.[2]

47. Differentia and Proprium: Connexion of Proprium with Essence.—*Differentia:* One concept is said to be the *differentia* of another when it gives the attribute (or group) which distinguishes the former from other co-ordinate *species* of the same *proximate genus.* It is therefore the *excess of connotation in the concept of a species over that of its proximate genus.* Seeing that the same class of objects may find a place in different schemes of classification, a different part of its connotation will stand as *differentia* in each case. Thus if we compare "man" with other "animals" his *genus* will be "animal," his *differentia* will be "rational" and his definition "rational animal"; whereas, if

[1] Clarke, *Logic*, pp. 182-4.

[2] See the same fallacious line of argument in Zigliara, *Logica* (5), vi., where the author denies that dogs, horses, fishes, birds, etc. are true *species* "quoniam in illis speciebus peculiaribus differentia est accidentalis, non essentialis"—an observation which applies equally to classes above the *species infimae.* *Cf.* Joseph, *op. cit.*, p. 93.

we compare him with other beings endowed with intelligence—"spiritual beings"—his *genus* will be "spirit," his *differentia* "embodied," and his definition "embodied spirit".[1] In such cases the difference in definition is not a real difference, for so long as the *species* remains the same so will the specific nature or connotation. Moreover, in all such cases there is *some one genus* into which the *species* in question *naturally* falls, and it is *that genus* with the *appropriate differentia* that is regarded as the *definition* of the *species* in question (51).

The *differentia* we have been considering is the *differentia specifica*—that which differentiates the various *cognate species* of a given genus. In reference to these latter, the *differentia* which marks off their *proximate genus* from *its* cognate classes under a higher genus, is called their *generic differentia*, *i.e.* the *differentia* which, being common to them all, marks off their *proximate genus* from other classes. Thus "life" would be the *generic differentia* of "plants" and "animals"—marking off their *proximate genus*, "living things," from "inanimate things".

So, also, we might give the name of *differentia individua* to the whole group of characterizing traits and qualities (called inseparable accidents of the individual[2]), which serve to distinguish a given individual from all other individuals in the same *species infima*. Such "individual accidents" are therefore really as essential to the concrete individual (considered as really existing in concrete time and space) as the *differentia specifica* is to the *species*. It is not in relation to the individual that they are accidents or coincidences, but to the general characteristics we have already apprehended in the individual (48).

Proprium: The predication is said to be in the predicable called *proprium* when the predicate gives an attribute (or group), not included in the essence or connotation of the subject, but following necessarily from it. Such an attribute is called a *property* of the subject of which it is affirmed. Again, we may distinguish between *generic*, *specific*, and *individual propria*. For instance, "endowed with a nervous system" might be regarded as a *generic property* of man, inasmuch as it is *always* found in *every* man, though not in man alone, but in all the members of his proximate genus "animal": sensibility or sense consciousness (which is the *differentia* of "animal") involving the

[1] WELTON, *Logic*, i., p. 109; JOSEPH, *op. cit.*, pp. 68, 69.
[2] CLARKE, *Logic*, p. 183.

function of a nervous system. The "time and place of his birth" might be regarded as *individual properties* of any particular person. But the most important are the *specific properties*, which are necessarily connected with the specific nature of the class and with the connotation of the class name. Thus "capable of using language," "tool-using," "food-cooking," "capable of learning," and other such attributes, necessarily following from the possession of *reason*, are all *specific propria* of *man*.[1] They are found *always* in *all* men, and in men *alone*. "Proprium dicitur quod convenit *omni, solo et semper*"—*i.e.* that which is found *always* in *every member* of the species, and in them *alone*. Since, as we have already seen (32), there is a certain conventional element in fixing the connotation of the name of any species—in drawing the line between what is "*directly* implied" or "meant" by a name, and what is only "*indirectly*" if necessarily involved therein—so, also, it may be sometimes a matter of little moment which of two or more attributes we regard as the *differentia*, and which as *propria*,[2] of a given class: whether, for instance, we regard "equality of sides" or "equality of angles" as the *differentia* of an equilateral triangle. As a rule, the attribute which serves best to identify members of the class is selected as *differentia;* all others flowing necessarily from it being regarded as *propria*.

Connexion of Proprium with Essence.—A more important question is this: By what kind of *necessity* must an attribute be connected with the specific nature of a thing in order to be and be called a *proprium?* It must flow from the *essence* "as effect from cause,"[3] "by some law of causation".[4] "The connexion of a proprium with the connotation is a necessary one; that is, its not following would be inconsistent with some law which we regard as part of the constitution either of the universe or of our minds or of both";[5] for "propria are attributes which flow from the whole, or part, of the connotation either as effect from cause or as a conclusion from premises".[6] These answers are apparently meant to convey that the necessity of the connexion in question must be an *absolutely inviolable necessity*. That is to say, the proprium "must be not only inseparable in *fact* but also inseparable in *thought*"[7] from the specific nature of the thing, and hence from the connota-

[1] Ἴδιον δὲ ἐστὶν ὃ μὴ δηλοῖ μὲν τὸ τί ἦν εἶναι, μόνῳ δ' ὑπάρχει καὶ ἀντικατηγορεῖται τοῦ πράγματος. Οἷον ἴδιον ἀνθρώπου τὸ γραμματικῆς εἶναι δεκτικόν. Εἰ γὰρ ἄνθρωπός ἐστι, γραμματικῆς δεκτικός ἐστι, καὶ εἰ γραμματικῆς δεκτικός ἐστιν, ἄνθρωπός ἐστιν.—*Top.*, lib., i., c. 4, n. 5.

[2] This is mainly true in the mathematical sciences where our concepts are clearly defined and the relations between them clearly seen to be necessary and universal. Since the strict *proprium* is coextensive with the *differentia* it will serve equally well to mark off the species.

[3] CLARKE, *Logic*, p. 173. [4] *ibid.*, p. 174. [5] WELTON, *Logic*, i., p. 85.
[6] *ibid.* [7] CLARKE, *Logic*, p. 183—referring to the *differentia*.

tion of its specific class name. It must therefore be something which can be inferred from the connotation by the absolutely necessary laws that govern our thought processes ; something whose absence would in the first instance involve a *contradiction in thought*, a violation of "some law which we regard as a part of the constitution . . . of our minds". But, since the laws of thought are not *purely* subjective (15) ; since, too, the *truth* of knowledge (10) depends on whether or not we fix the connotation of our concepts and terms *in conformity with things*, in other words, on whether or not there *are* things corresponding to our concepts, on whether things *are what we assert them to be* when we predicate such concepts of them in judgment : it follows that the absence of a proprium, by violating some *subjective* or *mental* law, would, by that very fact, be also violating some *objective* or *real* law, "some law which we regard as part of the constitution . . . of the universe," as well as "of our minds," that is to say, some law which is a law "of both".

The knowledge, therefore, that a certain attribute constitutes a *property* of a given class of objects is much more than a knowledge of the *subjective* implications of certain arbitrarily fixed notions and names. It is knowledge about the nature of real things. The essences revealed to the human mind in the intension of the concepts and names by which it thinks and expresses its knowledge, are not merely "nominal essences" but also "real essences" ; so too are the properties discerned in these essences real, and not merely nominal.

A further inference is that real knowledge of things, objectively valid information about the nature of reality, is conveyed to our minds not merely in and through that class of judgments called *synthetic* or *accidental*, in which the predicate is an *accidens* of the subject (85), but also in that other class of judgments called *analytic* or *essential*, whose predicates reveal some *essential* attribute or *necessary property* of their subjects.

The distinction between *property* on the one hand, and *specific essence* (as constituted by *genus* and *differentia*) on the other, is easily intelligible in deductive sciences like geometry and mathematics. For these sciences deal with magnitude and multitude, *i.e.* with the spatial and numerical aspects of material reality ; and since the universal laws or conditions of space and number are ever the same, constant and unchanging, we can take as *definitions* such concepts as will bring these quantitative aspects of reality before our minds (*e.g.* the definition of a triangle), and proceed to deduce or draw out and demonstrate from these, other more complex aspects which we will regard as *properties* of those objects—*e.g.* that the three angles of a triangle are equal to two right angles. It is in these sciences, then, that we have copious examples of *properties* in the strictest sense of this term.

But the concrete objects, organic and inorganic, with which the natural and physical sciences deal, do not lend themselves so easily to definition, to the detection of what is fundamental in their natures. Nor do the constant changes to which they are subject, the endless diversity and variety of conditions in which they manifest themselves, permit us to demonstrate their *properties* with the same rigour as in the case of the purely abstract sciences of mathematics and geometry. "Hence for definition, such as we have it in geometry, we must substitute classification ; and for the demonstration of properties, the discovery of laws."[1] Those classes are formed, as we shall see

[1] JOSEPH, *op. cit.*, p. 89 ; *infra*, 66,

later, by the possession of a few common characteristics, but each class is indefinitely complex and comprehensive of variety in detail; each class presents many characteristics common to all its members; these are called properties in a looser sense; and it is the aim of the *inductive* sciences to account for each "property" by discovering and establishing the law of its manifestation, *i.e.* its connexion with other properties of the class in question or of kindred classes. "If a species, for example, is keen-scented, that must depend upon conditions which are but a small part of what would be included in a complete account of its nature. In order to find a commensurable subject of which a property is predicable we must abstract from all in the species which is not relevant to that one property; and our subject will not be a concrete kind, but a set of conditions in the abstract. The property whose conditions we have found is not of course the property of those conditions, but of anything that fulfils those conditions; keen-scentedness, for example, is not a property of a particular construction of the olfactory organ (though we should call it an effect of this), but of an animal in whom the olfactory organ is thus constructed; the laws of organic life suppose of course that there exist organisms in which they are exhibited. We may still speak therefore of properties of kinds; but the demonstration of them considers the nature of the kind only so far forth as it concerns the property in question. The property is not common and peculiar to the kind, if other kinds, as may well be the case, agree with it in those respects on which the property depends; or if it depends on conditions which cannot be fulfilled except in an individual of that kind, but are not fulfilled in every individual thereof."

"Such reflections led the Schoolmen to distinguish four senses of the term property."[1] These were already distinguished by Porphyry[2] as follows: (1) What belongs to a certain species *alone*, though not to *all* its members, as to be a "doctor" or a "surveyor" is to the human species; (2) What belongs to *all* the members of a certain species, but *not* to them *alone*, as "biped" in regard to man; (3) What belongs to a certain species *alone* and to *all* its members but *not always*, as "white-haired" in regard to old people; (4) What belongs *always* to *all* the members of a certain species, and to it *alone*, as the faculty of "laughing" in man, or that of "neighing" in the horse.

"In all the uses of the term property the notion of a necessary or causal connexion is retained; but commensurateness with the subject is not insisted on in all. No doubt a commensurate subject for every predicate is to be found; but only by specifying the precise conditions (in an organism or whatever it may be) on which the property depends; but the concrete thing is the subject about which we naturally make propositions, naming it after its kind; and

[1] JOSEPH, *op. cit.*, pp. 89, 90.

[2] "Proprium vero quadrifariam dividunt. Nam et id quod soli alicui speciei accidit, etsi non omni, proprium dicitur: ut hominem esse medicum vel geometram. Et quod omni accidit etsi non soli, quemadmodum hominem esse bipedem. Et quod soli, et omni, et aliquando: ut homini in senectute canescere. Et quod soli et omni et semper: quemadmodum hominem esse risibile; nam etsi non semper rideat, tamen risibilis dicitur, non quod semper rideat, sed quod aptus natus sit ad ridendum; hoc autem ei semper naturale est, quemadmodum et equo hinnibile. Haec autem nominantur vere propria, quoniam etiam convertuntur, quicquid enim est equus, hinnibile est, et quidquid est hinnibile est equus."—PORPHYRY, *Isagoge*, cap. 3.

kinds being complex may agree together in some points while differing in others with intricate variety; so that when we have distinguished the species to which objects conform, and the attributes which they possess, we cannot divide the latter among the former without overlapping." [1]

48. ACCIDENS: SEPARABLE AND INSEPARABLE ACCIDENTS. —When the predicate of a judgment gives an attribute which neither forms part of the connotation of the subject nor has any necessary connexion with the latter, the predication is said to be *accidental* or *contingent*—to be in the predicable called *Accidens*. An *Accidens* is, therefore, an attribute to whose presence or absence the subject is indifferent.[2] We may distinguish between the *Accident of an Individual—e.g.* "sleeping" or "waking," "well" or "ill," in reference to any individual—and the *Accident of a Class, e.g.* "red-haired," "learned," "virtuous," in reference to the class of human beings.

Of course when we speak of an attribute as being an "accident" of an "individual," we do not mean that the attribute is unessential to the individual in the concrete state in which we conceive him as possessing it—if we say that "John is (here and now) asleep," it is *essential* to him here and now to be asleep if our proposition is true. What we mean is that the attribute "asleep" is accidentally or contingently conjoined with *the other general characteristics* which we have already included in *our concept of John*, so that this concept—the "*ratio subjecti*," in the words of Cajetan—would remain unaltered even if John were not here and now asleep.

That reality, whatever it is, in any individual, which distinguishes that individual numerically from other individuals of the same species, is called the Principle of Individuation—"*Principium Individuationis*". The problem as to what precisely that reality is, has long been discussed in the schools; and it is a gratifying sign of the impartial study which present-day thinkers are beginning to devote to scholasticism, that Mr. Joseph recognizes in this question "a serious philosophical problem".[3] His own view, that individuation depends not on *substantial form* nor upon *matter* conceived in the abstract, but perhaps upon matter somehow related to *mass*, appears to us to come very near the mind of St. Thomas himself upon this problem. It is, however, a question for metaphysics rather than for logic.

A more important distinction is that between *separable* and

[1] JOSEPH, *ibid.*, p. 91.

[2] "Accidens, id est, accidentale praedicatum, id esse dicitur, quod indifferenter affirmari et negari contingit absque hoc quod subjecti ratio destruatur."—CAJETAN, *Liber Praedicabilium*, cap. 4.

[3] *op. cit.*, p. 76.

inseparable accidents. The former may be entirely separated or removed from the class or the individual, the connexion being altogether contingent and reversible, as in the instances just given; they may be absent without changing the *conceptual sameness* or identity of the class or of the individual; they are *de facto* found to be absent from some members in the case of a class accident—as "white" in regard to "swans"—or to be sometimes present, sometimes absent, in the case of an individual accident. The inseparable accident, however, is some quality which is found to belong *invariably* as a matter of fact to all the members of a class (or to be *always* present in an individual; a certain group of such attributes (47) constituting what may be called the *differentia individua* of the individual). Thus "cloven-hoofed" is an inseparable accident of "ruminants," "blackness" of the "raven" or of an "Ethiopian".

When, then, it may be asked, is a certain quality or attribute which is *always* and invariably found in *all* members of a class—and perhaps of that class *alone*—to be regarded as a property, and when as an inseparable accident? The answer is that we are to regard it merely as an inseparable accident as long as we are unable to discover any *necessary connexion* between it and the specific nature of the class in question—as long as we can even conceive it to be absent "absque hoc quod subjecti ratio destruatur" in the words of Cajetan, or, in modern language, "without interfering with the connotation or other known properties of the class". Mere *observation*, therefore, may reveal to us the *actual*, *invariable* presence of an attribute in a class of subjects; but we cannot call it a property in the strict sense until we have so *analysed* the nature and known properties of the subject in question as to be able to pronounce that the attribute *must be* present, as being necessarily involved in, or connected with, those more fundamental thought-objects[1] which we have already regarded as constituting the specific nature of the subject and as yielding the connotation of its specific class name, or with some property or properties already known to follow necessarily from this specific nature. As soon, therefore, as the attribute becomes inseparable not merely in fact but *in thought* from what we have already conceived as constituting the nature and properties of the subject, that attribute becomes for us a *property* of that subject.

In the purely abstract sciences, such as geometry, there is really no room for "accidents": "in geometry there are no happenings, no conjunctures":[2] the attributes we discover and demonstrate from our definitions and axioms are all properties. But in the concrete sciences it is different. For example, the attribute we call "mortality," and by which we understand the subjection of a living organism to decay, corruption, and death, we see to

[1] We cannot properly call those fundamental realities *attributes* of the thing; for they constitute its very essence and enter into its definition.

[2] JOSEPH, *op. cit.*, p. 84.

be an invariable characteristic of all mundane living things; it is, therefore, *at least* an inseparable accident of these things. But when we have analysed the nature, structure, and functions of the living organism, and have come to see that *by virtue of its composite nature* and of the *physical conditions* in which it subsists it is necessarily liable to dissolution and death, and will actually die unless known physical agencies are suspended or modified in its regard, then we have a right to regard "mortality" as a *physical property* of all living organisms (87). So, too, as an invariable rule, animals that chew the cud have been found to be cloven-hoofed, but unless and until the physiologist finds some reason why the possession of cloven hoofs should necessarily accompany the chewing of the cud, we have no right to regard the former attribute as a *property* of ruminants. With the progress of science attributes may and do pass from the class of inseparable to that of separable accidents—witness the discovery of coloured swans in Australia—or to the class of properties in the stricter sense.

The whole inquiry into the nature of the distinction between *accident* and the other predicables, as well as into the concept of *property*, has a very close connexion not only with the distinction between *necessary* and *contingent* judgments (85-88), and between *metaphysical* and *physical* truth, but also with the whole question of the relation between *cause* and *effect* in Induction.[1]

JOYCE, *Logic*, chap. viii. JOSEPH, *Logic*, chap. iv. CLARKE, *Logic*, Pt. I., chap. ix. WELTON, *Logic*, I., iii. VENN, *Empirical Logic*, pp. 269 *sqq*.

[1] *Cf.* JOSEPH, *op. cit.*, pp. 62 *sqq*.

CHAPTER III.

DEFINITION.

49. DEFINITION GIVES DISTINCTNESS TO OUR IDEAS AND LEADS TO THE AXIOMS OR PRINCIPLES OF THE SCIENCES.—In the preceding chapter we examined the various relations that may obtain between our concepts in the mental act of judgment, and, consequently, between the terms of a logical proposition. Since all human knowledge is or may be embodied in *judgments*, it is important that the terms of this comparison be in all cases as clear and definite as possible. Hence the mental processes by which we seek, while advancing in knowledge, to make both the application and the implication of our concepts clear and distinct, need to be carefully examined and analysed. The study of the different predicables has prepared us for this task.

The mention of an entirely strange term or name brings before the mind no concept of any object beyond the term itself: it leaves the mind *in the dark* as to the object referred to: the concept of the object—if the concept may be said to exist at all—is totally *obscure*. Or again, it may of itself, or through some other experience, bring into the mind a *vague suspicion* of the class of object referred to, but not a sufficiently *clear* idea to enable us to distinguish the object or class of objects denoted, from other objects or classes of objects. The concept is said to be *obscure* as long as its application is doubtful; it is *clear* when the application is certain. A concept may, however, be clear in the sense that it enables us to distinguish the objects it denotes from other objects, without at the same time bringing distinctly before our minds the various attributes it implies; without, therefore, giving us more than a vague, indistinct knowledge of the nature of the objects denoted. Such an idea, though *clear*, is said to be *indistinct* or *confused*. If, on the contrary, it brings distinctly before our minds the attributes which constitute the nature of the object, then the idea is not only *clear* but *distinct*. A child's

idea of a cat or a dog may be *clear*, though quite *indistinct;* the skilled zoologist's idea would be *distinct* as well as clear.

It is the process of *Logical Division* which directly serves to make our ideas *clear* in the technical sense just explained, while the process of *Logical Definition* makes them *distinct* as well as clear. The former process will be dealt with in a subsequent chapter. The present chapter will deal with *Definition.*

The Scholastics defined (Real) *Definition* as an "*oratio qua quid sit res aliqua explicamus*": a statement by which we explain what a thing is: a statement which answers the question "*Quid est illa res? What is that thing?*" by the happiest combination of fulness, accuracy, and brevity. They therefore connected *Definition* very closely with another process called *Demonstration*, and which modern logicians sometimes call *Scientific Explanation* (255). Regarding (real) definition as the final outcome and crystallized product of scientific research, they enumerated "demonstration," "definition," and "division," as the *tres modi sciendi* —the three great means of acquiring scientific knowledge (201). Modern logicians give expression to the same view in regard to the function of definition as a mental process when they emphasize the fact that its real value lies in the laborious work of seeking for accurate definitions, rather than in the finished product when found. "It involves careful observation, comparison, and analysis of the things observed, abstraction of the mind from their differences, and generalization, besides the power of distinguishing primary from derivative properties."[1]

Besides, therefore, contributing to the clearness and distinctness of our ideas, definition serves the equally important purpose of so prosecuting the analysis of these ideas as to lay bare the ultimate and unanalysable notions which form the very foundations of all the human sciences. "The principles of the sciences," writes Aristotle,[2] "are indemonstrable definitions. Definition

[1] WELTON, *Logic*, i., p. 108. *Cf.* VENN, *Empirical Logic*, p. 284: "To decide the relative importance of the attributes demands a delicate discrimination among their respective claims, and often presupposes the choice of some important leading principle in virtue of which they are to be judged. Each new attribute, therefore, instead of being lightly accepted, has to be carefully tested, and when it is accepted, must be compared and valued against the others. In a word, Definition is the outcome of a great amount of research on the part of the framer, and consequently a most important means of instruction on the part of the learner."

[2] Τὰ πρῶτα ὁρισμοὶ ἔσονται ἀναπόδεικτοι. Ορισμὸς μὲν γὰρ τοῦ τί ἐστι καὶ οὐσίας· αἱ δ' ἀποδείξεις φαίνονται πᾶσαι ὑποτιθέμεναι καὶ λαμβάνουσαι τὸ τί ἐστιν, οἷον αἱ μαθηματικαὶ τί μονὰς καὶ τί τὸ περιττόν, καὶ αἱ ἄλλαι ὁμοίως. Ὁ ὁρισμὸς οὐσίας τις γνωρισμός. —*Anal. Post.*, ii., 3, (10).

brings to light *what the thing is;* thus mathematics commences from the exposition of what *unity* is, what an *uneven number* is, and so on."

As an illustration of the work of analysis and synthesis involved in forming definitions, the following few examples may prove instructive. In deductive sciences like mathematics, whose principles are got by direct intellectual intuition (5), the process of definition is mainly synthetic, an accumulating of the simple attributes so obtained into more and more complex wholes. Each apart, being simpler than the complex whole, is also wider in extent than the latter: the extent is gradually *limited* (ὅρος, ὁρισμός) *by definition.*

Suppose we are in possession of the simple arithmetical notions of unity, number, prime number, odd number, even number; of the series, one, two, three, four, etc. How should we define the number *three*, for example? Which of our stock of notions will help us?

Three is an *odd number*, not divisible by two. Three is a *prime number*, in the twofold sense that it is neither a multiple of any other number, nor resolvable into other numbers.[1] *Three* is therefore defined as an *odd prime number.* Each of these notions apart is found in other numbers; all together are found only in the number *three.* They define it, therefore. The notion "number" is true of odd and even alike. The notion "odd" is true of *five*, *seven*, *nine*, etc. The notion "prime," in the sense of not being a multiple, is true of five, seven, eleven, thirteen, etc. The notion "prime" in the sense of "not resolvable into other *numbers*" is also true of the number *two.* But the combination "*odd*, *prime number*" is true only of the simgle number *three.*[2]

In the inductive sciences, *synthesis* is preceded by a long and often laborious work of *analysis.* Let us take an example from psychology, the science of life. To define life we begin by observing "living" things. Here is a rose-tree in full bloom: it is *alive.* Here is a puppy, barking and frisking: full of *life.* Here is a man, hard at work: he is *alive.* Why do we say of all these that they *live?* Why ascribe an identical attribute to them? The rose-tree assimilates nourishment, grows, and perpetuates its kind by seeding; and so of all vegetables. The puppy is endowed, moreover, with powers of sensation and locomotion; and so are animals generally. Man, furthermore, has intelligence and free will. Have all these various activities —nutrition, growth, reproductlon; sensation, locomotion; thought and volition—anything in common? If not, each group should have a name of its own, nor should we be justified in giving all a common name. But they have a common characteristic, which we may detect by abstracting from their differences; and this common feature is that all these activities have their term within the agent, which they perfect: they are not transitive but immanent; the definition of life is, therefore, *immanent activity.*[3]

The elimination of the differentiating attributes from the classes compared—the rose-tree, the puppy, the man—was simultaneously a process of

[1] Unity is not regarded as a "number" but as the "principle" of number, the latter notion implying *plurality.*

[2] ARISTOTLE, *Anal. Post.*, ii., 8. [3] MERCIER, *Psychologie*, Pt. i., ch. i.

aggregating individuals, of grouping or classifying the classes in question under the wider class of "living things".

"There are two contrasting ways," writes Mr. Joseph,[1] "in which we may attempt to construct a definition. We may take instances of that which is to be defined, and try to detect what they have in common, which makes them instances of one kind, and on the strength of which we call them by the same name. This is the 'inductive' method.[2] We might thus define 'snob,' comparing those of our acquaintance to whom we could apply the name, or those whom Thackeray has drawn for us; and if we thought that among all their differences they agreed in prizing rank or wealth above character, we might accept that as our definition. The other method is that of dichotomy [60], and in this we try to reach our definition rather by working downwards from a genus, than upwards from examples. Some genus is taken to which the subject we wish to define belongs. This genus we divide into what possesses and what does not possess a certain differentia. The differentia taken must be something predicable of the subject to be defined; and if genus and differentia together are already commensurate with that subject, the definition is reached; if they form only a subaltern genus predicable of it, this subaltern genus must be again divided in the same way: until we reach a commensurate notion. At every stage of our division the differentia taken must if possible be a modification of the differentia next before it, it must at least be capable of combining with those that have preceded it in the construction of one concept in such a way that we are throughout specifying the general notion with which we started [62]; and there should be as many steps of division as there are stages which our thought recognizes as important in the specification of this concept. At every stage also we precede by dichotomy because we are only interested in the line that leads to the subject we are defining; all else contained within the genus we thrust aside altogether, as what does not exhibit the differentia characterizing that subject." Thus, to revert to Aristotle's example, we work downwards to the definition of "three" from the genus "number" by dividing the latter into "odd" and "not-odd," and then dividing "odd" into "prime" and "not-prime". Porphyry's tree, leading from the *summum genus*, "substance," to the definition of "man," is another example of this method.

But in ultimate analysis the two methods are not fundamentally different: both alike involve comparison, division, classification. When, for instance, by the *inductive* method, we take examples of "snobs" and look for that which "differentiates" them, we are really trying to divide the genus "man" into snobs and people who are not snobs. "Let us take the prizing of rank or wealth; if that by itself does not constitute a snob, we need some further differentia, to distinguish snobs from other men who prize rank or wealth; say they are distinguished by prizing these beyond character; we then have a definition of a snob, but in getting it, we have taken note of a wider class of men within which they are included."[3]

50. Nature of Definition: Per Genus et Differen-

[1] *op. cit.*, pp. 111, 112.

[2] It was extensively employed and illustrated by Socrates and Plato.

[3] *ibid.*, p. 114.

TIAM.—In apparent opposition to the scholastic account of definition as the "*oratio quae explicat quid sit res aliqua*"—the explanation of what a thing is,—we find the more modern description of definition as the "explicit statement of the connotation of a term".[1] The opposition is apparent rather than real. For terms are the outward expression of concepts, which, in turn, are mental representations of things. Whence it follows that whatever explains the connotation of a term, or of the corresponding concept, explains likewise the nature or essence of the thing—the reality represented by the concept.

In order to emphasize this essential and inseparable connexion of term, thought, and thing, it would be well, perhaps, to describe *Definition* as the *Explanation of the nature of an object of thought by explicitly unfolding the connotation of the term by which we refer to that object.* Both the mental process, and the verbal formula which sums up its result, are called *definition.* Etymologically, the term would signify a process of *marking the boundaries* or *limits* of the implication of our concepts—and, by way of consequence, of their application also—thus enabling us to *distinguish* between the various objects of our thoughts.

Definitio per genus et differentiam.—If definition, then, is the explicit unfolding or analysis of the connotation of a term, the question arises, How far are we to go in our analysis of attributes, in setting forth a definition?

Firstly, we need not, and should not, go beyond the connotation, or proceed to enumerate the *properties* (47) of the object in question. We must not imagine the *ideally perfect* definition to be one which would give us the *fullest possible knowledge* we can have of the nature and properties of the thing to be defined.[2] Such an exhaustive analysis goes beyond the scope of *definition*, involving *metaphysical analysis*, *demonstration*, or *scientific explanation.* No one would think of including in the definition of a right-angled triangle all the properties which geometricians have ever, by pure deductive reasoning, discovered in it. Such a catalogue would constitute what has been more accurately described as the "metaphysical analysis"[3] of the object. The true aim of definition is rather to include "the minimum number

[1] WELTON, *Logic*, i., p. 107.

[2] See DE WULF, *Scholasticism Old and New*, pp. 12-14, 142, for some interesting attempts at framing a definition of *Scholasticism.*

[3] WELTON, *op. cit.*, p. 126.

of properties necessary for identification rather than the maximum which it is possible to include".[1] Hence the analysis in question should be confined to the connotation of the name of the object.

Secondly, it has never been regarded as necessary to analyse the connotation *into its simplest, ultimate factors*, and to set forth a complete list of these as the definition of the term. Such a process would be needlessly long and cumbrous. And as a matter of fact it has been traditionally regarded as sufficient for the purpose of definition to go back *one step* in the analysis of the connotation, by setting forth explicitly the *genus proximum* and the *differentia specifica* of the object to be defined.

In order, therefore, to define any object of thought, we must find out and indicate its *proximate genus*—the next highest class into which it *naturally* falls[2] (47)—and the attribute or group of attributes which distinguishes it from other *cognate species* of the same *genus*. Care must be taken that the *genus* selected be the *next* highest; otherwise the definition will prove faulty by omitting portion of the connotation, and applying to other things besides the thing defined. Thus, were we to define a *square* as a "*parallelogram* having its adjacent sides equal," we should be omitting the attribute "right-angled," and our definition would include not merely squares but lozenges.

The reason for limiting the analysis required in definition to *one step*, is sufficiently simple. The aim of definition—from the point of view of logic—being to secure distinctness in our ideas, it ought to avoid lengthy analysis which would lead rather to confusion; and we ought to assume that, if our concept of the thing to be defined is not already distinct, at all events the simpler

[1] KEYNES, *Formal Logic*, p. 55.—This minimum, however, must be not merely *any* minimum that will serve to identify the class, but *that particular minimum* which contains the *fundamental factors* included in the connotation (47). For identification of objects is not the primary aim of definition: it aims rather at giving us information about their nature (53). This, however, we must bear in mind, is an ideal that is not always attainable. In the classificatory sciences of botany and zoology the natural kinds of things are so complex that it is extremely difficult to say which attributes are of "fundamental importance" (66), and so we have often to follow the "identification" ideal in our definitions: "our differentiae are intended as much to be *diagnostic—i.e.* features by which a species may be identified—as to declare the essential nature of the species".—JOSEPH, *op. cit.*, p. 116.

[2] One and the same class may fall into different *proximate genera* according to the point of view from which we reach a knowledge of it and classify it. In such cases we may define the class—if it have the same name in more than one scheme of classification—in different ways: what is its *differentia* in one case being a *property* in the others (51). Such cases are, however, rare.

concepts of its *proximate genus* and *differentia specifica* are distinct, and will enable us to understand *distinctly* the object to be defined. In other words, we should assume that if by accident we do not know the meaning of a given class name, we *do* know the meaning of the next highest one, and can be made to understand the former by means of the latter.[1]

It is in fact an assumption of *formal* logic that people know the meaning or connotation of the terms in common use, and of the concepts they form about things; and those logicians who confine their investigations to the formal departments of the science, acting on this assumption, decline to deal with definition at all.[2] Were the assumption in question always verified in practice, definition would be "obviously uncalled for and useless".[3] But the assumption is not verified. "Many persons are constantly diverging, and all of us are occasionally diverging, from the common consensus of sound opinion about the meaning of words. Accordingly, definitions are in practice very often of extreme value"; [4] and this although we "have no *right* to a definition at all: the mere fact that [we] ask for one is in itself an admission of the general truth of our postulate about language" [5] [that we know the meaning of the terms in common use].

51. FIXITY OF DEFINITION.—Since definition is the analysis and explicit statement of connotation, its fixity will depend altogether on the fixity of the latter. It will have the same conventional element and will be subject to the same accidental fluctuations and changes owing to progress of knowledge or change in point of view (52). The fixing of the connotation of our concepts and terms and the framing of our definitions have a most important influence on advance in knowledge, and are influenced in turn by the latter.

Not only progress in knowledge may modify our definitions by changing what we previously regarded as *accidents* or *properties* into *differentiae:* a change in the point of view may effect a similar modification—and often does, especially in the mathematical sciences, where *Genetic Definitions* (55) are of such frequent occurrence. Thus an ellipse may be defined either as a "conic section right across the cone and not parallel to its base," or as

[1] "If we are not to suppose that people know the meaning of the terms they use, we will keep as near to this supposition as we can by assuming that they know the meaning of every term except the one in question, and there is then all the requisite propriety and completeness in the offer of the *genus* and *species* by way of definition."—VENN, *Empirical Logic*, p. 302.

[2] Dr. Keynes, for example, deals only with *Verbal, Formal and Real Propositions* in this connexion, not with *Definition* proper.

[3] VENN, *op. cit.*, p. 280. [4] *ibid.* [5] *ibid.*

"a plane curve produced by a point so moving that its distance from a fixed line always bears the same ratio to its distance from a certain fixed point".

Again, owing to the fact that denotation and connotation influence each other, while the denotation often remains vague because there seem to be no hard and fast lines in Nature—*Natura non facit saltum*, and so we may be doubtful about the boundary lines of our classes, how we are to classify new and doubtful specimens—it may and does happen that our class limits fluctuate somewhat and that as a consequence the connotation and definition of our class names may gradually vary.[1] The discovery of new objects always raises the problem of their classification. And this may conceivably disturb the denotation and connotation of some familiar class name. We might illustrate the point by the fanciful supposition that some planets were proved to be inhabited by real rational animals—like those encountered by Gulliver in his travels: Would we classify these as "men," or continue to define men as "rational animals"?[2]

52. LIMITS OF DEFINITION.—If definition unfolds the connotation of a term it follows that non-connotative terms (37) have no definition.

(*a*) *Proper Names*, therefore, cannot be defined.

(*b*) Furthermore, not all *connotative* terms can be defined. So long indeed as the connotation is capable of further analysis into simpler attributes—generic and differential—definition is possible. But all complex concepts are built up from simpler ones, and analysis inevitably brings us to a multitude of notions—those which enter into and compose the self-evident axioms of the sciences—which defy further analysis, and therefore cannot be defined. Hence the ten Aristotelian categories or *Genera Suprema* (71) do not admit of strict definition: because they have no higher *genus* of which they could rank as *species*. "Being" is not a *genus* for them, because it is not differentiated in them by *differentiae* the concepts of which would differ in intension from that of "Being" itself: no object of any concept is distinct conceptually from the object "Being". As substitutes for definition, recourse is had to explanations and descriptions. Again, the names of simple, unanalysable feelings, such as *pain* or *pleasure*,

[1] *Cf.* what was said above about generalization and specialization of names, 22, 27; also VENN, *op. cit.*, pp. 286-7.

[2] *Cf.* CLARKE, *Logic*, p. 214.

for example, defy definition: we must suppose that these terms refer to identical kinds of experience in all men's minds. Similarly, the abstract names of simple attributes, such as *redness*, *sweetness*, *duty*, *goodness*, cannot be defined, because although we may deal with them after the analogy of concrete objects and so find a *genus* for them (36)—*colour*, *taste*, *moral feeling or intuition*, *virtue*, in the examples given,—we can find no *differentiae* that will convey any intelligible notion of them to a mind not otherwise acquainted with them.

The Scholastics regarded classes lower than the *species infimae* (46) as incapable of definition in the strict sense—*per genus et differentiam*,—because they refused to recognize the sub-classes and their distinguishing attributes as *species* and *differentiae*. There does not seem, however, to be any sufficient reason for refusing these titles, in their modern *logical* meaning, to such sub-classes and attributes, or for declining to regard as a *definition* of *negro*, for example, the statement that he is a *black man*. So, also, if we set forth the implication of a *significant individual name* (28) by assigning its *infima species* as *genus* and its group of individualizing properties and accidental attributes (47, 48) as *differentia*, we may not unreasonably call such a statement a *definition*.

(*c*) A little reflection will convince us that while technical, scientific terms and notions—such as triangle, artery, planet, volition, etc., in which the implicational side of the meaning, the reference to attributes, is more prominent—lend themselves more easily to definition, the names and concepts of familiar objects such as book, dog, house—in which the applicational reference entirely overshadows the connotation—are difficult, if not practically impossible, to define.

53. Exemplification or Extensive Definition or Definition by Type.—Now, in all the cases just enumerated (52, *a*, *b*, *c*), since the concept or term has an object, be this an attribute or a thing, there ought to be *some* means of securing between different minds *identity of reference* to such objects. The process of definition, so far described, is concerned directly and primarily with the *connotation* of our ideas and terms. It thereby indirectly fixes their reference to things, since *denotation* follows *connotation*. What, then, about non-connotative and simple abstract terms? How is identity of application, and of implication, respectively, to be secured in these? And are there not, perhaps, ideas and terms whose application to objects can be more easily and directly fixed than by appealing in the first instance

to their connotation? If in answer to the former of these questions we refer back to cases (*a*) and (*b*) above, we may observe that the only way to disclose to one who does not already know it, the reference of a *proper name*, is by pointing out to him, or otherwise bringing into his experience, the actual individual object to which the name refers;[1] and that, in like manner, the only way of conveying to others a notion of what we mean by a simple, unanalysable abstract name, such as "redness," for example, is by describing it as "the colour of a field of poppies, hips and haws, ordinary sealing-wax, bricks made from certain kinds of clay, etc.".[2]

In answer to the second question, a glance at the things referred to in (*c*) will convince us that there are innumerable class names with whose denotation we are much more familiar than we are with their connotation. Were a child asked for a definition of *animal*: "What do you mean by an *animal?*"—it would be far more likely to answer, "I mean dogs, horses, cows, and all that sort of thing,"[3] or, pointing to a dog or some other animal, "I mean something like that";[4] than to make any attempt at analysing the connotation of the term, and defining *animal* as a *sentient living organism.*

Perhaps, therefore, instead of first fixing the connotation, and allowing the ordinary (or *intensive*) definition to determine for us indirectly the denotation, of the term, it might in many cases be more convenient to have recourse to a process which would show *directly* the *denotation* of the class name. To fix the latter directly in this way would be to determine the membership of the class *from within*, by taking some individual object as centre or *type*, and grouping others around it according to their degrees of resemblance, before the outside limits of the class (and with them the *connotation* of the class name) have been definitely decided. This process, which is called "Classification by Type," is the one which in all probability has often actually occurred "in the case

[1] When, for instance, we ask "What is *Carn Tual*?", we wish to know what is the thing or object for which it stands, of which it is the name; and the answer "*Carn Tual* is the highest mountain in Ireland" would be the nearest approach to *definition*. The process by which we impose a proper name on an individual—"This child is to be called John"—is not a definition. Neither is the judgment by which we identify an object by means of its proper name, or its class name—"That blue point is Snowdon" (VENN, *op. cit.*, p. 211), "That object in the distance is a horse"—to be regarded as a definition.

[2] KEYNES, *op. cit.*, p. 35. [3] *Cf.* CLARKE, *Logic*, p. 216.

[4] WELTON, *op. cit.*, i., p. 122; VENN, *op. cit.*, p. 306.

of natural groups in the animal, vegetable, and mineral kingdoms,"[1] where the classes "are not separated from each other by rigid and definite lines"[2] but "shade off into each other by imperceptible differences".[3]

"Men form classes out of vaguely recognized resemblances long before they are able to give an intensive definition of the class name, and in such a case, if they are asked to explain their use of the name, their reply will be to enumerate typical examples of the class. This would no doubt ordinarily be done in an unscientific manner, but it would be possible to work it out scientifically. The extensive definition of a name will take the form: *X is the name of the class of which* Q_1, Q_2, . . . Q_n *are typical.* This primitive form of definition may also be called *definition by type.*"[4]

These remarks call for a few observations. *Firstly*, in thus *classifying by type* we are not classifying things *at random*, but are guided by their *resemblance* to some central, *typical* example: the type becomes an *ideal* to which every object must conform *in a certain degree*, that is to say, by the possession of *a certain group of attributes*, before we admit it into the class; so that even in this process *intension*, and not *extension*, is the fundamental guiding factor.[5] *Secondly*, although *definition by type* may be only "a rough and ready means of making others recognize the objects of which we are speaking,"[6] and although it does not give us any knowledge of their *nature* or *essence* (50), as ordinary (or intensive) definition does,[7] yet it contributes not a little to securing *clearness* in our ideas (49) and to removing ambiguity in language by safeguarding *identity of reference* in our terms. *Thirdly*, if we choose from any given class of things, say *metals*, a *smaller* group composed of members *so numerous and so different from one another*—such as aluminium, antimony, copper, gold, iron, mercury, sodium, zinc—*that they possess in common those attributes and those only which are common to all metals* (thus forming the *known comprehension* of the term *metal*), our smaller group will form a perfect *typical definition* of the larger group. Such a typical collection, thus chosen from the denotation, is called the *exemplification* of the class name.[8]

We remarked above (50) that the ordinary definition *per genus et differentiam* will explain a thing only to one who already understands the *genus* and the *differentia* of the latter: so, too, *extensive* definition would be useless to one who had no previous acquaintance with the typical members chosen to exemplify the class in question.

54. "NOMINAL" AND "REAL" DEFINITIONS; VERBAL DISPUTES.—We have stated (49, 50) that *all* definition is at once

[1] KEYNES, *op. cit.*, p. 34. [2] WELTON, *op. cit.*, p. 144.
[3] *ibid.*, p. 113, quoted from Professor CAIRNES, *Logical Method of Political Economy*, pp. 139-141. *Cf. supra*, 51.
[4] KEYNES, *op. cit.*, p. 34. [5] *ibid.*, footnote. *Cf.* WELTON, *op. cit.*, p. 144.
[6] WELTON, *op. cit.*, p. 122.
[7] Dr. VENN says (*op. cit.*, p. 306) that "from the logical point of view it is less of a true definition than is the so-called description". *Cf. ibid.*, p. 279.
[8] KEYNES, *op. cit.*, p. 33.

nominal, *conceptual*, and *real*, inasmuch as term, thought, and thing, are inseparable. At the same time, it must be admitted that while in some definitions the *sole* concern *seems* to be with *names*, in others it *seems* to be no less decisively with *things;* and the distinction between *Nominal Definition* and *Real Definition* has been always recognized by logicians, though perhaps never quite satisfactorily explained.[1] The formula for a nominal definition is pointed out to be "The word '——' *means* '. . .'" as distinct from the formula for real definition "'——' *is* '. . .'"; and the Scholastics distinguished clearly between *Real Definition*, which *explains the nature of a thing*, and *Nominal Definition*, which *explains the meaning of a name or term.* But this seems to be, at least to some extent, a distinction without a difference; for, how can we explain the meaning—whether the implicational, or even merely the applicational, meaning—of a term, without *some* reference to the *reality* which is the object of the concept expressed by the name, whether this reality be a thing or an attribute? The most we can allow, therefore, in this direction, is that some definitions aim primarily at securing the correct use—especially the correct *applicational* use—of *names*, and may thence, perhaps, be called *nominal;* while others refer us more directly to the *nature* of *thing* which they explain, and may thence, perhaps, be called *real.*

Those who seek to detect a deeper difference are not unanimous in the explanations they offer us.[2]

Hamilton's view[3]—that the nominal definition "clears up the relation of *words* to *notions*," while the real clears up the relation "of *notions* to *things*"—establishes a distinction which is more apparent than real, for no definition of the *notion* can abstract from the relation of the latter to the *thing* which is its object.

Ueberweg[4] explains the distinction by pointing out that whereas the nominal definition merely analyses the *notion*, without determining whether the latter has for its object something *real* or something merely *conjured up* by the individual mind, that is to say, without determining or guaranteeing the *objective validity* of the notion; the real definition, on the contrary, always carries with it the proof or the implication that the object of the definer's notion is not a mere mental fiction of his own, but has an objective existence in some particular sphere (33)—besides the existence it has in his passing thought and the existence it has in other people's thoughts as

[1] Compare the analogous distinction between *Formal Division* and *Material Division*, next chapter.

[2] *Cf.* VENN *op. cit.*, pp. 273-8; WELTON, *op. cit.*, pp. 118-20; CLARKE, *op. cit.*, pp. 197-200.

[3] HAMILTON'S *Reid*, p. 691. [4] *Logic*, pp. 164-7.

implied by the very fact that it has got a name. In other words, real definition is of notions that are *objectively valid.*

This does not seem to differ substantially from Mill's view [1]: "in some definitions, it is clearly apparent, that nothing is intended except to explain the meaning of the word; while in others, besides explaining the meaning of the word, it is intended to be implied that there exists a thing, corresponding to the word"; —exists, that is, in some particular sphere other than that of the thoughts of those who offer or dispute the definition. This view is represented by Prof. Welton [2] as "practically, that set forth originally by Aristotle that Nominal Definitions are those in which there is no evidence of the existence of the objects to which the name is applied". And it is an intelligible view; it shows us why the same definition is accepted by some and repudiated by others: because, namely, there is difference of opinion as to the sphere in which the thing defined exists, as to whether it *really* exists as such in the sphere in which *it is believed* to exist by those who propose the definition, and in which its existence is supposed by all to be implied by the definition: there could be no other intelligible reason for such controversies as scientists, philosophers, and theologians, are wont to carry on about certain definitions. This view represents the function of nominal definition as being merely to secure a common understanding about the attributes we include in, and the things we denote by, the terms and concepts which we use, without implying in any way the actual existence or occurrence of these things or attributes in any particular sphere. This explains the Scholastic adage: *Initium disputandi, definitio nominis: Discussion about anything must be opened by nominal definition:* which will secure identity of meaning in our language and save us from *verbal disputes;* the insinuation being that it is only at the *end* of our discussion and investigation that we can frame a *real definition:* which will explain the *nature of the thing* for us and *guarantee its real existence in its appropriate sphere.*

"Before looking for a thing," says Satolli,[3] "we must evidently know what we are looking for": which may mean either that we must agree as to what our term or notion *connotes* before we can proceed to find out whether any object with such a group of attributes really exists in any sphere, and not merely in our own thought; or that we must agree as to what our term or notion *denotes* before we can proceed to explore the *nature* of the object denoted; either preliminary convention being presumably nominal definition, intensive in the former case and extensive in the latter. The former carries with it no implication of the existence of the thing defined. The latter, however, starting with the denotation (as explained above, 53) seems to carry with it such implication of existence; though for another reason (there indicated) it is more emphatically nominal in its character than intensive definition is: inasmuch, namely, as it merely identifies the object referred to by the name, without giving any information—at least any definite information—[4] about the nature of the object, while intensive definition does give us some such information.

[1] *Logic*, bk. i., ch. viii., § 5. [2] *op. cit.*, i., p. 120.

[3] *apud* MERCIER, *Logique*, p. 274.

[4] It *presupposes* some information, as intensive definition does, and it *gives* such vague information as that upon which the common name is given to the objects classified (53).

But, while this implication or non-implication of existence, in the definition, is perfectly intelligible and in itself all-important, it may well be doubted whether it *can* form the basis for a clear division of definitions into two distinct classes; and also whether it *does* mark the actual line of distinction between definitions that are set forth as nominal and others that are set forth as real. It scarcely *can* furnish such a basis; for all definition—even the most distinctly nominal—carries with it a reference to reality from which language cannot be divorced, and by which it must be constantly moulded and directed;[1] while no definition, even the most decisively real, carries with it, of itself, a *proof*, but only a more or less strong *presumption*, of the existence of the thing defined.[2] And no matter how strong the implication of such a presumption may be, its presence "cannot be collected from the mere form of the expression".[3] Accordingly, some logicians would be inclined "to finally discard this distinction from logic" on the ground that "the province of logical definition is not to verify, or to disprove, this presumption [of the existence of the thing defined]".[4]

It is, we believe, permissible to retain the distinction: regarding as *nominal* those definitions whose primary aim is obviously (*a*) to fix the connotation we attach to the terms we use, without raising any possibly contentious question as to whether, or in

[1] *Cf.* WELTON, *op. cit.*, p. 120; VENN, *op. cit.*, p. 274-6.

[2] "My own view," writes Dr. Venn, "is that we shall do best if we rid Logic altogether of this distinction between real and nominal definition. The best general account of the matter would then be this. All language, as a broad *primâ facie* presumption, carries with it the implication that the speaker believes in the reality . . . of the things corresponding to the words which he uses. People do not speak with an intention to mislead, nor do ordinary adults talk habitually of non-entities. The mere use of a word, therefore, raises the presumption of the . . . reality of the objects answering to the word. . . .

"We may say then that every definition . . . raises a presumption of the . . . reality of the objects to which it refers. But it certainly ought not to claim more than to raise such a presumption (unless, of course, reality is formally incorporated into it). Nor indeed is it easy to see how a definition could intimate such a claim. . . .

"A definition is merely the interpretation of a name. In and by itself it has no warrant to convey one kind of reality rather than another, nor has it any known means of doing so. It stands in fact on precisely the same footing in this respect as a term or name. If one of these is uttered we have to judge, by the context of the subject-matter, to what order of existences it belongs, and we must do the same in the case of definitions. . . .

"It is, therefore, strictly correct to refer the Definition to the name, that is, to make all Definitions nominal, *provided we do, what we have expressly undertaken to do, viz. refer the name to the thing.* The name contains the limited group of attributes which always is, or in careful thought should be, present to the mind. This is subjective or conventional; and is all that we can possibly undertake to expound in any formal kind of science such as logic. The logical Definition, therefore, confines itself to this analysis. *But it can only safely trust itself to do so,—if it is to be an instrument for inductive research and judgment,—so long as we always strenuously assert that the names have an archetype behind them.*"—VENN, *op. cit.*, pp. 275-8. We have italicized the passages which lay down the conditions on which it may be lawful to regard *all* definitions as *nominal*.

[3] MILL, *ibid.* [4] WELTON, *ibid.*

what particular sphere, objects will be really found possessing such attributes; or (*b*) to remove ambiguity as to what are the objects to which we wish our terms to refer, by pointing out or in some way exemplifying those objects, without raising any analogous question as to what precisely is the *nature* of those objects or what the proper attributes to include in the connotation of their class names: and regarding as *real* those definitions —whether intensive or extensive—which appear from the context or subject-matter to carry with them the implicit (and often highly contentious) assertion that *there does really exist*, in the sphere to which we claim such definition to refer, an object or class of objects possessing precisely the attributes contained in our definition.[1]

Some light may be thrown on the question, whether or not definitions imply the existence of the things defined, by a glance at what are called *Verbal Disputes*. A purely verbal dispute is of course a dispute that arises solely from a misunderstanding about the meaning of words, where there is no difference of view as to facts between the disputants : the sort of disputes which the Scholastics sought to avoid by enforcing the maxim, *Initium disputandi, definitio nominis*. Logicians have held the most widely divergent views about the extent of such disputes, some maintaining, with Locke, "that the greatest part of the disputes in the world are merely verbal," others, with De Quincey, that "they have never in the whole course of their lives met with such a thing as a merely verbal dispute".[2] The truth lies much nearer the latter extreme than the former, for when different people attach different meanings to the same term the cause of such difference of usage will almost invariably be found to be a difference of view about facts. In fixing the connotation of names, in attaching meanings to terms, people are guided by what they consider to be facts (31, 32), and by their interpretation of the latter: and it is just precisely because all do not agree in their admission of alleged facts, and in their interpretations of admitted facts, that differences in connotation and definition—leading to ambiguity, equivocation, and so-called verbal disputes—arise. But it is an undeniable fact that many disputes

[1] In case of *Exemplification* the claim would be that the selection of individuals made, *is* a typical selection: that the class exemplified *does* possess in common those attributes, and those only, possessed in common by the smaller group.

[2] VENN, *op. cit.*, p. 295. *Cf.* KEYNES, *op. cit.*, p. 50, footnote.

which are *partly*, and often even *mainly*, verbal, owe their existence to the misunderstandings that arise from looseness and vagueness in the use of terms. We might instance the dispute as to whether proper names are connotative (37).[1] Mere controversy, mere arguing, will never settle such questions: with a little attention to clearness and precision in our use of language, they will settle themselves.

It will, however, be found that in most cases the real cause of these "verbal" disputes is a difference of view as to the reality or existence of the objects defined, in the sphere in which existence is claimed for them. So long as a definition is not supposed to carry with it any implication of the existence of the thing defined, in some appropriate sphere, contentions about its limits will of course be necessarily verbal and only verbal; they will have for object the number of notes or attributes we shall agree to include in the connotation of the term. But it is precisely because connotation is *not fixed arbitrarily*, because it is *not merely a matter of convention*, because definition carries with it a *general assumption* that the terms defined are the names of *realities* existing in some sphere other than that of the individual's own inventive thought, that definitions are so constantly challenged and disputed, and with a conviction on all sides that the debate is about matters of vital reality, and not about mere conventions as to the use of words.

A little reflection on the manifold and conflicting definitions we encounter in regard to such terms as "The True Religion," "The Christian Religion," "The Christian Church," "The Catholic Church," etc., is all that is needed to convince us how very intimately people's views as to facts influence the meanings they attach to words.

About the intensive definition of "True Religion" as "a religion believed to be true by those professing it," there would scarcely be any dispute; but about the sphere of its denotation the atheist would differ from the theist (33), the former referring religion to the sphere of people's *beliefs* the latter to the sphere of *reality;* and about its denotation in the sphere of reality the atheist, the indifferentist, and the Catholic, would disagree, the first denying the existence of *any* true religion in the real sphere (making the denotation *zero*), the second holding the existence of several more or less true and

[1] Were we to remove successively the handle, blades, etc. of a penknife, replacing each by a new part similar to the part removed, we might dispute whether we should be still in possession of the same penknife. Our dispute would here rest on our conception of what it is that constitutes individual sameness, and how the latter is lost; this conception determining the meaning we attach to the word "same".

equally good and useful religions (making the term *general*), the third holding the existence of *only one* true religion (making the term *singular*).

Similar doubts and differences exist, in regard to denotation and its proper sphere, in the case of "Christian Religion" and "Christian Church"; some, who admit that Christ founded a *Religion*, denying that He instituted a *Church;* and those who admit that He founded one or both differing among themselves as to what or where it is, or by what marks it is to be identified—differing, therefore, as to identification of the object or objects denoted by these terms. Such differences as to denotation arise of course from different convictions as to the *essential characteristics* of the Church or Religion established by Christ (and hence, as to the *connotation* of the terms).

About the denotation of "The Catholic Church" there is usually no ambiguity. Notwithstanding the fact that Anglicans call themselves "Catholics" and members of "a branch of the Catholic Church," the term "Catholic Church" is commonly understood to denote the Church whose visible head is the Pope and whose centre of authority is at Rome, and the term "Catholic" to denote a member of this latter society. The divergence of view commences with all attempts to enumerate the essential characteristics of this particular religious society, to determine whether it actually possesses the features understood to be essential to constitute it "The True Church of Christ," both by those who defend and by those who impugn the claims of the Catholic Church to this latter title.

From these examples it will be seen that agreement as to the *application* of a term does not exclude, but often only clears the way for, disputes about the nature of the object referred to (and hence about the intensive definition of the term); while agreement as to the *implication* or intensive definition of a name leaves open to dispute the question whether or in what sphere there exists an object that will verify the definition (*i.e.* what is the denotation of the term, or where we are to look for it).

The second doubt suggested above—as to what, *de facto*, is the test applied by those logicians who recognize the distinction between nominal and real definition—will perhaps be best cleared up by enumerating those classes of definition which are usually accepted as nominal. We shall see at the same time whether or to what extent they will fall into the classes suggested under (*a*) and (*b*) above (52).

55. Some Nominal Definitions.—Among nominal definitions are enumerated, *firstly*, those which explain the meaning of a word according to its etymology, *e.g.* "Sycophant," *a shewer of figs* (σῦκον, φαίνω); "Angel," *a messenger* (ἄγγελος); "Pagan," *a villager*. This meaning may or may not be the commonly accepted meaning; and the definition may seek to connect both meanings by considerations of a philological character.

Secondly, translating for another a word from a language unknown to him, to the language he knows; or, explaining to him the meaning of a word he does not understand, by means of a simpler synonym which he does understand: these processes

may be called nominal definition; *e.g.* "Political Autonomy is *Home Rule*"; "The Irish word *Gort* means a *tillage-field*". The explanations of terms "peculiar to certain classes, the slang of thieves, schoolboys and sporting persons, and the whole vocabulary of peculiar expressions required by sailors, miners, and indeed most classes of workmen,"[1] belong to this class of definitions. In this as in the former classes there is scarcely any reference to *things* present to the speaker's mind; his chief concern is with words.

Thirdly, what is called *Private*, or *Special*, or *Conventional*, or *Technical Definition*, is mainly nominal in its character. It consists in a person's assigning to some term a perfectly definite sense in which he is about to use the term in a given context—in a discussion, essay, treatise, etc. The meaning assigned may be different, and is usually partly different, from the commonly accepted meaning; or, rather, as a rule, it is the vagueness and ambiguity of the latter that creates the need for the former.[2] The sole aim in thus assigning a definite meaning to terms at the outset of any reasoning process is to secure identity of reference and avoid ambiguity. This was what the Scholastics intended by the maxim "*Initium disputandi, definitio nominis*". All science abounds in terms having definite meanings attached to them, usually somewhat different, often entirely different, from the meaning attached to them in ordinary usage. We may instance the words *Substance*, *Accident*, *Form*, *Extension*, in philosophy.

56. Some Substitutes for Definition.—It has been already observed that it is not always possible to give a strict definition *per genus et differentiam*, that the application of a term is often determined by some such process as *exemplification*, or by a direct appeal to experience. Sometimes, too, a clear and distinct idea of the object is conveyed by the "exercise of the imagination".[3] This is frequently resorted to in the mathematical sciences. *A description of the process by which we may imagine an object to be produced* is called a *Genetic* or *Constructive Definition*. For example, "*A circle is a curve generated by one*

[1] Venn, *op. cit.*, p. 282.

[2] Hence *legal* definitions, *i.e.* those embodied in Acts of Parliament and judicial decisions, are commonly regarded as *nominal*. *Cf.* Joseph, *Logic*, pp. 188, 189.

[3] Venn, *op. cit.*, p. 279.

extremity of a straight line revolving in a plane around the other extremity fixed". "*A cusp is a curve traced by some fixed point in a circle travelling along a straight line.*" "*A ring is the solid figure obtained by allowing a circle to revolve around a fixed axis in its own plane but outside it.*" In definitions of this class, it is fairly obvious that the reference is to the thing rather than to the name.

Again, it is evident from the nature of a *property* in the strict sense (47) that an object may be identified by assigning a property, though perhaps not always so easily[1] as by assigning the *differentia*. *A statement which enables us to identify an object by assigning one or more of its properties* (instead of its *differentia*), is called a *Distinctive Explanation.* For example, "*Man is an animal endowed with the faculty of articulate speech*"; or "*Man is a biped who cooks his food*". Such "definitions" are most commonly met with in the physical and natural sciences—chemistry, botany, zoology, etc.—their purpose being to identify individuals by the possession of certain natural properties.

Were it the sole aim of definition to secure unfailing accuracy in the identification of the objects defined, any of the devices now under consideration would suit as well as strict definition *per genus et differentiam;* and our ultimate test of the adequacy of any such device would be "to try whether by conceivable variation of circumstances we can cause it to break down. . . . Thus to recur to a very venerable old logical joke, when it was proposed to define "man" as "a featherless biped," a plucked cock was exhibited by way of confutation".[2]

We may also identify an object by assigning, instead of its *differentia*, one or more of its *inseparable accidents*, or such a *collection* of its *separable accidents* as is found in that object alone. This process is called *Description.* It is the only way we have of defining *individuals.* Though of great practical utility, the device is a precarious one, for it is liable to be vitiated by change of circumstances. Thus, we may *describe* "midday" as "the time when the sun is due south," but the description will be correct *only in the northern hemisphere.*

The process sometimes called *Physical Definition* or *Physical Division* or *Physical Partition* is nothing more than the *description* of a physical object by an enumeration of the parts of which this is composed. For example: "*A ship is an object composed of*

[1] *Cf.* JOSEPH, *op. cit.*, pp. 78, 83 *sqq.* [2] VENN, *op. cit.*, p. 304.

hull, masts, sails, rigging, etc."; "*Man is a being composed of a rational soul and an organized body*".

The so-called "definitions" to be found in dictionaries belong for the most part to one or other of those classes of substitutes: they are *descriptions*, or at best *distinctive explanations*, rather than *definitions*. And the same is largely true of what passes for definition in literature, prose as well as poetry. It is only in scientific works we may hope to find the precision of thought which it is the aim of *logical definition* to secure.

Although it is not the duty of logic, but of each special science in its own department, to *construct* definitions, it *is* the duty of logic to *control* the process, and to secure that all definition contribute to *clear* and *distinct* thought by prescribing certain general canons or rules with which definition should aim at conforming.

57. RULES OF DEFINITION.—The following rules, traditionally set forth in logical treatises, give the *conditions* which a good definition should fulfil, and at the same time the *characteristics* which serve to distinguish an accurate from a defective definition:—

I. A definition should contain neither more nor less than the connotation of the term to be defined; or, *should be coextensive with the thing to be defined.*

II. It should be in itself clearer and simpler than the thing to be defined, and, therefore, should not contain ambiguous or figurative language.

III. It should not contain terms synonymous with the name of the thing to be defined.

IV. It should as far as possible be expressed by positive rather than by negative terms.

Briefly, then, a definition should be (1) *adequate* and *precise*, (2) *clear* and *simple;* and it should *not* be (3) *tautologous*, or (4) *negative.*

A very brief word of explanation will suffice for each of these rules.

Rule I.—Should the definition contain, in addition to the *differentia*, a *proprium* or an *inseparable accident*, this will not in any way unduly limit the denotation of the term, but it may mislead by suggesting the existence of a really non-existent class of things. To define "equilateral triangle" as "A triangle having three equal sides and three equal angles" suggests the

existence of a class of triangles having one of these attributes only, though no such class could exist. Should the definition contain a *separable accident* in addition to the *differentia*, it will unduly lessen the denotation which we desire to ascribe to the class in question. To define "labourer" as "one who performs manual work for wages" would exclude slaves and others who, though they may not work *for wages*, are certainly *labourers*.

On the other hand, should the definition give only part of the connotation (by assigning a *remote* instead of the *proximate genus*; or by assigning the latter alone, without the *differentia*), it will unduly enlarge the denotation of the class name by including groups never meant to be included (50). This is, perhaps, the most common of all the mistakes liable to be committed in forming definitions.

Rule II.—The terms which give the *genus* and *differentia* respectively, must, of course, be *less complex*, *more simple*, than the name of the class to be defined, seeing that the connotation of each separately is less than that of the latter. But the terms of a definition are often by no means so *familiar* as the term under definition; provided, however, they convey, when understood, a more explicit knowledge of the nature of the thing than the name under definition does, the definition is a good one. Familiarity with the application of a term is often unaccompanied by any definite knowledge of its implication. Such statements as that a "net" is "a reticulated fabric, decussated at regular intervals, with interstices and intersections," that "the lion is the king of beasts," that "bread is the staff of life," that "logic is the medicine of the mind," violate the rule under consideration; the first of them violating the third rule also.

Rule III.—The violation of this rule—against tautology—is called a *circulus in definiendo*, defining in a circle. The fault is committed whenever the term to be defined, or any other term synonymous with it, is introduced into the definition. It is likewise committed when we define relative terms (42) by their correlatives: it is the *ground* of the relation that must be given in such cases. Thus, "virtue" is not properly defined as "the opposite of vice".

This rule may easily be violated in English owing to the numerous synonyms which the language has derived from Teutonic and Latin sources. Of course, such flagrant violations as "Truth is veracity," or "Veracity is truth," would not be seriously

committed. But it is by no means uncommon to find a less evident, though not less real, synonym, involved in a so-called definition: *e.g.* "Life is the sum of vital functions"; "An archdeacon is one who exercises archidiaconal functions"; "A feeling is *pleasant* when it is *desired* because of itself; we *desire* only what we in some way *represent to be good*; the sensibility *takes that to be good* which *warrants or promises pleasure* . . . the *desire* rests on *pleasant feelings*";[1] all of which comes to this, that "the *pleasant* is the *desirable*, and the *desirable* is the *pleasant*."

It must be remembered, however, that not all terms are capable of definition, that where strict definition is impossible recourse may be had to other devices, and that one of the means of explaining the import of a strange word is by offering simpler synonyms.

Rule IV.—Definition should tell us what the thing *is*, not merely what the thing *is not*.

It is not easy, however, to observe this rule in defining opposites. Only concepts that are negative, or arrived at by a process of negation, may have negative definitions. And we have seen that often when the form of the term suggests a negative concept, the latter is really positive (38). "Intemperance," for instance, is a more positive concept than "temperance," for what it really denies or removes is the *due limit* implied in the latter term; it is therefore properly defined as "excessive indulgence". On the other hand, terms that are apparently positive often express concepts reached by negation: "A bachelor is an unmarried man, . . . A stool is a seat for one without a back to it."[2] In regard to all such concepts, it is easier to take exception to negative definitions—such as Euclid's definitions of a "point" as "that which has neither length, breadth, nor thickness," and of "parallel straight lines" as "those which lie in the same plane and which, being produced ever so far both ways, never meet"—than to suggest suitable substitutes for them.

Of course, *purely negative* concepts, and *privative* concepts, cannot be really defined except as the negation or privation of what is connoted by the correlative positive concept: "inequality" is "the absence of equality"; "blindness" is "the absence of sight in a subject capable of vision". In such cases, the positive con-

[1] Quoted from UEBERWEG'S *Logic* (p. 175) by WELTON, *Logic*, i., p. 117.
[2] JOSEPH, *op. cit.*, p. 99.

cept itself should be defined independently of the negative or privative concept.

In regard to what are called material contradictories (38) or "counter-alternatives"[1]—such as male and female organisms, odd and even numbers, real and personal property, straight and curved lines, citizens and aliens, etc.—if we define one member of any such pair as the contradictory of the other—"alien," for instance, as "one who is not a citizen of this country"—we must define this other independently of the former; else we violate *Rule III.* by a *circulus in definiendo.*

VENN, *Empirical Logic*, chap. xi. WELTON, *Logic*, bk. i., v. CLARKE, *Logic*, pt. i., chap. x. JOSEPH, *Logic*, chap. v., pp. 188-90. KEYNES, *Formal Logic*, pp. 31-40.

[1] JOSEPH, *op. cit.*, p. 99.

CHAPTER IV.

DIVISION AND CLASSIFICATION.

58. General Character of Logical Division.—As definition dealt with the *intension*, so does division deal with the *extension*, of concepts and terms. Logical division may be defined as the *Analysis of the extension of a more general concept into less general concepts*. It is the distribution or splitting up of a class into its sub-classes. It is *not* the distribution of a *lowest* sub-class or *species infima* into the *individuals* which constitute the denotation of the latter: this process is called *Enumeration*. It is only a *genus*, therefore, that can be logically divided. This genus is called the *totum divisum*, or *totum dividendum*, and the constituent sub-classes are called the *membra dividentia*, the *dividing members*—because they embody the generic concept by modifying it each in a different way. Starting with the generic concept, we trace downwards the various forms or modes in which it is differentiated in the things wherein it is embodied. In order, therefore, to divide a genus into two or more co-ordinate species, we must obviously think of some peculiar modification (of the generic concept) possessed by some members of the genus and not by others, or possessed in clearly and definitely varying degrees by different groups of members of the genus, and make this specific mode the *basis* of the act or process of division. Such an attribute, thus serving as the reason, or basis, or ground, of a division, is called the *Fundamentum* (or *Principium*) *Divisionis*. Thus, taking "number" as *totum divisum*, and *divisibility by two* as basis of division, we divide numbers into odd and even; taking "conic sections" as genus, and the *direction of the plane through the cone* as *differentia*, we divide conic sections into the ellipse, the parabola, and the hyperbola; taking "triangles" as *totum divisum*, we may select *equality of length of sides* as *fundamentum divisionis*, yielding three sub-classes: equilateral, with all three sides equal; isosceles, with only two sides equal; scalene, with no sides equal.

Or, again, we may take as *fundamentum divisionis* for triangles the *size of the largest angle*, thus also arriving at three sub-classes: obtuse-angled, having one angle of more than ninety degrees; right-angled, having one angle of ninety degrees; and acute-angled having each angle less than ninety degrees.[1] From the latter example it will be seen that the same genus can be divided in various ways according as we take different *grounds of division*. Such distinct processes of dividing the same genus are termed *co-divisions;* each yields sub-classes which may partially or totally overlap those of the others. The sub-classes yielded by any one act of division are called *co-ordinate species* of the genus divided. Each of these sub-classes may be itself logically divided on some new[2] basis, into two or more narrower classes, and these again on another basis into other still narrower groups, and so on. This continued application of the process is called *sub-division.* It may be carried from a *summum genus* right through any *predicamental line* down to the *infima species* which has only individuals under it (46).

59. Relation to Definition and Kindred Processes.—Just as definition serves to introduce *distinctness* into our ideas by setting forth their connotation, so division introduces *clearness* by marking the boundaries of their denotation (49). The two processes are complementary and inseparable from each other. We define a species by assigning its proximate genus and its specific difference. The latter becomes a *fundamentum divisionis* for the genus and suggests the division of the latter into the species defined and one or more co-ordinate species. The essential function of both processes is to trace out the different embodiments or realizations of our generic concepts in the things of experience. And while their combined application enables us to arrange, compare, co-ordinate, and subordinate, our general concepts of

[1] In each of these examples we can see from the very nature of the genus in question that there can be no other alternative modes or species of it besides those enumerated; we divide the genus "with a perception that the species revealed in experience are such as must necessarily have existed in that genus" (Joseph, *op. cit.*, p. 119). But this is owing to the peculiar clearness of the abstract intuitions of space with which geometry deals. In the concrete, physical sciences, we have no such *a priori* conviction that the divisions we make must be exhaustive. Here we must wait on experience for an *a posteriori* verification of our classifications (62).

[2] *i.e.* new in the sense that it cannot be a mere repetition of the previous basis, but must be a special modification of the latter, or another and distinct modification of the genus as already specified by the previous basis. *Cf. supra*, 43; *infra*, 62; Joseph, *op. cit.*, pp. 112, 116 *sqq.*

things, it also leads us up to the clear understanding of those widest and simplest notions which form the principles and axioms of all the various human sciences. The importance of the function of logical division in the sciences and in philosophy will be apparent from what we shall have to say in the present chapter on *Classification*, and in the next chapter on the *Categories*.

Definition itself is a sort of *division* or *analysis* of the *implication* of the name or notion, while logical division is concerned with the *application* of the latter. Every concrete general name or notion has those two aspects (30, 36, 37). It is an intensive whole, sometimes called a *Totum Comprehensionis* or a *Totum Metaphysicum*, resolvable into abstract *attributes* by *Definition*, or by *Metaphysical Analysis* (50). It is likewise an extensive whole, sometimes called a *Totum Logicum* or *Totum Extensionis*, resolvable by *Logical Division* into smaller "logical parts," subclasses or groups of individuals, which may be comprised under it.

Of each and every one of these sub-classes, comprised by logical division in the extension of a given genus, the attributes connoted by the name of the genus may be predicated. That is to say, we can predicate the "logical whole" of its "logical parts"; we can say "men are living things," "horses are living things," "trees are living things," etc.[1] This shows that the relation of "logical whole" to "logical parts" is quite different from the relation of any *real* whole to its real parts. We cannot assert the whole connotation or comprehension of a term, as predicate, of any portion of the same as subject, or *vice versa;* we cannot assert that "the capacity of rational speech is man" or *vice versa;*[2] nor can we, in case of *physical partition*, or the *physical description* which expresses it (56), predicate the part of the whole, "the ship is the mast," or *vice versa*.

Finally, division as well as definition has to do with *names* or *terms* inasmuch as it is only through these it can deal with concepts and things. We have seen in what sense definition is of names, and how far verbal disputes turn on equivocation or ambiguity in the use of terms (54). The process of distinguishing between the various meanings attached to an equivocal or ambiguous term is sometimes described as *Verbal Distinction* or *Verbal Division*. Here, too, we can predicate what is *apparently* the same "whole" of what are *apparently* its "parts". We can say, "This tool is a vice," and

[1] Similarly, we can predicate the parts of the whole, though only *indefinitely* (93), *e.g.* "some animals are men".

[2] We can, however, predicate the metaphysical parts *concretely* about the whole, *e.g.* "Man is *capable* of rational speech".

"This fault is a vice," but in reality a *different* logical whole is expressed in these cases by the same verbal symbol as predicate.

60. FORMAL AND MATERIAL ASPECTS OF LOGICAL DIVISION: DICHOTOMY.—We have already seen (50) that *formal* logic assumes the definition or connotation of terms to be known by those who make use of them, and that the treatment of definition (in *formal* logic) is justified only because, and in so far as, this assumption is not verified in real life. We have now to observe that without some *additional* knowledge of a class concept, *over and above what is contained in its connotation*, we could never subject it to the process of *logical division*. In order to divide any genus logically, we must know some note or aspect which is a *separable accident* of that genus, *i.e.* which modifies the genus in a certain way as found in some, and not at all, or in a different way, in other members of the genus, and which will serve, therefore, as a *basis* or *fundamentum divisionis*. And, seeing that our knowledge of any such basis is not contained in what is postulated by *formal* logic, *viz.* the connotation of the genus, but must come from an outside source, it follows at once that the process of logical division is *always a partly material, never a purely formal*, process: unless, indeed, starting with a class A, real or imaginary, we *imagine* an attribute B to be possessed by one, and not by another, group of the class A, and on this basis divide A into B and $\overline{B}$[1]; and proceed similarly to divide B or $\overline{B}$ on the basis of another imaginary separable accident C, into BC and $B\overline{C}$, or into $\overline{B}C$ and $\overline{B}\overline{C}$; and so on indefinitely. Were we to carry on such a process, it might indeed be called *formal* inasmuch as it would make no appeal for information to any source independent of the concept with which we started, and of our own imagination for a basis of division;[2] but, obviously, we should have no guarantee whatever for the reality of our imagined sub-classes; their real existence would be purely hypothetical; and in case we *did* appeal to facts in order to check and verify our imaginary divisions, these would cease to be hypothetical and formal, and would become absolute and material, or real, precisely in the measure in which we made that appeal.

The division of a class, whether real or imaginary, into a pair of sub-classes, the one denoted by a positive, the other by a purely negative, term (39), is called *Dichotomy* or *Division by Dichotomy.*

[1] $\overline{B}$ = *not-B.* [2] *Cf.* KEYNES, *Formal Logic*, p. 449.

It is purely formal whenever the grounds of it are purely imaginary and give rise to merely hypothetical sub-classes ; it is material in so far as it deals with a real class and has for its grounds certain attributes found in some members of that class and not in others. Porphyry's Tree (46) furnishes a good illustration of a number of successive dichotomous divisions (of the successive *positive* class concepts), all of which are material or real, and not merely formal or hypothetical.

Although dichotomy, based as it is on the principles of contradiction and excluded middle, necessarily secures sub-classes which are mutually exclusive, and collectively exhaustive of the class divided, it is, nevertheless, useless as a means of dividing real genera into real sub-classes, since it does not guarantee the existence of all its sub-classes. It does not represent as co-ordinate, classes which really are co-ordinate. It fails to exhibit its sub-classes as so many different positive characterizations, or modes of realization, of the genus divided ; for half of its sub-classes are wanting in any positive specifying character, being expressed by purely negative terms ; and, besides, at each step it takes as a distinct *fundamentum divisionis* what is really only a part of one single *fundamentum divisionis*, thus reaching by a cumbrous and roundabout method a number of *infimae species* which could have been reached equally well by one single step. If a genus is seen to fall naturally into three, or four, or fourteen, co-ordinate sub-classes, on a certain basis, it should not be divided first into two, and so on, dichotomously, until all the sub-classes are reached. It is more natural to divide triangles into equilateral, isosceles, and scalene, than into equilateral and non-equilateral, the latter into isosceles and non-isosceles, and the latter again into scalene and non-scalene (if any). Moreover, if we are not certain whether we are including all the sub-classes into which the genus falls, whether we are perhaps omitting some real sub-class—and this is often the case—dichotomy will not help us by its ultimate hypothetical negative class. Without the aid of dichotomy we can, in such doubtful cases, add to our *infimae species* a hypothetical class of "others, *if any*".

The only fruitful application of dichotomous division to *real* classes of things is that referred to in the last chapter (49)—its application for the purpose of discovering and formulating the *definition* of some class.

It can be applied, however, in a mechanical, mathematical way,

within the domain of intellectual concepts, abstracting altogether from the existence or possibility of any objective counterparts for those concepts. This process is known as *Purely Formal Division.*

61. PURELY FORMAL DIVISION.—The development of dichotomy along purely formal lines, *i.e.* through imaginary and hypothetical grounds of division, leads to the conception of the positive and negative members so obtained, not as existing classes, but as (possibly *full* or *empty*) *class compartments:* a conception which has proved exceedingly useful and fruitful in the peculiar treatment of certain logical problems, which has come to be known and described as *Symbolic Logic.*

In this purely formal, dichotomous division of a given class name, the number of subdivisions made will depend exclusively on the number of new terms successively introduced as foundations for the successive steps. Thus, taking any universe of discourse X, we may first divide it into S and $\overline{S}$ ($\overline{S}$ being a shorter way of expressing *not-S*); each of these, next, into M and $\overline{M}$; and each of the resulting four into P and $\overline{P}$. We have now reached eight *class compartments* into which existing classes may be fitted. The utility of this process will become apparent when we learn later on that it is possible to interpret every universal proposition as denying the existence of a certain class, or, in other words, as asserting the emptiness of a certain compartment. Of the eight compartments S M P, S M $\overline{P}$, S $\overline{M}$ P, S $\overline{M}$ $\overline{P}$, $\overline{S}$ M P, $\overline{S}$ M $\overline{P}$, $\overline{S}$ $\overline{M}$ P, $\overline{S}$ $\overline{M}$ $\overline{P}$, the universal proposition "No M is P" would empty the compartments containing M P, *i.e.*, $\overline{S}$ M P and S M P; and the universal proposition "All S is M" would empty the compartments containing S $\overline{M}$, *i.e.* S $\overline{M}$ P and S $\overline{M}$ $\overline{P}$; the combination of both propositions thus leaving S M $\overline{P}$, $\overline{S}$ M $\overline{P}$, $\overline{S}$ $\overline{M}$ P, $\overline{S}$ $\overline{M}$ $\overline{P}$, as the only classes capable of existing compatibly with the truth of both propositions.[1]

Were the divisions thus obtained supposed to represent existing classes, and not merely class compartments, the process would be misleading and invalid; for, not every combination of attributes represents a class capable of existing, *e.g.* the combination "right-angled-equilateral-triangles" represents an impossible class, an empty compartment.

In contrast with this purely formal process, the development of logical division on the basis of attributes *found in really existing things, e.g.* along *real* or *material* lines, is known as *Classification.* Before dealing further with this latter, it will be convenient to formulate certain conditions to which *all* logical division must conform, and which are commonly known as the *Rules of Logical Division.*

[1] *Cf.* WELTON, *Logic*, i., p. 133.

62. RULES OF LOGICAL DIVISION.—I. *Each act of division must have only one basis.*

II. *The sub-classes must be together co extensive with the divided whole.*

III. *In a continued division each step should divide a class or sub-class into its proximate sub-classes.*

In other words, logical division must (i) *not be cross-division*, but give results that are "mutually exclusive"; it must (ii) *be exhaustive*, or give results "collectively exhaustive" of the denotation of the divided whole; and finally, if continuous, it must (iii) *be step by step: Divisio non faciat saltum.*

These rules are variously stated. Sometimes superfluous rules are given, as *e.g.* the rule that *only class terms can enter into a logical division.* This is involved in the very definition of division. Sometimes, also, a rule is laid down to the effect that *none of the dividing members must be equal in extent to the divided whole.*[1] This rule must be observed in *real* or *material* division, or, as it is commonly called, *classification;* for here the sub-classes are supposed to be groups of really existing things, and if one sub-class be coextensive with the whole, the others are non-existent, and there is no real division at all; but the rule cannot be insisted on in purely formal or dichotomous division, in which the sub-classes are not necessarily supposed to exist, but are regarded as mere hypothetical possibilities of existence—class compartments which may be full or empty without detriment to the formal accuracy of the process.

RULE I.—A *cross-division* is one in which some member or members fall into more than one sub-class, so that these are not mutually exclusive but overlap. This cannot happen unless, in the act of division, we fail to adhere to one and the same basis of division. There is no danger of this when the division is dichotomous; and very little danger when the genus is seen to be such that it must yield, on a given basis, a *small* number of alternative groups, as in the examples given above (58) from mathematics and geometry. But when the immediate result of applying a given ground of division would be to divide the *genus* into a large number of co-ordinate sub-classes, there is a danger that before all are set down we may inadvertently modify the ground of our division, partially or totally. If we do, the result may be (*a*) to include some individuals twice, or oftener, or (*b*) to leave out some individuals altogether, or (*c*) to commit both faults; or, finally, we may (*d*) accidentally escape both faults and reach an accurate result. For example, we commit both, if, intending to

[1] CLARKE, *Logic*, p. 236. *Cf.* KEYNES, *Formal Logic* (4th edit.), p. 445.

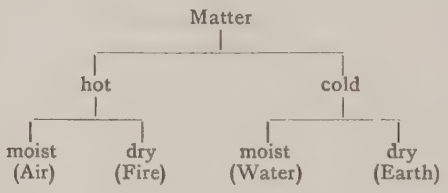
divide triangles on the basis of *equality of length of sides*, we set down two members of the division, equilateral and isosceles, and then, passing to another basis, *viz. size of angles*, we set down as a third member, right-angled triangles: for we thus include right-angled isosceles triangles twice, while excluding obtuse-angled-scalene and acute-angled-scalene altogether. On the other hand, a division of triangles into equiangular, isosceles, and scalene, *happens* to be accurate; though made on a twofold basis, because the two principles simultaneously, employed (relation of angles and relation of sides) happen to give the same sub-classes. So, too, the division of animals into cloven-footed and non-ruminants happens not to be a cross-division simply because the two bases, employed simultaneously, happen to coincide in their results. These divisions are, nevertheless, logically unsound, because by confusing different grounds in the same act of division they lead to sub-classes which do not exhibit, as they should, their mutual exclusiveness.

It is sometimes stated,[1] in connexion with the rule under consideration, that we should preserve the same basis, not merely throughout any single act of dividing a genus into its proximate co-ordinate species, but *as far as possible throughout all the subsequent stages of sub-division of these species;* not, of course, the same identical mode of that basis, but some variation of the latter (58): so that on the predicamental line leading from the genus to any one of its *infimæ species*, each *differentia* will appear as a more definite and specific mode of the preceding *differentia:* "When the division is carried further than one stage, the same *fundamentum divisionis* should be retained in the later stages which was used in the first".[2] It is certainly desirable to exhaust in this way the potentialities of a given basis of division as far as may be feasible: it will have the advantage of showing all the sub-classes thus derived to be "alternative developments of a common notion, or variations on a common theme";[3] thus tracing the generic notion downwards through its specific embodiments. It will have this advantage, too, that a *differentia* which is a modification of a preceding one can never be predicated of any other class co-ordinate with the preceding genus[4]; whereas

[1] *Cf.* JOSEPH, *op. cit.*, pp. 104, 112, 116.
[2] *ibid.*, p. 104. *Cf. supra*, 49. [3] *ibid.*, p. 108.
[4] "*Biped* is not applicable to *footless*, the other member along with *footed* of the genus *animal*."—JOSEPH, *op. cit.*, p. 116.

when a "new" basis is introduced into a scheme of division, it may, for all we know, be predicable of some other class, co-ordinate with some higher genus.[1] Aristotle seems to have regarded the former as the more proper sort of division (*κατὰ τὸ ὀρθόν*), counting all the subordinate *differentiae* between the genus and its *infima species* as *one;* and to have regarded any division which separated the *infima species* from the genus by a number of distinct *differentiae*, unrelated to one another so far as our knowledge goes (67), as less proper and less scientific (*κατὰ τὸ συμβεβηκός*).[2]

But this method of division is not always possible in dividing the *summa genera* of things in the world around us into sub-classes, or in grouping together (or classifying) species into genera. We are forced repeatedly by facts to recognize, between sub-classes of things, *differentiae* which we cannot see to be modifications of, or in any necessary way connected with, the higher *differentiae* of the higher genera to which these sub-classes belong. In all such cases, what we may call the *total basis* on which the genus is differentiated into an *infima species* is *the combination* of all the various differentiating concepts employed. Thus, Aristotle based his division of matter into four fundamental elements on the combination of two attributes, temperature and humidity:—



RULE II.—This rule may be violated in two ways: by omitting some of the sub-classes, or by including some that are not sub-classes, of the genus: in other words, by making the division *too narrow* or *too wide.* In either case it is not the supposed genus that is divided, but that genus *plus* or *minus* some additional group or groups of objects. We are not likely to violate these rules in simple cases: to divide triangles into equilateral and isosceles, omitting scalene; or to divide coins into gold, silver, copper, and banknotes. But the accurate logical division of the

[1] "*Feathered* and *featherless* might be applicable to *quadruped* no less than to *biped*" (*ibid.*); *rational* might be predicable not only of *man* but of *incorporeal substances* such as angels (*cf.* p. 73 *n.* 3).

[2] *ibid.*, p. 116.

classes of things existing in the world around us, and expressed by the common names of ordinary language, is not at all so easy a matter. It is in fact an ideal which can at best be only imperfectly realized. Our success in making an exhaustive division of any given genus, politicians, for example, or styles of architecture, will depend upon the extent and minuteness of our information about these departments. Any attempt to divide such classes into their *real* sub-classes—as distinct from mere hypothetical, fanciful sub-classes—will show us at once how such division is dependent on material considerations, on our knowledge of matters of fact.

RULE III.—If each genus be not divided into its *proximate* species, some intermediate species may be omitted, thus making the division too narrow. Thus, if I attempt to divide rectilinear plane figures *immediately* into such remote species as equilateral triangles, squares, parallelograms, pentagons, etc., I am running considerable risk of omitting one or more sub-classes. If I give one or more proximate species, *e.g.* triangles and polygons, simultaneously with one or more remote species, *e.g.* parallelograms and squares, I am making a *disparate* division, which is pretty sure to be confusing even when it is not also inaccurate.

63. "MATERIAL" DIVISION OR "CLASSIFICATION".—Let us now turn from the purely formal aspect of logical division (61), and inquire how we are to put its principles into practice, how we are to group together logically the objects which form the material of all our knowledge, and thus introduce order and clearness into our ideas.[1] We may say at once that this is already done for us at least in a rough and ready way *in the language* we possess.[2] The formation of the common names of ordinary language has involved in it *generalization*, *grouping*, *classification*. The chaotic mass of data revealed to us in sense experience furnishes the raw material for the problem of *Classi-*

[1] It must be borne in mind that *all* logical division is a *mental* analysis, an arrangement of our *ideas* according to their greater or lesser extension. Even when an actual arrangement of *objects* (in space) or of their *names* (in a catalogue) results, as it often does, from the mental process, it is this latter *alone* we call logical division.

[2] "Ages before the logician or anyone else who deals with systems, had a hand in the matter, the necessities of common life had been at work prompting men to group the things which they observed. All names imply the recognition of groups, and a great number of names imply a subordination of groups, so that at the earliest stage to which we can transfer ourselves we find that we are already in possession of a rudimentary classification; and that we cannot even talk or think about things, without an appeal to this."—VENN, *op. cit.*, p. 322.

fication. Given the whole, or a more or less extensive portion, of that material—the universe of living things, for example—the problem of classification is to arrange (our ideas of) the things[1] which constitute this universe into a regularly related hierarchy of coordinate and subordinate groups of classes. When we have apprehended the extent and boundaries of our territory, we may either begin *above* by some great, broad division of the whole into a few departments—into plants and animals, for example,—and then, by *subdivision* of these, proceed gradually *downwards* towards the lowest and narrowest classes discernible; or, without looking first for any great line of division suggested by a view of the whole field, we may begin *below* with some of the narrowest, most obvious, and most clearly marked, groups that may happen to fall under our notice—robins, potatoes, greyhounds, oak-trees, human beings, or the like—and proceed *upwards* gradually by a process of *aggregation* of larger and larger groups, until we have exhausted the whole sphere of living things;[2] or, finally, we may combine the *upward* and the *downward* processes, working as best we can in both directions. But, wherever we begin, one thing is perfectly clear, namely, that even supposing the group to be divided, the *summum genus* as it is often called (living things), and all the *lowest groups* or *infimae species* (daisies, rabbits, wasps, herrings, etc.), to be given us in the very language we use, as data, as *fixed* groups, still between these lowest and the one highest class there will be practically endless ways of framing and arranging the intermediate classes: plants, for instance, will be classified very differently by the physician, the agriculturist, and the botanist. From which we infer that our system or hierarchy of classes will depend *on the attributes we choose as grounds of our divisions.*

64. ITS GROUNDS DETERMINED BY ITS PURPOSE: THIS EITHER "GENERAL" OR "SPECIAL": HENCE "NATURAL" AND "ARTIFICIAL" CLASSIFICATIONS.—Hence arises the question: How, in classification, are we to determine our *fundamenta divisionis?* What considerations ought to influence us in our selection? Needless to say, logic cannot tell us what attributes, as grounds

[1] Or *events*, as the case may be: the problem of classifying natural events or phenomena is by way of discovering their *natural causes*, the *laws* according to which they take place: which is the main problem of physical induction.

[2] "In the physical process of sorting shot or gravel into a number of packets according to size, it would come to the same thing in the end whether we made use of the sieves by beginning with the finest or with the coarsest."—VENN, *ibid.*

of division, will yield the best results in any particular department of science; nor whether—when we have made a number of divisions and subdivisions, selecting at each step the basis we have thought best,—the results of our process are exhaustive. What, then, ought to guide us in selecting the grounds of our divisions? Obviously, *the end we have in view* in making the classification. The *general* aim of all classification is, of course, to give us clear ideas, definite, well-ordered knowledge, control over facts, increase of power in retaining and communicating our knowledge about them. But every single department of facts will be found to yield on investigation several widely distinct and very *special* kinds of knowledge, in addition to what may be described as general knowledge of that department. And hence we may have one or other of two possible purposes in approaching any sphere of classifiable data: we may wish to classify the contents of the sphere in question with a view to obtaining some *special kind* of knowledge about them, with this *special* object in view; or, without any such particular preoccupation, we may approach it with a view to acquiring *general* knowledge, general information about them. The former process is called *Classification for a Special Purpose*, the latter, *Classification for General Purposes.*

When the physician classifies plants on the basis of their medicinal properties, or the agriculturist on the basis of their utility as food for animals, or of their suitability to different soils, we have instances of classifications for special purposes; while the botanist's classification of plants is for the general purpose of promoting our knowledge of their origin, nature, and general relations and properties.

These two kinds of classification are also called respectively *Artificial* and *Natural:* because in the former we deliberately group the objects (mentally) for a certain definite, arbitrarily selected purpose, by means of a basis determined, if not even invented, by ourselves; while in the latter we rather recognize differentiating attributes already existing in the facts, and so, instead of *inventing new* mental divisions for *practical* purposes, we *discover existing* classes, and so secure a better *speculative* knowledge of the latter.

Many modern logicians take exception to the use of the terms *artificial* and *natural* in this context: pointing out that *all* classification is *artificial* inasmuch as it is a *voluntary* arrangement *of our ideas*, not a segregation *of*

the objects themselves as if these were given to us, or disposed by us, in distinct groups, separated in time or in space in the real world; and that *all* classification is, or ought to be, *natural*, in the sense of conforming to the facts and suiting the special purpose in hand. However, notwithstanding the force of these reasons, it is impossible to deny the propriety of describing (65), for example, the mental scheme that directs the alphabetical arrangement of a library catalogue according to the initial letter of each author's name, as an "*artificial*" classification of the known contents of the library, and the genealogical schemes of the botanist and zoologist as "*natural*" classifications of known plants and animals. And although, as we shall see from the various examples given below, it is impossible to draw any definite line of demarcation between the two kinds of classification, to say where "nature" begins and "artifice" ends, still we shall be able to point to ample reasons for calling the one kind "natural" and the other "artificial" classification.

65. CLASSIFICATION FOR SPECIAL PURPOSES.—This is the simplest kind of classification, and its "artificial" character will be apparent from a few characteristic examples. The names of the topics dealt with in a book are, for example, sometimes classified in groups following the order of the alphabet, in an *index* at the end of the book, with references to the pages on which the treatment of them will be found. The *special purpose* there is to facilitate the study of these topics by saving time and trouble in searching for them; and the alphabetical classification is the best for the purpose. So, also, in the arrangement of a library catalogue. And the likelihood that a person who consults a book on any topic may need to consult others on the same topic, at once suggests a new basis for arranging the books on the shelves: that all books on the same topic should be arranged in the same place, side by side on the same shelf or shelves. To arrange the books according to *size*, or to *binding*, or to *language*, would be perhaps considered more artificial than the arrangement according to subjects; while all alike would be perhaps considered less artificial than the alphabetic arrangement of the authors' names in the library catalogue.

Again, suppose we are interested in the study of botany and have at hand a copy of Bentham's *British Flora*. *Hearing the name* of a strange flower, we go to (*a*) the alphabetical index at the end of the volume, and are there referred to various places in the body of the book, where we find the name figuring in (*b*) a system of classifications[1] quite other than the alphabetical one

[1] This system is experimentally illustrated as far as possible in the arrangement of the flower-beds and plant-plots in a botanic garden: plants of the same

at the end. Suppose, however, we *meet a specimen* of a strange flower *the name of which we do not know*, we have recourse to (*c*) a third scheme of classification standing at the beginning of the volume and known as the *Analytical Key*. This latter scheme has been constructed for the precise purpose of enabling us with the least possible trouble to make a "*diagnosis*," or, in other words, to discover the name and identity of any specimen we have before us. It is a dichotomous or bifurcate scheme, which takes as *fundamenta divisionis* some great, broad, obvious characteristics, the presence or absence of which is easily detected in any specimen: dividing flowers into compound and non-compound; flowers with one seed and flowers with more than one; aquatic and land flowers; creepers and non-creepers, etc., etc. This analytical key is a *more* "natural" classification than the alphabetical index, though its object is, by discovering the name, to send us to the latter, and thence to the *really natural* or botanical classification in the body of the work.

Although *special* or *artificial* classes may be made for their own sake, and without any ulterior object, yet they are usually made as a means to the discovery and knowledge of other "natural" lines of division. This is manifest in the case of analytical keys and alphabetic arrangements: "we seldom or never want to refer to [these] for their own sakes but only as a help to the identification of the ultimate [classes]. We have not the slightest interest, for instance, in the names which begin with S, taken as a whole: we just rest on this as on a sort of landing for an instant, on our way towards reaching Smith or Scott or Sykes. Similarly with the highly artificial classes in the analytical key: 'trailing plants with evergreen leaves' is one element of a disjunctive alternative which happens to make itself useful at one of the steps in the course of deciding between the *Periwinkle* and other plants, but as a class for any other purpose it is never recognized. But *directly we begin to broaden the ultimate aim of our arrangement, the substantive importance and independence of the intermediate classes begins to be established.*"[1]

What are those "ultimate" classes which we *do* seek "for their own sakes"? those classes which have a "substantive importance and independence"? and by what scheme of classification do we establish them? They are what are known as "natural" classes—the classes arrived at by a scheme of "classification for general purposes".

66. CLASSIFICATION FOR GENERAL PURPOSES.—How are we so to classify the objects investigated by the various human sciences as to contribute most and best to our general knowledge

natural class being placed near one another—as books dealing with the same subject are placed on the same library shelf.

[1] VENN, *op. cit.*, p. 331 (italics ours).

of them by such classification? The vast extent of the problem constitutes its difficulty. What general classification of plants or animals, for example, will give us the clearest and deepest insight into their nature, constitution, characteristics, relationship, origin, utility, and so forth?

The duty of the logician here is merely to make suggestions of a general nature. The value of these will be tested by the facility or difficulty of putting them into practice, and by the practical results reached by acting on them.

(*a*) We are told, for example, that if we begin with the *summum genus*, proceeding downwards, we should aim at selecting as basis of division, at each stage of the process, the *most important* attribute,[1] meaning by "most important" that which is most important for the purpose in hand: and the purpose here being the *acquiring of general knowledge*, the "most important" attribute will be the one which will enable us *to make a maximum amount of aggregate assertion with a minimum number of propositions* about the classes so obtained.

The aim here is intelligible enough. The grouping together, in a Trade Directory, of people's names beginning with S, does not enable us to make a single other statement about them. The arrangement according to *streets* will probably give us a basis for numerous statements and inferences about those dwelling in a given street; the arrangement according to *trades* will be still better in this respect, and so on. This example, from "artificial" classification, shows how one method of classifying may enable us to affirm economically about the members of the classes, much more than another method does. So, too, will the "natural" botanical classification, as compared with the analytical key, and this again as compared with the alphabetical index. And so, again, the classification of flowers on the basis of similarity of structure, as compared with that on the basis of similarity of colour.

But is the recommendation in question as practicable as it is intelligible? That the degree of power and facility obtained for embodying a maximum amount of knowledge about the classified objects in a minimum amount of language, is a good test of the value of any scheme of classification, *when that scheme has been made out*, is undoubtedly true; for the individual members or groups constituting each such class or sub-class will be

[1] *Cf.* what was said about *Proprium*, *supra*, 47.

seen to have the richest known stock of *common* attributes, and this already implies the best possible contribution to our knowledge of the objects so classified.[1] But to determine beforehand, when still face to face with the undivided *summum genus*, what attribute *is* the "most important," in the sense of involving the greatest number of distinct and dependent modalities within it (62, R. I.), and to proceed to divide on this basis—is not by any means an easy undertaking. Attributes of objects are not numerically distinct entities that may be weighed, measured, and counted mathematically. Nor does the *real importance* of an attribute depend on the "number" of other attributes involved in it; for the "number" of attributes we may discern in any object depends largely on the depth of our own mental analysis of that object, on the number of distinct points of view from which we regard it. And while some of these attributes may open up rich and fruitful vistas of knowledge, others may not carry us a single step beyond themselves. Considerations, therefore, of mere number are useless here. Again, then, how are we to know what attributes are "most important" in view of general knowledge, in order to base our classification on these?

(*b*) John Stuart Mill's answer is instructive. They are, he says, "those which contribute most, either by themselves or by their effects, to render the things [in any class] *like* one another, and *unlike* other things [of other classes]; which give to the class composed of them the most marked individuality; which fill as it were the largest space in their existence, and would *most impress* the attention of a spectator *who knew all their properties but was not specially interested in any*".[2]

The concluding words of the extract just quoted show clearly what Mill means by the "most important" attributes; but, unfortunately, since we can never say of any objects that we know "all their properties," we can never be certain that the attributes which "most impress *our* attention" are really the "most important". The words also show, however, that the more we do know about the properties of the objects to be classified, the more likely we are to make a *good* general or natural classification—provided we are "not specially interested," or *biassed*, in respect of any of these properties or any special interpretation of them: a condition which, as we shall see presently

[1] And the recognition of a stock of common attributes is simply the recognition of identity amid diversity, *i.e.* of *real resemblances*—thus ultimately resting on the "resemblance" test given below.

[2] MILL, *Logic*, Bk. iv., chap. vii., § 2, *apud* WELTON, *op. cit.*, p. 142 (italics ours).

(in reference to Evolution), may considerably influence the whole trend and significance of our attempts at classification.

We have been looking for some tangible test whereby to select "important" attributes on which to frame a classification for general purposes, and we have at last, as it seems, found something tangible, in the opening words of the extract just quoted from Mill. The important attributes are those which make the members of a class "*like* one another and *unlike* other things," those which will secure "that the members of each class may *resemble* each other in as many points as possible,"[1] while, presumably, they will *differ* as much as possible from those of immediate co-ordinate classes, and more and more widely from those of more remote classes. In other words, our guiding principle in natural classification is to be *Degree of Resemblance*, *Similarity*, *Affinity* in the objects to be classified. Individuals are to be brought together into a group, and groups into wider groups, according to *Degree or Amount of Resemblance*, *Similarity*, *Affinity*.

Here we have a principle which is not merely intelligible, but—apparently at least—easy of application. It appears, moreover, to be suitable to our purpose: the gaining of general knowledge about the objects classified. For, what other way have we of advancing in our knowledge of things, than by comparing them with one another, observing their points of agreement and difference, arguing by analogy from the known characteristics of one class to the existence of like characteristics in a similar class?

In applying the principle, however, we are constantly exposed to the danger of what is known as "judging by appearances". The common advice, "Do not judge by appearances," would, of course, be unmeaning if it were interpreted literally; for, after all, what have we to judge by except appearances? The real meaning, obviously, is that we must beware of hastily interpreting points of resemblance (or difference) which may be only superficial, and of no significance or importance towards the advance of our knowledge by indicating profound and far-reaching affinities (or divergences) between the things under examination. And in this sense, the advice, though somewhat of a paradox in the form in which it is expressed, is both sound and useful.

67. Why Classification for General Purposes is called "Natural" Classification.—When we thus en-

[1] Welton, *ibid.*, p. 140 (italics ours).

deavour to classify, according to their general resemblances, the objects of visible nature—of the material universe, animate and inanimate (as distinct from the artificial things produced by, and under the control of, our own activity)—we feel that we are not *inventing* classes, or *constructing* grounds of division, but rather *discovering* classes by *recognizing* (in the existing resemblances and affinities of the objects of animate and inanimate nature) grounds of division already existing there. We feel that we are following nature, that nature itself has differentiated class from class, roughly, perhaps, but very extensively, if not indeed universally, in every domain. And this is why we describe as "natural" the classification which aims at following the broad lines of demarcation that are undeniably traced out between things independently of our own activity, whether mental or physical. Such classes, moreover, we call *Natural Kinds*—the *species infimae* of the Scholastics (46, 52). If we compare them with the artificial classes resulting from grounds of division deliberately chosen for some special purpose, we shall find that while the latter, or artificial, classes differ from one another *merely by the attribute we have chosen*—as when names differ in not beginning with the same letter, or flowers, by having or not having a certain colour,—and while nothing further can be predicated of the things on account of such a basis of difference, the former, or natural, kinds differ *very much* in many ways, *by a deep and comparatively unexplored mine* of distinguishing characteristics,[1] the gradual analysis of which will enable us to predicate very much about the classes so divided.[2]

It used to be commonly believed by natural scientists that in the animal and vegetable kingdoms these "natural kinds" were the respective groups which came by organic descent from first representatives—or pairs—brought into existence originally by distinct acts of creation.[3] This belief was confirmed by the comparatively fixed character of such "natural kinds"; for

[1] Which appear as "new" or "non-continuous" differentiae in our subdivisions (62, R. I.).

[2] "According to Mill's well-known analogy, what we find to be the boundary in such cases as these is not so much a shallow trench which we can dig for ourselves, but an apparently bottomless crevasse which has been placed where we find it by nature."—VENN, *op. cit.*, p. 335.

[3] In the inorganic or mineral kingdom, too, certain profoundly different "types" or "kinds" of matter, were regarded as differing "by nature" *ab initio*, as having been created "different". These "chemical elements"—their number still unknown—were regarded as irreducible to one another and underivable from one another; though they could combine with one another to form *new kinds* of material substance.

these were regarded as "fixed" in the sense that no one could originate, or be itself an off-shoot of, another or other groups; though each was recognized to be capable of "varying" within certain limits, thus giving rise to "varieties" within its own sphere. This "fixity" of any group of living organisms, the fact that its members continue to "breed true," to retain its characteristics and to resist all attempts to merge these, by interbreeding, with those of adjacent groups, became the *test* of a "natural kind" or "species," as distinct from a "variety," among organisms. For the fixed and unchangeable groups botanists and zoologists adopted the name "species" (though "genus" would have been more naturally suggested by the idea of descent from a common stock), while to the smaller, changeable groups within each "species," they gave the name "varieties". The higher and wider groups into which the species themselves were gathered on account of their varying degrees of similarity in structure, function, and general constitution, were called "*genera*," "natural orders," etc. (46). This relationship of similarity between the various "natural kinds" was described as "affinity": at first, perhaps, without any suspicion of a *real* affinity, *i.e.* a connexion by descent from a common stock, the term being used in a *merely analogical sense*, based on the resemblances that are so easily and universally recognized between those living things that are really cousins, or members of the same family.

However, biologists nowadays claim that a closer examination of the unexplored and "apparently bottomless" rifts of cleavage, which mark off the so-called "natural kinds" from one another, has brought to light two important facts: (1) that in very many cases these lines of demarcation are exceedingly vague and difficult to trace, and (2) that there are evidences against the internal fixity and external independence of many so-called "species," and in favour of the theory that these too are somewhat variable, and that many of them have reached their present comparatively stable and differentiated condition by evolution from a common primitive stock.

As regards the first of those two points, we have already recognized the fact that, although there are innumerable clear and unmistakable "differences" or "dissimilarities," or *grounds of division*, between groups of objects both in the organic and inorganic kingdoms of nature, nevertheless these lines are oftentimes not hard and fast,—*Natura non facit saltum*,—but such that the groups seem to shade off imperceptibly into one another (53); so that in the process of *aggregating*, or grouping individuals according to their resemblance to some one or other of certain typical specimens, chosen as *nuclei* for the formation of natural classes, we often reach a borderland of doubtful individuals, any of which may apparently fall just as *naturally* into one as into the other of two neighbouring classes: in which cases alternative classifications will give rise to alternative definitions.[1]

68. How Classification may be Influenced by Hypothesis.—Such extreme or "limiting" examples as those just referred to will serve as tests to determine which of the competing schemes of classifica-

[1] We have already called attention to the fact that when we speak of the *Definition* of a thing as being *per genus et differentiam* we mean to refer to the *genus* and *differentia* which it is found to possess in a *natural* scheme, not in any of the possible artificial schemes, of classification (50, 51). *Cf.* Venn, *op. cit.*, pp. 226, 336; Joseph, *op. cit.*, pp. 85-92.

tion is, in a given case, the more "natural," and will thus raise once more the ultimate difficulty: How are we to judge and decide between conflicting "resemblances" or "affinities" in a case in which some individual object resembles those of one group in the attributes *a*, *b*, *c*, and those of another group in the attributes *d*, *e*, *f*? Here is where our judgment, *as between the appearances*, will be inevitably influenced by our own beliefs and convictions, by our own views, theories and prepossessions, whether these be helpful or the reverse towards an approximation to a *true* knowledge of the things classified. How are we to decide whether, for this purpose, *a*, *b*, *c* are more or less important than *d*, *e*, *f*? Will not our decision depend upon what we regard as the most valuable kind of knowledge about the things? And if we take into account the second great fact referred to above (67): that naturalists claim to have detected evidences of the gradual evolution of "species" from a few parent stems, evidences of the mutability of "species" in living things: will we not regard as "most important" grounds of classification the presence or absence of such attributes as would seem—however otherwise insignificant and likely to escape notice—to point to a common descent? This is what has actually happened. The Evolution hypothesis has determined scientists to regard *affinity* between species—no less than between individuals—as indicative of a common original ancestry; to supplement the knowledge of actual living forms by the knowledge of fossil remains; to look for traces of the supposed transformation and evolution of "species" in surviving fragments from bygone, prehistoric ages; to discard any more obvious and striking resemblances that may seem to be merely superficial and to have no close connexion with the origin and descent of the living forms—colour, for instance, in the case of flowers,—as of much less importance than other less obvious and less easily detected resemblances—as, for instance, in the structure of certain minute organs,—resemblances which, however hidden from view, point to identity of origin—to descent from a common stock.

Although, under the influence of the evolution hypothesis, "conceptions of the nature and functions of classification [of plants and animals] naturally underwent some alteration," yet "as regards the details of actual classification very little change has been introduced".[1] The conception of any such hypothesis, and its application to facts, invariably stimulates scientific inquiry and leads to valuable results, and this even though the hypothesis itself may turn out afterwards to have been partially or wholly erroneous.[2] It is not the function of logic, but of the respective sciences in which such hypotheses are employed, to discuss the latter on their merits. The evolution hypothesis has been mentioned here only on account of the bearing it has on the interpretation and application of the general law which logic seeks to formulate for "natural classification" or "classification for general purposes": *that it must be carried out on the basis of* IMPORTANT *attributes*.

69. Scientific Nomenclature and Scientific Terminology.—To carry on successfully the work of classification, two distinct systems of names or terms are indispensable: (*a*) a

[1] Venn, *op. cit.*, p. 338. [2] *Cf. infra*, bk. iv., ch. v.

system of names for the groups, of which the classification consists; and (*b*) a set of terms for describing the nature and characteristics of the individual objects composing those groups. The former collection constitutes what is technically known as *Scientific Nomenclature*, the latter what is known as *Scientific Terminology*.

(*a*) The interaction between thought and language is close and constant (22). Accordingly, it is only in such sciences as zoology, botany, and chemistry, where classification has been carried out more or less completely, that anything like an adequate nomenclature is to be found.

As one and the same individual object may enter into several different artificial classifications, it might in the abstract be considered an advantage to have a different class name for it in every such system; but as a matter of fact such advantage would be much more than counterbalanced by the endless confusion that would arise from having several names for the same object. The practical needs of life demand that as far as possible one and the same object should be referred to by *only one* name. Hence, special names are not usually sought, for the groups of every ephemeral artificial classification; while, for the popular, rough and ready classification of things, based on their broad resemblances, we have the existing nomenclature in which these classes are already embodied and expressed in ordinary language. But when the scientific study of the things of any particular department—minerals, plants, animals—leads to a much more complex and comprehensive classification than that which satisfies popular needs, then we must aim at helping our memory of these numerous classes, and of their relations to one another, by fixing upon them *names expressive of these relations*.

Endeavouring to accomplish this purpose, we usually start with the existing names which we find in ordinary language for the highest, the lowest, and certain intermediate, broadly-marked classes—or with as many of these names as will suit our purpose,—and we may form names for the new classes in either of two ways.

(i) We may, as in zoology and botany, combine the name of the higher class with a term descriptive of some distinctive attribute, in order to form the name of the next lower class.

On the assumption that the connotation and definition of terms, though to a certain extent conventional, should nevertheless aim at "following nature," in the sense of embodying the most fundamental and important attributes of

the thing defined (31, 32, 57, 67), we should imagine that such a combination of *genus* and *differentia* would give us a *logical definition* of each such sub-class; and so it would, were the classifications in question regarded as "natural". But we are warned not to imagine that the "modern zoologist consciously and avowedly makes his scheme of classification a genealogical [*i.e.* what he would regard as a 'natural'] one";[1] and "in Systematic Botany we are expressly reminded that [the 'differentiating' attribute] has no significance, that whatever may be its apparent meaning or its actual etymology we are to regard it as a mere unmeaning label. Thus 'perennis' as an adjunct [in 'Bellis perennis'] does not *mean* that the daisy in question lives for several years."[2] Both botanists and zoologists have presumably been content to fix on qualifying titles which either indicate some prominent feature of the sub-class,[3] or are connected with some other easily remembered fact; they have not endeavoured in all cases to get names expressive of the "natural" *differentia* of the sub-class. The result is that the sum of all the *differentiae* leading from a *summum genus* to a lowest sub-class would give, not a *definition* of the latter, but rather a sort of *description*, since "we can hardly consider that the meaning of the term could be stretched so as to include all these attributes"; the latter constituting "a curiously heterogeneous group . . . some . . . really important in themselves . . . others . . . very superficial".[4]

(ii) A second method of establishing a nomenclature expressive of the mutual relations of the sub-classes, is the method illustrated in the science of chemistry. Here the relations are expressed by *changes in the termination of the words.* "Thus the new chemical school spoke of sulph*uric* and sulph*urous* acids; of sulph*ates* and sulph*ites* of bases; and of sulph*urets* of metals; and in like manner, of phos*phoric* and phos*phorous* acids, of phos*phates*, phos*phites* and phos*phurets*";[5] of ox*ides* and hydr*ates*, etc. "In this manner a nomenclature was produced, in which the very name of a substance indicated at once its constitution and place in the system."[6]

(*b*) It is no less essential to have a language in which to *describe objects*, than to have names for the classes to which they belong. This descriptive language is called *Terminology.* The terms used are all *general* terms; and they are *combined* to describe the *individual*, its *parts*, and its *properties.* In the use of such technical terms, accuracy and precision are of the greatest

[1] VENN, *op. cit.*, p. 340. [2] *ibid.*, p. 336.

[3] Constituting a "*diagnostic*" definition. *Cf. supra*, 65; JOSEPH, *op. cit.*, p. 116.

[4] VENN, *ibid.*, p. 326.

[5] WHEWELL, *Novum Organon Renovatum*, p. 308; *apud* WELTON, *Logic*, i., pp. 148-9.

[6] *ibid.*

importance. The new names coined in each science avoid ambiguity better than ordinary names in current use. When these latter are adopted as technical terms in any science, their exact meaning there should be clearly set forth; and this setting forth of the technical meaning of a familiar term is one of the processes described as *Nominal* (or *Conventional*) *Definition* (54-6). As a rule, their exact meaning "can be made intelligible only by presenting to the senses that which the terms are to signify. The knowledge of a colour by its name can only be taught through the eye. No description can convey to a hearer what we mean by *apple-green* or *French-grey*. . . . In order to derive due advantage from technical terms of this kind, they must be associated *immediately* with the perception to which they belong; and not connected with it through the vague usages of common language [*cf.* 52, *b*]. The memory must retain the sensation; and the technical word must be understood as directly as the most familiar word, and more distinctly. . . . In all cases the term is fixed to a peculiar meaning by convention; and the student, in order to use the word, must be completely familiar with the convention, so that he has no need to frame conjectures from the word itself. Such conjecture would always be insecure, and often erroneous"[1]

WELTON, *Logic*, i., chap. vi. JOSEPH, *Logic*, chap. v. KEYNES, *Formal Logic*, Appendix A, pp. 441 *sqq*. VENN, *Empirical Logic*, chaps. xii., xiii. MELLONE, *Introductory Text-Book of Logic*, chap. v.

[1] WHEWELL, *op. cit.*, pp. 314-5; *apud* WELTON, *ibid.*, pp. 152-3.

CHAPTER V.

THE CATEGORIES OR "PRAEDICAMENTA".

70. THE PROBLEM OF CLASSIFYING OUR WIDEST CONCEPTS.— In the preceding chapter we examined the general principles of division and classification, as well as their main applications to some of the special sciences.

We also pointed out that both definition and division, by analysing the intensive and extensive aspects of our ideas, discharge the very important function of bringing into clearer light those simplest, ultimate notions which form the self-evident axioms or principles of the various human sciences. Each of these principles is the formulation of a self-evident relation between two simple notions.[1] Every special science has its own proper first principles. But the sciences themselves are interrelated. They form a hierarchy in which the more special and derivative are subordinate to the more general, and these in turn to the one widest science which seeks to unify them all. This widest science is called *Metaphysics* or "*Philosophia Prima*": its principles are the widest of all, and the notions that form these are the simplest of all: notions of *being*, *substance*, *cause*, *action*, *time*, *space*, etc. It is on a comparatively small number of such notions that, in its ultimate analysis, all our knowledge is based; of these it is built up, so to speak; into these, ultimately, it is resolvable. But all our knowledge is formulated *internally* or *mentally* in acts of *judgment*, by which we *predicate*, *i.e.* affirm, or deny, something about something, and *externally* or *verbally* in propositions which express those judgments.

The question now, therefore, arises, whether it is possible to make out a general, all-embracing classification of all possible predicates, so as to bring these under one or other of a small

[1] We refer, of course, to principles of *Thought* or *Knowledge*, *logical* principles; not to principles of *Being*, *ontological* principles: these latter are the fundamental causes or constituents of *Real Being*, and form part of the subject-matter of metaphysics.

number of widest groups. It is a question of applying the principles of a natural or general classification, not to this or that particular department of human knowledge, but to the whole sphere of knowable reality.

The problem has evidently two quite distinct, though inseparable, aspects; for a classification of those direct universal ideas, by means of which we represent and interpret things for ourselves, is necessarily, *eo ipso*, a classification of the *objects* of those notions, a classification of things, beings, realities. Were we to follow up the problem from the latter standpoint, to see what light such a classification might possibly throw upon the nature and attributes of real being, our investigation would be *metaphysical;* were we to pursue it with a view to the better ordering of our knowledge within our own minds, for sake of the light it might possibly throw on our processes of conscious thought, especially on the mental act of judgment, our study would be *logical* in character. The latter, therefore, is our point of view at present.

71. ARISTOTLE'S "CATEGORIES": THE SCHOLASTIC "PRAEDICAMENTA."—As a matter of fact, repeated attempts have been made at such a classification, from the days of Aristotle. It is indeed the first problem we encounter in Aristotle's logical treatises (25). His classification terminated in *ten* highest groups which he called *Categories.* The word—which also gave its title to the treatise in question: *Αριστοτέλους κατηγορίαι*[1]—is from the verb *κατηγορεῖν to predicate, to assert* (affirm or deny).[2] The

[1] The entire treatise—*κατηγορίαι*, *Liber de Praedicamentis*—embraces three parts. It opens with certain preliminary remarks on the names by which we designate the things of common knowledge: called "*Antepraedicamenta*" (chaps. i.-iii.). Next come the "*Categories*" or "*Praedicamenta*," with a detailed study of their characteristics, especially those of the first four; for the latter six are but briefly treated (chaps. iv.-ix.). Finally, we have six chapters (x.-xv.) devoted to what are called "*Postpraedicamenta*". Aristotle treats, under this head, of certain corollaries which he derives from his classification of the categories, certain concepts whose contents are common to most, if not all, of the categories. Of these notions he enumerates five: opposition, priority or succession, simultaneity, motion or movement, and attributes expressed by the verb *to have: de oppositis, de priori, de simul, de motu. de habere.* With the first of these we have dealt already. The second and third will come up for discussion in connexion with *Judgment* and with *Induction.* The fourth and fifth will call for a few remarks in connexion with certain of the categories themselves.

[2] The word primarily meant *to speak against, to accuse: κατηγορία* meaning a charge, an accusation. Legally, the accused person was the *subject* against whom the heads of accusation, the charges, were preferred: about whom they were made. Transferred to logic, the things that were asserted about the logical subject of the judgment or proposition came to be called *categories, heads of predication* (*Cf.* PRANTL, *Geschichte d. Logik*, i., pp. 184-210).

Categories of Aristotle are, therefore, *a classification of all the possible predicates* by which we may formulate our knowledge about any individual subject. His classification is *logical* rather than *metaphysical:* a classification of notions rather than of things; an aid—in so far as it is successful—to clearness and order in our ideas, in the knowledge we already possess; and therefore, also, an aid in the pursuit of further knowledge, "an aid to the due examination of nature," since his categories serve "to recall points of view from which questions may be put in regard to the objects of inquiry that present themselves".[1] His categories are ten great headings, or schemes, or types, of predication (*σχήματα τῆς κατηγορίας*; *τὰ κοινῇ κατηγορούμενα*), under some one or other of which he would find a place for every possible general notion we can conceivably use in *interpreting*, or *judging* the individual things or subjects which come up for investigation in the course of our whole conscious or mental experience.[2]

In Scholastic logic these ten categories are called "praedicamenta" (from *praedicare*, to predicate); and, since they are ultimate classes, to one or other of which every conceivable notion can be referred, and beyond which "it can get no further, hence has arisen, by a strange freak of language, the familiar expression of 'getting into a predicament,' to express the unpleasant situation of one who has involved himself in circumstances from which he would fain escape but cannot".[3]

Aristotle, mindful of the metaphysical aspect of his division, called the categories *γένη τῶν ὄντων*, *γένη κοινά*, *τὰ πρώτα*; and his Scholastic commentators in the Middle Ages, treating the categories from the same metaphysical standpoint, called them "*suprema genera rerum*," at no time, however, losing sight of the primarily logical character of the division.

The scope and aim of Aristotle's classification have been sometimes misunderstood. The question as to the relation of the *logical* "categories" to the *metaphysical* "genera suprema" on the one hand, and to the *grammatical* "parts of speech" on the other, is a phase of the fundamental philosophical problem of the relation between language, thought, and thing.

[1] WELTON, *op. cit.*, i., pp. 94, 97, quoted from LOTZE, *Metaphysics*, i., pp. 24-5.

[2] The *γένη τῶν κατηγοριῶν*, classes or kinds of predicates, are therefore classes into one or other of which any simple predicate must fall (a complex predicate may be referred to two or more of them, *e.g.* "Socrates *instructed* his disciples, *in the market-place, yesterday*"); they are not classes into one or other of which *the individual things* about which they are predicated must fall. The latter may be referred to any of the categories, according to the predicates we apply to them. It is by such application we enter into conscious possession of whatever intellectual knowledge we have of individual things.

[3] CLARKE, *Logic*, p. 190.

A criticism of Aristotle's treatment of the categories, and a comparison of the same with those of some more modern philosophers, will be better understood when we have given a brief exposition of the former.

72. THE CATEGORIES AND THE PREDICABLES.—We may, before analysing Aristotle's scheme, call attention to the difference between the predicables (chap. ii.) and the categories. While the former are a classification of all the possible *modes of predication*, *i.e.* of all the possible *kinds of relation* that may exist (in point of intension and in point of extension) between the predicate and the subject of a logical judgment, the latter are a classification of the *predicates themselves*. The former are a division of *logical relations* between our direct universal ideas; the latter are a division of these *direct universal ideas* themselves. The predicables express not so much the *material* of thought, but rather certain *relations* which we see, by reflection, to obtain between our thoughts: *logical universal concepts*, which are the product of mental *reflection* on our own direct thoughts and judgments. The categories, being a classification of our direct universal concepts of things, should not be called a classification of "relations". To describe them as such would expose the student to the risk of confounding them with the predicables. The categories are not primarily a classification of things, but neither are they primarily a classification "of the relations between things".[1] They are a classification of the concepts by means of which, as predicates, we seek to formulate for ourselves and others our knowledge about things. They are therefore a classification of *aspects* of things, aspects revealed through our concepts, rather than of *relations* between things. Nor are they a classification of *relations between concepts or ideas*, as the predicables are. At most they can be said to be such a classification of concepts as *necessarily involves relations* between the latter; for they are a classification of concepts *considered as predicates*, and therefore as standing in the relation of predicate to some logical subject. From the very fact that we obtain all our intellectual concepts by an analysis of the data of sense experience, these concepts all embody relations of various kinds to one another. This amount of relativity does enter into the logical classification of the categories: they are not classes of things pure and simple, of things considered in their real state, but of things considered as objects of our thought, related by our thought to one another.

[1] WELTON, *Logic*, i., p. 89.

73. THE ARISTOTELEAN CATEGORIES ENUMERATED—Aristotle's tenfold scheme gives ten heads under one or other of which we can classify any conceivable predicate, any notion or term which can fill up the blank in the statement "This is——". For example, "This is '*white*,' '*cold*,' '*soft*' . . . This is '*snow*'."

The latter predicate, "snow," represents in the abstract the *substance*, *essence*, "*quidditas*" or "*whatness*," of the thing in which we find inhering the various *attributes* expressed by the adjectives "white," "cold," "soft," etc. When we thus designate any *individual* thing that comes into our experience—the "this something," τόδε τι, "*hoc aliquid*," as it has always been called—by any concept or notion signifying its *substance or essence*, we are predicating of it the *first* of Aristotle's categories, ἡ οὐσία, τι ἐστί, "*substantia*," "*quidditas*," the category of *Substance*. The concrete individual thing itself, the "*hoc aliquid*," which forms the ultimate *subject* of all *logical* predicates in the order of thought or knowledge, and the underlying *substratum* of all real or *ontological* attributes in the order of things or reality, is called the οὐσία πρώτη, "*substantia prima*"; while the abstract, universal substance which we predicate of this, and which constitutes the first Aristotelean category, is called by way of contrast the οὐσία δευτέρα, "*substantia secunda*". The latter, or "categorical," substance, *i.e.* the substance as represented in the abstract and universal notion (4, 5)—"snow," for example—can of course become the logical subject of other logical predicates, *e.g.* "*Snow* is white, cold," etc.; but it is never their *ultimate* logical subject, for it must itself be referred for its meaning to the individual, concrete "this," or "that," in which it finds itself verified, and of which, ultimately, itself is predicated; whereas the "*hoc aliquid*," the individual "this," can never be itself a mere mode or state or attribute of another individual thing in the real order,[1] nor can it be properly the predicate of any subject in the logical order.

Since the "second substance," the *kind* or *nature* of an individual, is conceived as abstract and universal, it is *attributed to* the individual by the same sort of logical predication as the other categories, the "accidents," are predicated of the latter. And just as the accidents attributed to an individual

[1] It is, however, undoubtedly a determinate or definite mode of being, and is therefore referred by Aristotle to the category of substance, since this latter gives us what is essential to it in so far as we can have an intellectual concept of it at all. *Cf.* JOSEPH, *op. cit.*, p. 39, n.

man do not constitute, appertain essentially to, his nature *as man*, so his nature *as man* cannot be regarded as constituting, or as identical with, himself *as an individual*. Socrates and Plato are not each "human nature," for if they were they would be identical; they *have* human nature. What, then, individuates, or distinguishes numerically, each of them from the other? This is the metaphysical problem of the *Principle of Individuation*—an aspect of the great, fundamental philosophical question of the relation between the universal and the singular, in our knowledge and in reality itself (4, 5).[1]

About the "this individual thing" we can predicate not merely its substance or essence, as when we say "This is *snow*"; we can find other ways of *denominating* or *determining* it: we can attribute certain accessory realities to it—commonly called *attributes* nowadays, better and somewhat more appropriately known as *accidents* in Scholastic philosophy.[2] These *accidental determinations* that may affect an individual substance, Aristotle reduces to *nine* classes.

Such terms as "white," "small," "inferior," "pushing," "being beaten," "at home," "to-morrow," "standing," "armed," etc., express *realities*, modes or kinds of being—otherwise they could not be used as predicates;—yet they do not express the *substance* or *essence* of the individual subject of which they are affirmed, but modes of being that are *coincident*, or concomitant and co-ordinate, with the mode of being which gives the substance of the latter: they are "accidental" or *supervening* realities.

Some of the modes of being which may be thus predicated are *intrinsic* to the individual subject to which they are attributed, are *inherent* in it: two of them, *quantity* (ποσόν) and *quality* (ποῖον), inhere in the subject considered in itself, or absolutely; a third, *relation* (πρός τι), is affirmed of the subject when this is regarded in connexion with any thing or things other than itself. Other predicates represent something *extrinsic to the subject: place* (ποῦ), the measure or determination of quantity, and *time* (πότε), the measure of duration: *action* (ποιεῖν) and "passion" (*i.e.* "being acted on," *endurance*, πάσχειν) which are affirmed of a subject as principle of the former, as term of the latter; and, finally, the two categories κεῖσθαι and ἔχειν, usually

[1] *Cf. supra*, 48, p. 86; Joseph, *Logic*, pp. 41-4, 52 *b*, 67, 76.

[2] Very often the concrete individual thing gets its *substantive denomination* not from that which is its *substance*, but from some one or other of the "*accidental*" categories: when we call a thing a "gate" we do not give its *substance* (wood or metal), but the form or structure ("quality") of the latter. Even here, however, the substantive gives what is *essential* to the notion it conveys about the individual as thus denominated.

translated *situs*, or "posture," and *se habere*, *habitus*, or "habit," respectively—about the exact meaning of which (in the mind of Aristotle), even more than of the others, there has been considerable diversity of opinion.[1]

We thus obtain the following list of ten categories, each of which stands as a *summum genus*, branching downwards into a number of predicamental lines of sub-classes—*genera* and *species*—each of which can be naturally predicated of its subordinates, and ultimately of the individuals that embody its content or implication in them.[2]

1.	οὐσία	*Substantia*	SUBSTANCE or *Essence*, not Individual ("*First*") but *Specific* or *Generic*, *i.e.* Universal ("*Second*"); the answer to the question *e.g.* "Who or what is Socrates?" "Socrates is a *man*, an *animal*, a *living being*."
2.	ποσόν	*Quantitas*	QUANTITY; as "Socrates is *five feet high*, *ten stone weight*". Quantity is either (*a*) *discrete* when its parts are disconnected, as in *multitude* or *number*; or (*b*) *continuous* when its parts are connected; and these latter are either (1) *successive* and *transient*, as in *motion* and *time*, or (2) *simultaneous* and *permanent*, as in *space*, *magnitude* or *size* proper (length, breadth and depth).
3.	ποῖον	*Qualitas*	QUALITY; as "Socrates is *flat-nosed*, *virtuous*, *patient*, *brave*". Of Quality there are four subdivisions: (*a*) *Habits* and *Dispositions* of Mind or Body (*Habitus* et *Dispositio*); (*b*) *Strength* or *Weakness* of *Natural Power*, such as Memory, Intelligence, etc. (*Potentia* et *Impotentia*); (*c*) *Sense Qualities* whether transient or permanent, such as paleness through fright or through ill-health (*Passio* et *Patibilis Qualitas*); (*d*) *Form*, of artificial, and *Figure* of natural, things (*Forma* et *Figura*).
4.	πρὸς τι	*Relatio*	RELATION; "Socrates is a *father*, a *proprietor*, a *citizen*, a *teacher*, *smaller* than Plato".
5.	ποιεῖν	*Actio* (or *Facere*)	ACTION; *i.e.* the action expressed by the *active transitive verb*; as "Socrates is *digging* his garden, *instructing* his disciples".

[1] St. Thomas (in *Met.* v., lect. 9) sets forth his reasoned account of the Aristotelean division in practically the following brief terms: The entity affirmed of a subject either constitutes that subject or is superadded to it. If the former, we have the category of *substance*; if the latter, we have an *accident*. Accident is either relative (*relation*) or absolute. If absolute, it is founded either on the material, or on the formal, constituent of the subject. The former is *quantity*, the latter *quality*. These three have their source *in* the subject; but the superadded accident may be attributed to the subject in virtue of some reason or source outside the subject: in virtue of what fixes its quantity, *viz. place where*, or in virtue of what measures its duration, *viz. time when*; or, again, in virtue of an effect of which the subject is either the principle (*active*), or the recipient (*passive*); or, finally, in virtue either of the relative disposition of its parts in the place occupied by it, *posture*, or of the extrinsic things, such as clothing, immediately affecting it, *habit or clothing*.

[2] *Cf.* JOYCE, *op. cit.*, pp. 142-4.

6.	πάσχειν	*Passio* (or *Pati*)	SUFFERING, BEARING, ENDURING; expressed by the *passive* voice of the *transitive* verb; *e.g.* "Socrates is being *contradicted*, or *condemned to death*".
7.	ποῦ	*Ubi*	WHERE or WHEREABOUTS;[1] as "Socrates is *in the city*, *at home*, *in bed*".
8.	ποτέ	*Quando*	WHEN, at *what point of time*;[1] as "Socrates *is now*, or *was yesterday*, or *last year*, or *soon will be*, held in esteem".
9.	κεισθαι	*Situs* (or *Agere?*)	POSTURE; as "Socrates is *standing*, *sitting* (or any *immanent* or intransitive action expressed by the *intransitive* verb; as "Socrates is *meditating*, *walking*".)
10.	ἔχειν (καλῶς or κακῶς)	*Habitus* (or *Se Habere*)[2]	HABIT[3] (or *State* as expressed by the *reflexive* verb); as "Socrates is *clothed*, *armed* (or *is well* or *ill*, or *pleased with himself*)."

74. THE CATEGORIES AND LANGUAGE.—Aristotle's own brief resumé of the list just given is as follows: "The elements of speech are sometimes connected, as 'a man runs,' 'a man triumphs,' and sometimes disconnected, as 'man,' 'runs,' 'triumphs'. Any such disconnected element is either an *essence* (*οὐσία*), a *quantity*, a *quality*, a *relation*, a *place*, a *time*, a *doing* (something), an *undergoing* (something), an *intransitive action* (*κεῖσθαι*), or an *intransitive passive state* (*ἔχειν*)."[4] Seeing that his classification is the result of an analysis of the act of *judging*, *i.e.* of *predicating*, asserting or denying, and that the judgment is expressed in the *proposition*, it is natural that he should have approached the problem partly from the point of view of language. Yet his categories are not a *grammatical* classification of the *parts of speech*, but a logical classification of terms used as predicates (22). He analyses the *logical judgment*, not the *grammatical sentence*. Some logicians think that his categories originated in an examination of the grammatical parts of speech. This view has been put forward by Mansel, for example. But, as Professor Welton remarks, it "may be doubted . . . for that division of words was by no means sufficiently developed in Aristotle's time to favour this idea".[5]

[1] The answer to the question "How long?" or "How large?" *i.e.* amount or *quantity* (of time or space) belongs to the second category—QUANTITY.

[2] Not *Habere*, to *have* or *possess*, as it is interpreted (with eight sub-classes) in the treatise entitled *Categoriae Decem* (c. xvi.), wrongly attributed to St. Augustine.

[3] In the sense of *clothing* (*e.g.* in a riding *habit*), not in the sense of one of the subdivisions of *Quality* (*e.g.* "*Habits* are hard to break").

[4] Categ., c. 2. [5] *Logic*, i., p. 97.

The categories may nevertheless be related to the grammatical parts of speech, and the comparison may yield results both interesting and instructive.

The individual subject about which (ultimately) all predications are made, is expressed by the grammatical *substantive* or *noun* (or *pronoun*)—either the *proper* noun, or the common noun *individualized* by the *article* (definite or indefinite) or the *pronominal adjective* (definite or indefinite). In so far as these serve to indicate the *substantia prima*, or individual, they lie outside the categories, which embrace only predicates.

But a predicate may be (*a*) a *common noun* asserting the individual subject to be (i) some *kind* of *essence* or *substance—i.e.* something which subsists in itself and not merely by inhering in some other underlying thing of which it is a mere attribute or *accident*, as "Socrates is *a man*"; or something which, if it does not *really* subsist in itself, *is considered* apart from its subject, or *in the abstract*, as "The virtue under discussion is *justice*";—or (*b*) an *adjective* of (ii) *quantity*, (iii) *quality*, or (iv) *relation*;—or (*c*) a *verb* (v) *active*, (vi) *passive*, (vii) *intransitive*, (viii) *reflexive* or *neuter*;—or (*d*) an *adverb* of (ix) *place*, or (x) *time*.

Thus we see the ten categories embodied in just a few of the parts of speech. The chief of these, from the point of view of logic, are undoubtedly the *substantive*, the *adjective* and the *verb*: expressing the *substance*, its *properties*, and its *activities*. The *verb*, used as predicate, is, of course, to be analysed into the logical copula (*is*, *are*) and the participle, which then ranks as an *adjective*. Moreover, the verb, as expressing action in the widest sense of *actuality*, embraces existence itself. *Exist* is a verb: and existence is called, in philosophy, the "*first*" *act* (*actus* "*primus*") of a being, in comparison with all further happenings or phases—"*second*" *acts* (*actus* "*secundi*")—of its actuality.

Furthermore, we have said (22) that adverbs, being modes or qualifications of verbs, are syncategorematic, and therefore cannot of themselves stand as predicates; but, by qualifying the latter, they indirectly qualify the subjects of those predicates. Moreover, if the actions or events or attributes, which they qualify, be themselves made logical subjects, the adverbs can be predicated of these *in the manner of adjectives*; and this is particularly true of the *spatial* and *temporal* determinations of things or events, expressed by the adverbs of place and time, and always implying actual happening or existence: "*Where* was Socrates?" "He was (existing) *in Athens*"; "*When* (was he living, existing)?" "(He was living, existing) *in the fifth century B.C.*"

Prepositions (or inflexions) help to express the category of relation; *conjunctions* serve to combine simple into complex predicates; *interjections*, in so far as they have logical significance, are to be expanded into complete statements.

Most of the examples given above, to illustrate the various categories, are taken from Grote's *Aristotle*.[1] Having in view an analysis of the knowledge that men do or may possess about the ordinary, familiar objects of sense experience—particularly about man himself, about the human individual,—Aristotle gives

[1] pp. 77-8, *apud* WELTON, *Logic*, i., pp. 92-3.

us in his categories all the possible independent heads under which we may put questions, or seek information, about such objects: (i) What is Socrates? (ii) What *size* is he? (iii) What *sort* of man is he? (iv) How is he *related* to others? (v) What is he *doing?* (vi) What is *being done to him?* (vii) *Where* is he? (viii) *When* do you speak of him? (ix) In what *posture* is he? (x) What is he *wearing?*

The last two categories, interpreted in the narrower sense just suggested, can apply practically only to *living* things, and the last is practically confined to *human* individuals. They would, in this sense, be more complex or derivative determinations of *place* and *quality* respectively.[1] A wider sense,—in which they would be expressed respectively by the intransitive or neuter verb with an active signification, and by the reflexive verb with a passive signification,—has been suggested by Max Müller.[2]

The Greek word κεῖσθαι is not adequately rendered by the Latin *Situs*, or the English *Posture:* it may mean *state*, *condition*, *manner of being;* and, thus interpreted, it may fairly be made to embrace *mental states* of various kinds—sensations, emotions, etc. —for which Aristotle is wrongly accused by many moderns—Mill and Mansel,[3] for example—of having made no provision. Even if such states could not be brought under this category (κεῖσθαι), they could be included under the categories of *Quality*, or of *Passio:* they might fairly be described as ποιότητες or πάθη.[4] Furthermore, even if κεῖσθαι were confined to the meaning of *Situs* or *Posture*, *i.e.* relative dispositions of the parts of an individual to the whole, in the place occupied by the latter, this is clearly different from the category ποῦ, *Ubi*—at *what point of space*, whereabouts, is the individual?—with which Mill, in his apparent anxiety to discredit the Aristotelean scheme, confounds the former category.

The science of philology throws an interesting light on the relation of the remotest known language-*roots* to such fundamental concepts as are embodied in the logical categories. All predicates must be ultimately referred, as we saw, to some individual thing, or phenomenon, or event, which comes into consciousness through the senses. This something the understanding at first most vaguely designates as a *subsisting* thing, a *substance.* By repeated efforts it gradually removes the original indeterminateness of

[1] *Cf.* JOSEPH, *op. cit.*, pp, 38, 47.
[2] *Science of Thought* (London, Longmans, 1887), p. 430.
[3] WELTON, *Logic*, i., pp. 95, 97.
[4] GROTE, *Aristotle*, *apud* WELTON, *Logic*, i., p. 98.

the "something" by *attributing* successive "determining" or "specifying" *predicates* to it. If we analyse the complex names we use as predicates into their remotest *roots*, we shall find that these all express not *concrete*, *individual* data of sense, but *abstract*, universal elements of thought—*products of intellectual abstraction.* This is Müller's contention;[1] and he supports it by copious and instructive examples. Take, for instance, the English (and German) word *wolf*, French *loup*, Latin *lupus* (for *vlupus*), Greek λύκος: these are represented by the Sanscrit root *vraçe*, *to tear*, *to lacerate* (and the noun *vrka*, wolf). Similarly, the Latin *avis*, Greek οἰωνός, comes from a root signifying *to fly;* the English *fowl*, German and Flemish *Vogel*, from a root meaning *feathered*; while the Sanscrit, *andaja-s*, conveys the sense of something *issuing from a shell.* The derivative words have thus become restricted in denotation to classes of *concrete things*, while the original roots had the *abstract* meaning of *anything that rends*, *flies*, *is feathered*, etc. Nor are proper names an exception to this law, for they too appear to have been all originally *common* names of *abstract attributes.* Examples would be superfluous.

It appears, then, that we have the authority of philologists for this remarkable and significant fact, that *primitive language-roots* indicate *abstract concepts*, *predicates* of judgments.

And each primitive root in turn can be shown to yield—by various transformations, according to established philological laws of language development—quite a number of distinct cognate forms, appropriate to the modes or kinds of predicates embodied in the logical categories.

75. THE CATEGORIES AND REALITY.—Any attempt to reach a comprehensive classification of those widest concepts which enter into all our judgments, and which form, so to speak, the warp and woof of all our knowledge, must inevitably give rise to metaphysical problems about the nature of being or reality: and this no matter how avowedly "logical" the purpose of our classification may be. While we aim at establishing order in our knowledge by the arrangement of a system of broad and distinct heads of interrogation, we cannot avoid asking ourselves whether or how far the corresponding heads of predication represent modes or determinations of being, actually inherent in, and characteristic of, the material of our thought—the being or reality of things.[2] And it is what we might call the natural, spontaneous conviction of everyone, that those distinct determinations of our thought, which we call

[1] *op. cit.*, chap. viii. *Cf.* MERCIER, *Logique*, pp. 122-3.

[2] "Logical and metaphysical problems have a common root. We cannot reflect upon the features which characterize our thought about things in general, without asking how things can be conceived to exist; for our most general thoughts about them are just our conception of their manner of existence."—JOSEPH, *op. cit.*, p. 44.

categories, are grounded in the being or reality that is apprehended by our thought, and are produced by the former in the latter.

It is only because we conceive being or reality as endowed with certain modes, as made *determinate* in some way or other,—for the concept of pure being as such is absolutely vague and indeterminate,—that we can predicate of it the conceptual content of this, that, or the other, category. We cannot, for instance, make any intelligible use of predicates taken from the category of quantity in reference to mind, for we conceive mind as a sort of being or reality not endowed with quantity, as other than quantitative. It is only to material being, existing in space and time, that we can apply predicates belonging to, or derived from, this category.

And, conversely, we cannot make any predication whatever about being or reality except in some category or other: "That which was not conceived as a substance, or a quality, or a state, and so forth, would not be conceived at all; . . . and therefore the consideration of these distinctions belong to logic, since they characterize our thought about objects in general; and though logic is not interested in the indefinite variety of existing qualities—blue, green, sour, shrill, soft, etc.—(because an object, in order to be an object, need not have any one of these qualities in particular, but only one or other) yet it is interested in the *category* of quality, or in noticing that our object must have some quality or other: in the *category* of relation, or in noticing that it must stand in relations to other objects: and so on."[1]

Aristotle—and the Scholastics after him—assumed that this logical inquiry into the highest and widest definite or determinate concepts used in our interpretation of reality, *i.e.* in our logical judgments, was simultaneously and *eo ipso* a determination of the modes or forms in which reality actually exists.[2]

Perhaps this assumption—that the categories of thought represent modes of real being—is unwarranted? If that be so, then there is this other alternative, that the categories are so many purely subjective *equipments of the mind*—"primordial concepts of the pure understanding," as Kant (77) called them,[3]—a system of innate mental apparatus by means of which we *interpret* or *judge* a something or other that is supposed to come *from*

[1] JOSEPH, *op. cit.*, p. 45.

[2] "The idea underlying Aristotle's scheme of the categories may be expressed thus—to discover the forms of existence which must be realized in some specific way in the actual existence of anything whatsoever."—*ibid.*

[3] "Die Stammbegriffe des reinen Verstandes."—*Critique of Pure Reason.*

without, from the extramental domain (if there be such), into the sphere of conscious sense experience. But perhaps Kant is wrong, and the realist assumption in question well grounded : the assumption that the determinations of our thought are *objective* in the sense that they represent the nature of real being ? We think so, ourselves. This, however, is not the place to argue the question (which belongs properly to *Criteriology* or the *Theory of Knowledge*, *cf.* 77), but merely to show how an attempt to classify the fundamental concepts that enter into our judgments—in other words, to draw up a scheme of categories—is perhaps even more metaphysical than logical in character : a circumstance which, however, in no way forbids its treatment in logic, or lessens in any way its utility towards a better understanding of the "logical" or "truth" aspect of the mental act of judgment.

The determination of the categories in their metaphysical aspect, *i.e.* as modes of real being, is one of the most fundamental problems of philosophy. That there are certain ultimate categories of thought is a generally admitted fact. Whether any of these, or which of them, represent ultimately irreducible modes of real being, has been always a matter of dispute. The solution of this problem demands a deep and prolonged analysis of man's internal and external mental experience. And it is because philosophers have derived conflicting results from this analysis that the history of philosophy sets forth so many conflicting philosophical systems. The adherents of one system will not admit as ultimate the categories accepted as such by the adherents of another system. Thus, the philosophers known as Atomists, who endeavour to explain all human experience on purely mechanical principles, endeavour to eliminate the category of *Quality* by reducing it to other categories—to *Quantity* and *Motion*. "That a quality is not a quantity, writes Mr. Joseph,[1] is a truth which those overlook who think that sound can *be* a wave-length in the vibration of the air ; they forget that it is not possible to define terms of one category by another." To which he adds : "Except as terms in a derivative category involve terms in those from which it is derived".[2] And this, precisely, is what these philosophers maintain—that the category of quality *is* derivative, not ultimate and irreducible.

The philosophy of Hegel is an attempt to show that all categories are, identically, categories of thought and of being, and that all are gradually worked out in a process of self-evolution of Thought or Idea which is the one and only reality.

76. Limitations and Modifications of Aristotle's Scheme.—Some of the logical divisions of terms in Chapter I.—for example the divisions into concrete and abstract, into general and singular—are partial anticipations of the attempt to make out a classification of the highest categories of thought : like the latter, they are based on more or less fundamental differences in the ways in which we conceive things to exist. The differences between the various categories in Aristotle's scheme are more fundamental in some cases than in others. Thus, *Actio* and

[1] *op. cit.*, p. 46. *Cf.* Joyce, *op. cit.*, pp. 143-4. [2] *ibid. n.* 3.

Passio are merely different aspects of the same reality—*Change*, or *Motus*, in the wide, Aristotelean sense of the latter term. And the Scholastics embodied this view in the maxim: *Actio et passio sunt idem numero motus.* All three concepts are involved in the notion of *Causality*, which in turn suggests a concept that comes under the category of *Relation*, *viz.* the relation between cause and effect. And the question whether *Relation* itself (πρός τι) is ever in any case a mode of real being, distinct from the other modes or categories which give rise to it and are its foundations, or whether it is always, on the contrary, a mere mental mode of conceiving the latter, a mere "*intentio logica*,"—has been always a controverted point among Scholastics. Again, the categories *Ubi* and *Quando* are aspects or determinations of the category of *Quantity*, which they in a certain sense presuppose. So, too, the category of *Situs* (κεῖσθαι) presupposes Quantity, Place, and Relation; and the category of *State* or *Condition* (ἔχειν, *se habere*) presupposes those of Quantity, Action, and perhaps Quality and Relation.[1]

But even although some of Aristotle's categories may thus be possibly derivations from others, this is not a serious objection to their inclusion in the list. Even though certain of our commonly recognized determinations of thought and being may be found on deeper analysis to be reducible to others, still, so long as they are clearly distinct from others in men's thoughts generally, and are themselves sufficiently wide and simple and fundamental, they may justly claim recognition in a logical scheme the object of which is to procure and promote clearness in our thinking processes.

Many of the objections urged from various quarters against the Aristotelean categories are based on misconceptions of Aristotle's real aim, and therefore call for no explicit or detailed notice. It is much easier to criticize his scheme as cramped, or artificial, or arbitrary, than to suggest real amendments, or to substitute a better one.

[1] "Sed sciendum est quod praedicata diversificantur secundum diversos modos praedicandi. Unde idem, secundum quod diversimode de diversis praedicatur, ad diversa praedicamenta pertinet. Locus enim secundum quod praedicatur de locante, pertinet ad genus *quantitatis*. Secundum autem quod praedicatur denominative de locato, constituit praedicamentum *ubi*. Similiter motus, secundum quod praedicatur de subjecto in quo est, constituit praedicamentum *passionis*. Secundum autem quod praedicatur de eo a quo est, constituit praedicamentum *actionis*." St. Thomas, *In Metaph.* lib. xi., sect. 9. *Cf.* Joseph, *op. cit.*, pp. 46-7. Joyce, *Logic*, p. 141.

HAMILTON suggests a rearrangement of the categories, in which "Being" (*Ens*), as the sole *Summum Genus*, is divided immediately into *Substance* and *Attribute* (or *Accident*). But substance and accident are not two species, into which "Being," as genus, can be divided by any *differentia* other than itself; nor can it be predicated of them *univocally* (or *in the same sense*) as a true genus, as "animal" can be predicated of its species "man" and "horse": but only *analogically* (27), as ultimately different modes or determinations of being.[1]

It is true, however, that whatever we think of we conceive either as subsisting in itself or as qualifying something else in which it inheres, *i.e.* either as a *substance* or as an *accident:* a fact which has induced Hamilton to divide being into substance and accident, and then to subdivide the latter into certain subordinate species; but accident is not a genus divisible by *differentiae* distinct from itself into such subordinate species, and *univocally* predicable of the latter. The concept "accident" may be predicated of Aristotle's nine latter categories only in an analogical sense: as ultimate determinations of being they are each as immediately co-ordinate with substance as they are with one another.

The STOICS, subordinating the categories to the notion of real being (*τὸ ὄν*) of which they regarded them as highest classes, reduced them to four;

1. *τὸ ὑποκείμενον* . . . substrate or substance.
2. *ποίον* . . . essential property.
3. *πῶς ἔχον* . . . accidental property.
4. *πρός τι πῶς ἔχον* . . . relation.

This scheme has in turn influenced the enumeration of DESCARTES and SPINOZA: *Substance*, *Attribute* and *Mode;* and that of LOCKE: *Substance*, *Mode* and *Relation.*

MILL regarded the categories primarily as a classification of the modes or forms of real being. That is to say, he viewed them from the metaphysical standpoint. For him their study is an investigation into the ultimate nature of real being. In this he rather misread Aristotle's main purpose. It is, however, interesting to see how he dealt with them as modes of real being: gradually reducing them as species to higher genera, until finally, in admirable consistency with his own philosophical doctrine of phenomenal

[1] The notions of BEING *in itself;* of *Being as* ONE or undivided, or identical with itself and distinct from what is "Other"; of *Being as* TRUE, or as object of knowledge; and of *Being as* GOOD, or as object of desire: are called TRANSCENDENTAL notions, as *transcending* (or surpassing in range of application) all conceivable species and genera, and as being predicable, not *univocally* or in exactly the same sense, but *analogically*, *i.e.* in cognate or similar senses, of every known or knowable individual being.

idealism, he reduced *everything*, *all Being*, to "Sensations" *plus* "Permanent Possibilities of Sensations".[1]

Rejecting as invalid and insufficient, all grounds for believing in the reality of anything apart and distinct from the flow of his own transient conscious states,—his feelings and sensations,—he concluded that mind and matter were merely "permanent possibilities" of such feelings and sensations, and that all the categories could be reduced to these. Only, if he were thoroughly consistent he might have seen that on his own assumption those "permanent possibilities" could stand for nothing *real*, apart from the element of expectation in the transient feelings as they flowed onwards; and he should have therefore reduced all Being to one supreme category: FEELING.

77. THE KANTIAN CATEGORIES.—The philosophers whose schemes have been so far mentioned aimed at a classification of the modes in which the matter or objects of our concepts reveal themselves to our minds, *i.e.* a classification of concepts according to the *reality* represented by these concepts: and this is true even of those who, like Aristotle, regarded the latter not in themselves absolutely, but as *predicates*, as means of *interpreting* subjects to the mind in the act of *judgment*. Every objective concept present to the intellect appears there as a *definite* or *determinate* reality: it involves both *knowing* and *being*, and the modes that determine it those philosophers regarded as modes of *known being*, not raising the question whether the modes sprang from the knowing mind or from the reality. In other words, they did not distinguish between the *matter* of knowledge—whether sentient or intellectual—and its *forms*. No doubt, they recognized external and internal sensation, imagination, abstraction, comparison, judgment, etc. (1, 2, 3), as so many *modes* or *processes* or *functions*, in and through which we acquire knowledge. But they did not conceive conscious knowledge as the result of an application of certain subjective or mental grooves, or *forms*, to certain data or *materials* coming from the region of the extramental into the domain of consciousness.

The philosopher whose name is inseparably associated with this latter conception of sense cognition and intellectual judgment is the German philosopher, IMMANUEL KANT (1724-1804). According to him, the reason why our *external senses* apprehend their data as existing in *space*, and why our *internal senses*—imagination and memory—reveal to us all their data as existing in *time*, is because those sense faculties of ours are endowed with two innate grooves or forms of sense cognition—the forms of "space" and "time"—into which all sense data run. And similarly, the reason why our *understanding* is capable of *interpreting* or *judging about* these "spatial" and "temporal" products of sense-intuition, or, in other words, the reason why we are able to gain further knowledge about them (for knowledge is embodied in *judgment*), is because our understanding is able to apply to them a further and richer collection of *innate forms* or *grooves* with which *it* is equipped. These latter, subjective elements of knowledge, Kant calls the *Categories of the Understanding*. Whatever we call them, the essential feature of Kant's theory is that they belong to the *form* of thought, not to its *matter* or *content*; that they are subjective contributions to the total known

[1] *Cf.* WELTON, *Logic*, pp. 101-3.

product, not objective contributions; that they come from the mind within, and are imposed by it on the reality which comes from without. They are endowments which he argues to be antecedent conditions necessary for any act of conscious cognition whatsoever, though we are not conscious of possessing them until we actually make use of them by applying them in our acts of sense intuition and intellectual judgment. With space and time, the two supposed "forms" of sense cognition, we have no further concern here. Of the "forms" or "categories" of the understanding Kant enumerates *twelve*. These he professes to reach by an analysis of the various possible modes of interpreting reality, or, in other words, of the various forms of the judgment. Though they cannot be fully understood without a knowledge of some of the subsequent chapters on *Judgment*, it will be convenient to enumerate them here.

Judgments may be divided, according to Kant, by reason of their (*a*) *quantity*, (*b*) *quality*, (*c*) *relation*, (*d*) *modality*. Under each of these heads we find three distinct forms of predication, to each of which there corresponds a category of the understanding. We thus reach the following twelve categories:—

Forms of Judgment.		*Categories.*
(*a*) QUANTITY.		(*a*) QUANTITY.
(1) Singular	This S is P	(1) *Unity.*
(2) Particular	Some S is P	(2) *Plurality.*
(3) Universal	All S is P	(3) *Totality.*
(*b*) QUALITY.		(*b*) QUALITY.
(1) Affirmative	S is P	(1) *Reality.*
(2) Negative	S is not P	(2) *Negation.*
(3) Infinite	S is non-P	(3) *Limitation.*
(*c*) RELATION.		(*c*) RELATION.
(1) Categorical	S is P	(1) *Substantiality; Substance and Attribute.*
(2) Hypothetical	If A is B, S is P	(2) *Causality; Cause and Effect.*
(3) Disjunctive	S is either P or Q	(3) *Reciprocity; Interaction.*
(*d*) MODALITY.		(*d*) MODALITY.
(1) Problematic	S may be P	(1) *Possibility and Impossibility.*
(2) Assertoric	S is P	(2) *Existence and Non-Existence.*
(3) Apodeictic or Necessary	S must be P	(3) *Necessity and Contingency.*

Those twelve categories are represented in the Kantian philosophy as so many mental forms or moulds in which the understanding grasps, synthesizes, and interprets the data of sense intuition. We reach the first three, for instance, by reflecting that the understanding may contemplate those data from three distinct points of view: "as a single object, as distinguished from other objects, and as forming in conjunction with those others a complete

class or universe of all that is conceivable. We have thus the three *forms* (or, as they are called by Kant, *categories*) of *unity*, *plurality*, and *totality;* conditions essential to the possibility of thought in general, and which may, therefore, be regarded as *à priori* elements of reflective consciousness derived from the constitution of the understanding itself. . . . They are thus distinguished from the *matter*, or empirical contents, by which one object of thought is distinguished from another."[1]

We shall see later (88, *cf.* 15) how Kant attributes the *necessity* and *universality* of certain of our judgments to these subjective *forms* of thought, denying that those characteristics have any basis in the *reality* which furnishes the *materials* or data of thought, and concluding that those judgments do not reveal to us the nature of *real things as they are in themselves.* But his categories are really not mere subjective *forms* of thought: they classify its data, its *contents.* We may say of all of them what Mansel says of two of them: "the two most important—those of substance and cause—present features which distinguish them from mere forms of thought".[2] They are represented by Kantians as a collection of the forms in which the mind must interpret reality, and as arrived at by an analysis of the cognitive faculties, considered antecedently to all experience ("*à priori*"). But they are, in fact, based on the traditionally recognized heads or forms under which the understanding *does* predicate: on the logical forms of the judgment—and unfortunately, too, on a defective and unscientific arrangement of those forms. There is, for instance, no real ground for distinguishing *negative* from *infinite* judgments under *Quality;* nor for the threefold division under the head of *Modality.* They are, therefore, just as empirical, as dependent on experience, as Aristotle's were; but their advocates put forward for them an extravagant and groundless claim that was never seriously advanced for the Aristotelean categories: the claim to be an absolutely and necessarily *complete* enumeration of all the cognitive forms with which the mind must be equipped in order to make conscious thought possible.

But apart from the fundamental difference in the respective standpoints from which Aristotle and Kant approached the question of classifying the ultimate categories of thought, there is very little real diversity in the results which the two philosophers reached. Aristotle's standpoint was objective; Kant's subjective: "Aristotle had sought to enumerate the kinds of being found in the different things that were; Kant was interested rather in the question how there came to be for us objects having these diverse modes of being."[3]

Aristotle and the Scholastics worked on the general assumption that these "modes of being" were *real*, in the sense that they were *there in the reality*, and were apprehended or recognized—not created and superimposed on the reality—by the mind, though they were far indeed from accepting or propounding the "passive" view of the mind which is characteristic rather of the teaching of Locke, Hume, Mill, and the Empirical school generally. But Kant erred by excess in his reaction against the latter view: "He maintained that in the apprehension of them [the 'modes of being'] we are not merely receptive

[1] MANSEL, *Metaphysics* (pp. 192-3), *apud* WELTON, *op. cit.*, p. 105.
[2] *ibid.* [3] JOSEPH, *op. cit.*, p. 48.

and passive; on the contrary all apprehension involves on the part of the mind the relating to one another in various ways of the elements of what is apprehended; if the elements were not so related they would not be elements of one object; and they cannot be related except the mind at the same time relates them, since relation exists only for consciousness."[1] From this it has been inferred by later followers of Hegel's philosophy that "Relation" is the one supreme category of knowledge. But while it is true that all judging is a process of comparing or relating, it is also true that there are many distinct and irreducible *grounds* and *kinds* of relation.[2] And the ultimate question remains as between realism and conceptualism (4, 5, 6): Are those ultimate grounds *real?* are they *in the reality?* and does an insight into them reveal to the mind, so far as the latter apprehends them, *the nature of reality?* Or are they pure products of intellectual activity, mere mental forms, an insight into which would reveal to us merely *the nature of our own cognitive processes*, and not the nature of things?

Apart from this profound problem of the ultimate metaphysical or real significance of the categories, the latter are largely identical in the schemes of Kant and Aristotle. Indeed, they must be largely identical in all systems of philosophy, however philosophers may differ as to the derivative and reducible, or ultimate and irreducible, character of this or that particular category: concepts such as those of *substance*, and *quality*, and *relation*, and *causality*, and *time*, and *space*, are so broadly and clearly distinct, at the foundations of human thought, that no philosophical analysis is ever likely to eliminate any of them.

JOYCE, *Logic*, chap. ix. JOSEPH, *Logic*, chap. iii. MERCIER, *Logique*, pp. 96 *sqq.*, 118 *sqq.* WELTON, *Logic*, i., chap. iv.

[1] JOSEPH, *op. cit.*, p. 48.
[2] *Cf. supra*, 72, 75; JOSEPH, *op. cit.*, p. 38, n. 1.

PART II

JUDGMENTS AND PROPOSITIONS.

CHAPTER I.

NATURE OF THE JUDGMENT AND PROPOSITION.

78. NATURE OF THE MENTAL PROCESS OF JUDGMENT: STRUCTURE OF THE PROPOSITION.—Just as the mental act of *Simple Apprehension* or *Conception* produces the concept or idea, verbally expressed by the *Term*, so does the mental act of *Judgment* result in a product which is also called a *Judgment*, and whose verbal expression is called a *Proposition*. *A Logical Proposition* is, therefore, *a Judgment expressed in language*. It is a significant utterance which *announces* something about something: *Propositio est oratio enunciativa*—ἀπόφανσις, ἀποφαντικός, Aristotle calls it. "All intelligible or rational discourse, he writes, *signifies* something, but not all discourse *announces* something. A name or term *signifies* something but does not *announce* anything: so, too, the imperative mood of a verb, a command, a wish, an entreaty, these are significant words or phrases, but they make no statements: therefore they are not logical propositions. Every statement or proposition must take the form either of an *affirmation* or of a *denial*. In every proposition there must be a verb. Take, for example, the notion of 'man': unless you assert (or deny) something of him, that 'he *is*' or 'he *was*' or 'he *will be*' something, you have no statement, no proposition."[1] Such is Aristotle's brief analysis. The logical proposition must contain *two terms*, the "something" [*Subject*] about which the statement is made, and the "something" [*Predicate*] that is stated about it; and the verb *to be* which serves as logical [*Copula* or] connexion between the terms.

Sometimes subject, copula, and predicate are expressed by

[1] *Perihermeneias*, chaps. iv., v.

three (or more) distinct words, as, "Man is mortal—*Homo est mortalis*". Such a proposition was called by the older logicians a *propositio tertii adjacentis.* Sometimes copula and predicate are expressed by a single word, and the subject by another, as, *e.g.* "Men die—*Homines moriuntur*". This was called a *propositio secundi adjacentis.* Sometimes the whole proposition is expressed in a single word, *e.g.* "*Pluit*". This was called a *propositio primi adjacentis.* Perhaps the only example of this in English is the *exclamatory* proposition, such as "Fire!" For logical treatment, the two latter forms may be, and ought to be, reduced to the form of the *propositio tertii adjacentis.* Statements in ordinary discourse are not always, or even usually, in the latter form. But they should always be reduced to this form by the student before he can safely deal with their logical implications. In thus reducing them, considerable liberties may—and often must—be taken with their grammatical structure. But we must not exceed the limits of mere grammatical change by altering the *import* of the proposition (86).

Again, the student must learn to distinguish between *Subject* and *Predicate* in a proposition. As a rule this is not difficult. In many cases, however, there is room for doubt, inasmuch as the two terms or concepts, compared or related in the judgment, appear to be equally prominent in the mind, and equally important. [1] Needless to say, although the subject usually comes first, this is by no means necessary. [2] Inversion is common, especially in poetry. It is to *meaning* we must look, not to order of expression. That which is spoken about, explained, interpreted; that which is the first and more fundamental and central in the speaker's mind; that which he fixes or determines or qualifies mentally in some way—is the *Subject.* That which he says about the subject; the better known notion by which he explains, interprets, determines, qualifies the subject; the term that gives information about the latter—is the *Predicate.* In all cases of doubt the final appeal must be to the context, for the *meaning.*

That which presents itself to us through our sense experience for rational interpretation is *concrete* and *individual.* And we interpret it in judgment by means of abstract notions used as predicates. Hence, in the natural

[1] *Cf.* VENN, *Empirical Logic*, pp. 200-14.

[2] It has been already observed (22) that the *grammatical* subject of the sentence is not at all necessarily identical with the *logical* subject of the judgment formulated in the sentence. *Cf.* JOSEPH, *op. cit.*, pp. 150, 152.

order of thought the subject of the logical proposition is nearer to the individual, narrower in extension than the predicate. We naturally predicate an *attribute* or *quality* about the *thing* in which it inheres, not *vice versa*; and a generic or wider notion about a specific, or narrower notion, not *vice versa*. We say "Men are mortal," rather than "Some mortal beings are human". The former was called by the Scholastics a *propositio naturalis* or *ordinata*, the latter a *propositio innaturalis* or *inordinata*. So, too, for example, the propositions "The soul is immortal," "Something immortal is the soul"; "Justice is a virtue," "Some virtue is justice"; "Blue is a colour," "Some colour is blue". We may say naturally, of course, "A virtue may be justice," or "A colour may be blue," or "A stone may be a diamond"; for here we are not predicating the narrower notion of the wider, but of some individual instance or instances (of a stone, colour, or virtue) which we have in mind.

When, however, our judgment does not compare notions one of which is naturally subordinate in extent to the other, or one of which indicates a thing of which the other implies an attribute; but notions which imply equally important, or conceptually independent, attributes that may happen to coincide in the same thing; then there is no real distinction between logical subject and logical predicate. We may say with equal appropriateness "No lawyers are clergymen" or "No clergymen are lawyers"; "Some politicians are not poets" or "Some poets are not politicians"; "The Prime Minister is the First Lord of the Treasury" or "The First Lord of the Treasury is the Prime Minister".

The judgment in which the natural order of predication is inverted was described by Aristotle as predication κατὰ συμβεβηκός—"*per accidens*": "The proper subject of which to predicate attributes was in his view *substance*, and of which to predicate any genus, its species or the several examples of these. Where this order was inverted, the judgment did not state what its subject was in its own nature, but to what it was *incident*. Doubtless this is often what we want to state, as in such a judgment as 'The composer was Handel'."[1]

The following few general directions for discriminating between subject and predicate may be found helpful:—

(*a*) If both terms be exact synonyms, or both proper names, we may regard the first in order as subject and the second as predicate: that is, if we consider their conjunction as real predication, as forming a logical proposition at all.[2]

(*b*) If one term is a substantive and the other an adjective, the former is logical subject, the latter predicate: the function of the adjective is to qualify, explain, interpret.

(*c*) The predicate being usually thought of as an attribute, its *intension* is uppermost in the mind (100), whereas the *extension*

[1] JOSEPH, *op. cit.*, p. 239 n.

[2] *Cf.* 85, *infra.* WELTON, *Logic*, i., p. 160; JOSEPH, *op. cit.* p. 150 n.; VENN, *Empirical Logic*, pp. 211-13.

of the subject is usually the more prominent. This will often help to determine which of two common substantives is subject and which predicate.

(*d*) If one term be singular and the other general the former is usually subject, the latter predicate.

(*e*) If one of two general terms be used *explicitly in its whole denotation*, in an affirmative proposition, it is subject. Affirmative propositions do not make reference to *the whole denotation* of their predicates (91).

(*f*) Professor Welton observes [1] that the presence of a *limiting subordinate clause* will help us to fix upon the logical subject, since such clauses "always really affect the subject, even when it is not immediately apparent that they do so; for the subject is the more definitely determined term in every proposition". It is the more fixed and central element; not, however, the better known, for the function of the predicate is to interpret or explain it.

The example he gives is the following: "I have read all the books in this library which treat of Politics". This proposition illustrates well the ambiguity that may often arise about the logical subject.[2] If the speaker be answering a query as to what books *he* in particular had read, the logical subject would be "*I*". Were he boasting of the extent of his political reading, the logical proposition would run thus: "My *political reading* embraces or extends to all the political books in this library". In the absence of any information about the context we may fairly say that "All the books in this library which treat of Politics" form the subject-matter, the logical subject, about which he wishes to make a certain assertion, namely, that they were read by him. Thus the restrictive clause appears in the subject.

The above remarks apply primarily to the categorical judgment. In the hypothetical, we shall see that *antecedent* and *consequent* take the place of subject and predicate.

The structure of the judgment by means of subject, predicate, and copula shows us that in a certain sense the judgment necessarily involves both *unity* and *plurality*, both *synthesis* and *analysis*. There is a plurality of terms and concepts (subject and predicate); but the act of judgment effects a *union* of these, inasmuch as it is *one* mental act, *one* single interpretation of some *one* reality, a *synthesizing* or bringing together of separate ideas which represent separate aspects (reached by *analysis*) of that

[1] *Logic*, i., p. 177.

[2] *Cf.* example from Byron, *infra*, 94: which is logical subject—the crag or the Rhine?

single reality.[1] All judgment presupposes analysis of our sense experiences (2, 3), viewing and examining these from different standpoints, establishing *diversity*, *plurality*, of aspects. The affirmative judgment *identifies*, *unifies* these aspects in the reality, by the affirmative copula. Even the negative judgment *unites* or compares mentally two concepts, and in separating them by means of the negative copula, increases or makes more definite our knowledge of the *one* reality which it interprets (94, 98).

Hence all judgment involves both mental analysis and mental synthesis, and has been for this reason called *Compositio et Divisio*, or the *Actus Componendi et Dividendi*, by the Scholastics. The act of judgment, therefore, is *the mental assertion or denial of something about something*. This definition gives us the essence or inner nature of the process itself.

79. JUDGMENT AND TRUTH: CONCEPTION: INFERENCE.—There is, however, inherent in this process a *property* which is all-important from the point of view of logic: judgment is *always* the process, and the *only* process, in and through which *truth or error is attained by the mind*. Hence, we sometimes find the judgment defined as the *mental*, and the proposition as the *verbal*, *expression of a truth or falsity*. "Not all discourse," again writes Aristotle, "is an *enunciation* [ἀποφαντικός], but *only that in which we find truth or falsity expressed*. Now, truth or falsity are not found in some forms of discourse: prayer is a discourse, and yet it is neither true nor false."[2]

Of the judgment and proposition alone can we say: *that is true*, or *that is false*. Of the three logically distinct mental processes, conception, judgment, and reasoning (5), judgment is the most important. Conception gives us the notions which are the material of the act of judgment, and all reasoning processes compare judgments with one another in order to reach, and terminate in, new judgments. All human knowledge is embodied in the form of judgment; all truth is contained in it, and in no other mental act. And the reason is this: Knowledge is an *interpretation* of what comes into our experience, and all such interpretation is mentally formulated in acts of judgment. When I investigate any object of sense experience, any individual thing,

[1] "In the judgment the [logical] subject with which we start is modified or enlarged by the predicate and declared to be real. We end with the subject with which we began, differently conceived."—JOSEPH, *op. cit.*, p. 52.

[2] *Perihermeneias*, chap. iv.

οὐσία πρώτη, "*hoc aliquid*," by apprehending in it, and abstracting from it, through repeated acts of conception, the ideas *tree*, *trunk*, *branches*, *leaves*, *green*, *rough*, *large*, etc., I simultaneously refer or attribute these concepts to the individual sense-datum from which I derive them: that is to say, I *predicate* them of the latter, I *interpret* the latter through them, I try to diminish its original indeterminateness by means of them: I *judge* that it is all that they represent. Thus it is that judgment goes hand in hand with, and is naturally inseparable from, the conception or simple apprehension of abstract ideas.

This is a fact of consciousness which is confirmed and corroborated by the application of the common names of language (74). In fact, the process of *naming*, of fixing a common name on some individual "this," or "that," of sense experience, amounts simply to affirming of the vaguely known "this" or "that" in question, some abstract concept found to be embodied and verified in it: to give the name *wolf*, *lupus*, *Vrka* (meaning that which *tears* or *rends*) to an individual, is to predicate of it the attribute (or attributes) implied by the name. And if, after asserting "this" to be a *brown-coloured animal*, I go on to formulate the proposition, "This brown-coloured animal is a wolf," I am simply giving two names to the same thing and stating that the reality designated by both names, and embodying their joint meaning, is one and the same reality. Hobbes was right in saying that stating a proposition is simply expressing one's belief that the predicate is a name of the thing of which the subject is another name; that it is, as it were, giving two names to the same thing, a subject-name and a predicate-name.

These concepts, which form the materials of the judgment, are, if taken in themselves, neither true nor false. Logical terms which express them are neither true nor false: the words of a dictionary, for example, are neither true nor false. Truth or falsity appertains exclusively to the judgment, not to the concept, nor to the process of simple apprehension by which the concept is formed. This is all the more deserving of notice because the concept—particularly if it be complex—often *seems* to carry with it this characteristic of truth or falsity. Scientists, philosophers, and theologians often discuss and dispute whether certain *concepts* are *valid* or *invalid*: but a little reflection will show that in all such cases the controversy is about the truth or falsity of *a certain statement that is made or implied about the object of the concept in question*, the statement, namely, that *the concept represents an object which exists really* in some definite sphere. The dispute is always about the *existential import* of the concept, about the truth or falsity of the *judgment* which asserts that the

concept in question represents not a mere fiction of the thinker's mind, but a reality existing in some sphere beyond that of his present thought (54).

In the other direction, the process of *reasoning*, by which we *infer* one judgment from another, may indeed be consistent or inconsistent with itself, may or may not conform to the recognized canons of inference, and may thus be *formally valid* or *invalid*, but it cannot in itself be said to be *true* or *false*:[1] of the judgments alone which enter into it can we properly say that they are true or false.

80. THE TRUTH OF JUDGMENT IS OBJECTIVE, UNIVERSAL, AND IMMUTABLE.—The judgment, therefore, differs from every other mental process in this, that *it claims to be true*: it claims the *belief* or *assent* of the mind to it as true. Now this claim will be seen on a little reflection to imply of necessity a reference of the whole content of the actual subjective or mental synthesis to something beyond itself. A toothache, or a fit of anger, or a strong emotion or desire, are simply subjective, conscious states, which are neither true nor false, and which do not refer us to anything beyond themselves for verification: but the conscious mental process of affirming or denying, of asserting, judging, reveals itself as an *interpretation*[2] *of something distinct from itself*, something to which an appeal can be made for the *verification* of the mental process. This "something" is the *reality* (in the widest sense of the word), which the mind interprets, in and by its act of judgment: the reference of the judgment to some such sphere of reality—be this the visible world, or the world of mental states, or an invisible, spiritual world, or an imaginary, fictitious world of our own mental creation (33)—is called the *objective inference* of the judgment; and the conformity of the mental representation resulting from the act of judgment, with the originally presented[3] reality which it interprets, may be called the *objective validity* or the *truth* of the judgment in question.

This "objective" or "truth" aspect of the judgment is emphasized in some modern definitions of the latter. "JUDGMENT," says Mr. Bradley,[4]

[1] Of course every *inference* involves the *judgment* that the antecedent is a sufficient ground of the consequent, and, so far, may involve *truth* or *falsity* (*cf.* 148).

[2] Aristotle's treatise on Judgment is called Περὶ Ἑρμηνείας, *i.e. De Interpretatione*.

[3] The sphere of reality referred to, must be, of course, originally *presented* to consciousness in or through the operation of the external and internal senses; the *perceptive* or *presentative* faculties (2, 3)—or reached indirectly by the intellect through such sense presentations.

[4] *Apud* MELLONE, *Introd. Text-Book of Logic*, p. 370.

"IS THE ACT WHICH REFERS AN IDEAL [OR CONCEPTUAL] CONTENT, RECOGNIZED AS SUCH, TO A REALITY BEYOND THE ACT." This definition can be applied to all classes of judgments—including conditionals, such as "If the barometer falls it will rain". But although *All Reality* is in a certain sense the *ultimate* subject of all judgments (123), it is not the proximate subject actually thought about. This is always rather some limited portion of the sphere of reality than reality in general: a point which is well brought out in Bosanquet's definition of judgment as "THE REFERENCE OF A SIGNIFICANT IDEA TO A SUBJECT IN REALITY BY MEANS OF AN IDENTITY OF CONTENT BETWEEN THEM".[1] The mind's concomitant awareness, while announcing the judgment, that the latter is a *true* representation of *objective reality*, furnishes Ueberweg with what he regards as the most fundamental and essential feature of the judgment: which he defines as "THE CONSCIOUSNESS OF THE OBJECTIVE VALIDITY OF A SUBJECTIVE UNION OF CONCEPTIONS WHOSE FORMS ARE DIFFERENT BUT BELONG TO EACH OTHER".[2]

The test of this conformity—the *Criterion of Truth*, as it is called—must be sought, in each case, in that sphere of reality to which the judgment in question is understood to refer.[3] Whether, for example, it is true that "Hamlet killed Polonius," will be determined by reference to Shakespeare's play; whether "all dragons breathe flame," must be decided by reference to the whole literature of dragons; whether Napoleon conquered, or was vanquished, at Waterloo, I must discover by consulting history; whether it be true that the three interior angles of a plane triangle may be less than two right angles, I must determine by analysing the relation asserted in the judgment, and discovering whether there be anything impossible in it.

The logical truth thus claimed by the mental act of judgment has, therefore, this peculiar characteristic, which we call *objectivity*, this reference to a something beyond and distinct from the mental act itself, of the individual mind. The judgment of the individual mind claims to be true *for all minds:* it is itself an individual conjunction of two concepts in a single individual's mind, but it claims to represent something which must hold for everyone, for all minds. Thus, logical truth, because it is objective—or related to a something other than the passing psychological act of judgment—is also *universal*, in the sense that what is true for one mind is true for all minds, that truth is not relative to, or variable with, the variety of individual men or individual minds.

So, also, truth is *immutable*, in the sense that it cannot change with the time *at* which the judgment is made. Of course, all judgments which refer to objects existing, or events happening, *in time*, and which make assertions that are intended to refer to these only as they are at a particular point of time, need not be true of the latter as they are at any other point of time; for in these cases the particular point of time enters into, and becomes part

[1] *Essentials of Logic*, p. 70; *apud* MELLONE, *ibid.*, p. 372.

[2] *Logic*, § 67; *apud* MAHER, *Psychology* (4th edit.), p. 316. *Cf.* JOSEPH, *op. cit.* p. 147: "All judgments, besides affirming or denying a predicate of a subject, affirm themselves as true. But a judgment which affirms itself as true claims to express, so far as it goes, the nature of things, the facts, or the reality of the universe."

[3] *Cf.* VENN, *op. cit.*, pp. 28-33.

of, the predication; and to change the time is really to change the predicate, and so to change the whole judgment. The truth of a judgment, therefore, is not independent of "time *in* predication," and, consequently, "what is true to-day may be false to-morrow". That "Ireland has a native Parliament" is false of to-day, though it was true of the eighteenth century and may be true again of some future date. But this judgment—like every other such judgment—is not fully expressed unless the time referred to is explicitly predicated; and when this is done the judgment so formulated is true for ever, independently of the time *at* which it is formulated—independently of the "time *of* predication". Thus, it always was, and still is, and ever will be true, that "In the eighteenth century Ireland had a native Parliament": *Once true, true for ever.* Similarly it is "once false, false for ever" that "Ireland had a native Parliament throughout the nineteenth century".

Yet another corollary from the objectivity of judgment is its peculiar independence of our wish or will. We are conscious that if at any instant we are to judge *truly* about any matter, we cannot judge just as we please. Not indeed that we cannot gradually so choose and regulate our intellectual surroundings and influences as finally to modify our beliefs, thus making the wish "father to the thought": but that at any given instant *what is true* is not determined *by* us, but is rather determined *for* us, by that objective sphere of reality which our judgment seeks to interpret.

81. Matter and Form of the Judgment: Its "Abstract" Character.—The *matter* of the judgment is the reality about which we judge, and is embodied in the two concepts compared, the subject and predicate. The connexion made by the copula between these two elements, the interpreted and the interpreting, is called the *form* of the judgment. It is because the form remains the same while the matter may vary, that we are able to build up a *general* science of logic about our thinking processes (10, 17). But it would be a mistake to imagine that the form is *wholly independent* of the matter, or can be studied entirely apart from the latter in a "purely formal logic". To a certain extent, the modes in which we think and judge about things depend upon the nature of the things themselves: there are varieties of modes or forms in our thinking processes, and it is the duty of logic to attempt an analysis of all these. The form of the judgment, the mode of connexion between two elements of our thought in the interpretation of any thing, or portion of reality, is not necessarily the same in all cases. In a *hypothetical* judgment, for instance, "If any S is M, it is P," each of the two elements of thought connected together is already complex, "S is M" being one, "S is P" being the other, and the mode of connexion is not the same as in the *categorical* judgment, "S is M". In other words, though the general form of judgment is

the same throughout all judgments, *viz. a synthesis of two elements of thought*, this general form assumes a *variety of modes*, which will be studied in subsequent chapters; and this variety of modes must be accounted for, in ultimate analysis, as due to the variety of ways in which reality presents itself to our minds for interpretation.[1]

Again, it is impossible to separate the treatment of the mental act from that of its verbal expression—the judgment from the proposition. Nor is it desirable to treat the process of judgment separately, in logic, as it should be treated in psychology: for logic deals not with the process of formation of thought, but with the finished mental product; and this can be seized in its most exact and definite shape only in its verbal expression, the proposition. At the same time, it must be remembered that language is not a *perfect* instrument for the expression of actual thought; just as actual thought itself rarely if ever reaches the *ideal* of accuracy which logic, as a *normative* science, sets up for it. Logic deals with mental processes as they *ought to be*, not as—unfortunately—they more usually are. Again, it *abstracts* what may be called the "dry bones," the purely representative elements, the *truth and falsity aspects*, from the warm, living, palpitating, mental process, ignoring all the vital, emotional colouring of the latter.[2] Nay, even some of the truth and falsity aspects themselves are, of necessity, ignored by logic: all those subtle, delicate, semi-conscious movements of thought which exercise such an immense and undoubted influence on our convictions, are largely beyond its usual scope. Perhaps the larger part of those practical, ordinary, everyday assents and inferences, by which our conduct in life is shaped, are the outcome of such complex, manifold, half unconscious influences as could not possibly find *adequate* expression in any number of propositions, or of exactly formulated arguments.

Logic is sometimes called an *abstract* science for this reason, that the judgments and other thought processes with which it deals are divested, as it were, of the clothing they have in individual minds, and are *common*, *typical* of a class, falling short of a complete representation of any such process *in individuo*. This, however, calls for two remarks. Firstly, it is as true of every other science as it is of logic: "Just as thought is abstract [in all the sciences] in its dealings with reality, so logic is abstract in its dealings with ordinary thought".[3] Secondly, those necessary limitations of the scope

[1] *Cf.* JOSEPH, *op. cit.*, pp. 143, 153; also pp. 5-7.
[2] *Cf.* NEWMAN, *Grammar of Assent*, pp. 266-7.
[3] HOBHOUSE, *Theory of Knowledge*, p. 7; *apud* KEYNES, *op. cit.*, p. 69.

of logic must not be interpreted—as they sometimes are wrongly interpreted—in the sense of a reproach. They do not detract from the value and utility of logic, or from its claim to be the sole science which examines thought processes as means to the acquiring of truth. Writers who, like Cardinal Newman in his *Grammar of Assent*, devote special attention to the study of those multitudinous mental influences which are so evasive and intangible as to defy *adequate* expression in the stereotyped moulds of traditional logic, are inclined, naturally enough, to emphasize the shortcomings of the latter discipline; but it must not be forgotten that the difference between the fully conscious and the semi-conscious is one of degree, not of kind: that man has not two reasoning faculties, but one: that the processes of conception, judgment, inference, as leading to truth or error, must be tested at one and the same bar of *Rational Reflection*, and according to one and the same code of *Logical Laws;* that there is no such thing as a "logic of real life," at variance with, or even distinct from, the "notional logic" that has been built up by centuries of attention to the mental processes by which we attain to the knowledge of truth.

82. INTERPRETATION OF TERMS AND OF PROPOSITIONAL FORMS: FORMULATION: "MEANING" AND "IMPLICATIONS" OF THE PROPOSITION.—The proposition being the *verbal* expression of some truth which is *mentally* expressed in the judgment; ordinary language, too, being so often ambiguous and uncertain in its meaning; and, furthermore, what is *practically* the same truth being capable of equally exact expression in various forms of statement: we shall obviously be obliged, in the first place, to fix definitely the meaning we are to attach to the forms of expression that enter most frequently into logical propositions; and in the second place, to examine those among the ordinary forms of verbal statement or proposition which lend themselves most easily and most successfully to the logical treatment of the mental fact itself, the judgment.

For instance, in the traditional scheme of propositions, *All S is P*, *No S is P*, *Some S is P*, and *Some S is not P*, the interpretation of the *meaning* or *import* of each form will evidently depend on the exact meaning we assign to the terms "*all*," "*no*," "*some*". And again, we may inquire whether, perhaps, besides the traditional scheme of propositions just referred to, we might not find other schemes of great logical utility, to which all ordinary statements might possibly be reduced: such, for instance, as the *Existential Scheme*—"*S exists* [S > O]" and "*S does not exist* [S = O]"—into which many statements will be found to fall more naturally than into the traditional scheme (123).[1]

[1] *e.g. God exists. There are no such things as fairies. It is raining. The Resurrection of Christ is an historical fact.*—*Cf.* KEYNES, *op. cit.*, p. 218.

The reduction of an ordinary statement to some one or other of the propositional forms explicitly recognized in logic, is called *formulation*. Some of the common forms of statement fall more naturally into one logical scheme, some more naturally into another. But in all attempts at formulation we must have clear notions as to the exact *meaning* of the judgment to be "formulated," and the meaning of the propositional form to which we wish to reduce it. To *interpret* a proposition is to attach a *meaning* or *import* to it; and this will, of course, depend partly on the meaning (intensive and extensive) of the terms that enter into the *proposition*, and partly on the signification we attach to the *synthesis* or *connexion* of the terms, *i.e.* to the *copula*. Fixing the exact *meaning* of more or less ambiguous terms is, in a certain degree, a *conventional* or *selective* process; interpreting the meaning of a given propositional form is, likewise, in some measure optional: and the question whether a judgment expressed in a given propositional form is true or false, will depend not only on its matter, but on what is recognized by common agreement to be the import of its propositional form.

It was said above that what is *practically* the same judgment may be expressed in different propositional forms. The limitation is a necessary one, for a judgment often loses some of its import when its mode of verbal expression is altered. Hence arises the very important distinction between the *meaning* or *import* of a given propositional form, and what are called its *implications*, *i.e.* all the truths that are necessarily involved in it, and which may be extracted from it by careful and prolonged mental scrutiny. These mental processes and their products—*immediate inferences* as they are called—will be dealt with below (chaps. v., vi.). Here we wish to point out that, owing to neglect of this distinction between *import* and *implication*,[1] there has been much misunderstanding about the supposed conflicting claims of the various interpretations of propositional forms put forward, at one time or another, by logicians.[2]

[1] These technical terms have been used as *synonyms* in reference to the meaning of *concepts* and *terms*. *Cf.* Bk. i., chap. i., 30-34.

[2] "The assignment of meaning is within certain limits arbitrary and selective. But if element *a* necessarily involves element *b*, then *a* having been assigned as part of the meaning of a given propositional form, it is no question of meaning as to whether the form in question does or does not imply *b*, and there is nothing arbitrary or selective in the solution of this question."

"Sometimes the elements *a* and *b* mutually involve one another. It may then

It is often not easy in practice to distinguish between meaning and implication; and hence, in reducing ordinary statements to logical form, *i.e.* to some one or other of the recognized propositional forms, or in transferring a proposition from one logical scheme to another, we have sometimes to overstep the limits of mere verbal change, to have recourse to what is really *inference*, and thus erect into *meaning* what was originally only an *implication*.[1] Logic *postulates* the right of mere *verbal transformation*, the right to vary, as we find it necessary, the mode or form of expression, so long as we do not interfere with the *meaning* of the judgment expressed. Hence, the legitimacy of any change of wording which involves *inference*—which puts into the *meaning* of the new form what was only an *implication* of the old—must be determined by reference to the ordinary logical canons of inference.

JOSEPH, *Logic*, chap. vii. VENN, *Empirical Logic*, chap. viii. MELLONE, *Introductory Text-Book of Logic*, pp. 369 *sqq*. MERCIER, *Logique*, pp. 131 *sqq*. MAHER, *Psychology*, chap. xv. KEYNES, *Formal Logic*, pt. ii., chap. i.

be a question of interpretation whether *a* shall be included in meaning, *b* thus becoming an implication, or whether *b* shall be included in meaning, *a* becoming an implication."

"A failure to recognize what is really the point at issue in a case like this has sometimes caused discussions to take a wrong turn. Thus the question is raised whether the import of the proposition *All S is P* is that the class *S* is included in the class *P*, or that the set of attributes *S* is invariably accompanied by the set of attributes *P*; and these are regarded as antagonistic theories. If the implications of a proposition are regarded as part of its import, then the proposition may be said to import both these things. But if by import of a proposition we intend to signify its meaning only, then we may adopt an interpretation that will make either of them (but not both) part of its import, or our interpretation may be such that the proposition imports neither of them."—KEYNES, *op. cit.*, p. 71.

[1] *Cf.* KEYNES, *op. cit.*, pp. 422-3.

CHAPTER II.

KINDS OF JUDGMENTS AND PROPOSITIONS.

83. Problems on the Import of Judgment: Basis and Aim of Classification of Judgments.—The preceding chapter dealt with (*a*) *some* of the essential characteristics of judgment: its relation to reality, for instance, and the immutability and universality of the truth embodied in it. We also (*b*) pointed to the necessity of interpreting, or fixing the meaning of, propositional forms, and of the words ("all," "some," etc.) entering into these forms. And finally, (*c*) we indicated the possibility of selecting and comparing various schedules or schemes of propositional forms to which ordinary statements might be more or less conveniently reduced in order to admit of exact logical treatment.

In reference to the latter points, (*b*) and (*c*), it must be borne in mind that no one scheme of propositional forms will be found adequate to express in a *fully* satisfactory way *all* the different mental forms which the act of judgment may assume; that the determination and interpretation of verbal or propositional forms are to some extent matters of convention; and that, therefore, the choice between those various schemes is a choice between what is *more or less convenient or suitable*, not between what is *right and wrong* absolutely (p. 165, n. 2).

Questions, however, which fall under the first head (*a*), concerning as they do the nature of the act of judgment itself, *are* questions of right and wrong, and are independent of all conventions. Such was the question of the truth or objective reference of the judgment (80). Such, too, are the questions raised below, concerning the implication of existence in the judgment (123 *sqq.*), the *modality* of judgments and propositions (89, 90), the distinction between *necessary* and *contingent* matter in the judgment (85-88), the function of the negative judgment as compared with the affirmative (97-98). These would properly be described as "*problems relating to the import of judgments and propositions*";

though, under this rubric, discussions regarding the interpretation and comparison of propositional schemes (91-93, 99-109) are often likewise included. Both classes of questions will be gradually raised by an investigation of the various kinds of judgments and propositions which a little reflection will reveal to us as exemplified in ordinary human thought and discourse. What are *the various mental modes or forms* in which judgments can convey meaning as interpretations of reality? This is not an easy question to answer; but logicians are pretty generally agreed that an attempt to answer it reveals four distinct bases, or *fundamenta divisionis*, on each of which judgments may be divided, namely, *quality*, *quantity*, *modality*, and *relation*. On the details of the classification, however, there is some divergence of opinion and practice. We have already (76) encountered Kant's objectionable subdivisions of judgments under each of the four heads just mentioned. We purpose to substitute for them the classes set down in the following scheme.

Judgments may be divided on the basis of

I. QUALITY, into (*a*) *Affirmative, e.g.* . . . *S is P*[1]
and (*b*) *Negative, e.g.* . . . *S is not P*

II. QUANTITY, into (*a*) *Universal:*
(*α*) General, *e.g.* . . *All S is P*
(*β*) Singular, *e.g.* . . *This S is P*
and (*b*) *Particular* or *Indefinite, e.g.* . *Some S is P*

III. MODALITY, into (*a*) *Modal:*
(*α*) Apodeictic or Necessary, *e.g.* . *S must (not) be P*
(*β*) Problematic or Contingent, *e.g.* *S may (need not) be P*
and (*b*) *Pure* or *Non-Modal* or *Assertoric.*

IV. RELATION, into (*a*) *Simple:*
(*α*) Categorical (*Assertoric*),[2] *e.g.* . *All S's that are M are P*
(*β*) Conditional (*Modal*),[3] *e.g.* . *If any S is M it is P*
and (*b*) *Compound:*
a (1) Alternative (*Assertoric*),[2] *e.g.* . *Either A or B*[1]
(2) and its denial, the Remotive, *e.g.* . *Neither A nor B*
β (1) Conjunctive or Copulative (*Assertoric*),[2] *e.g. Both A and B*
(2) And its denial, the Disjunctive,[4] *e.g. Not both A and B*

[1] In the various examples given, *S*, *M*, and *P*, stand for terms, *A* and *B* for simple propositions.

[2] *Cf. infra*, 144. [3] *infra*, 135. [4] *infra*, 141.

γ { (1) Hypothetical (*Modal*),[1] *e.g. If A then necessarily B*
(2) and its denial, *e.g.* . . *If A then not necessarily B*

84. JUDGMENTS CLASSIFIED ACCORDING TO RELATION.—A glance at the various members under the last head of division will show how fundamental the division is, and how widely different are the types of judgment brought together under it. We have illustrated all that has been said about judgment so far by choosing as examples *simple categorical judgments*. It is with these in the main we shall continue to deal for the present, deferring to subsequent chapters our examination of the various kinds of compound judgments. The full import of the present division will be grasped only when we have analysed the nature of hypothetical and alternative judgments, and compared these with the categorical.[2] Here, then, we will be content with a brief explanation of the distinctions involved, and a simple definition of each of the members.

The simplest form of judgment is the *categorical:* it is an absolute, unconditional affirmation or denial of something (*P*) about something (*S*). It enters as an element into all compound judgments.

A *compound* judgment "may be defined as a judgment into the composition of which other judgments enter as elements".[3]

A *conjunctive*[4] (or *copulative*) judgment is one which asserts that two (or more) simple judgments are true *conjointly*, or *together*.

A *disjunctive*[5] judgment is one which *disjoins* or separates two (or more) simple judgments by *denying* that they are true together.

A *hypothetical* judgment is one which affirms (or denies) that the truth of one simple judgment (*B*) is a necessary consequence of the truth of another (*A*).

[1] *Cf. infra*, 138. [2] *infra*, 132-5, 146.

[3] KEYNES, *Formal Logic*, pp. 82, 478. His treatment is here followed. It is a distinct improvement on the triple division into *categorical*, *hypothetical*, and *disjunctive*. It is practically impossible, however, to find an entirely satisfactory nomenclature. The categorical judgment is *simple*, no doubt, as opposed to the types of judgment described here as compound, but it may itself be expressed either by a SIMPLE or by a COMPLEX (categorical) PROPOSITION, according as the latter has a simple or a complex term either as subject or as predicate (*ibid.*, p. 478).

[4] Some logicians use the term *conjunctive* as synonymous with *hypothetical*.

[5] Most logicians apply the title *disjunctive* to what we have called *alternative* judgments (141). Though the disjunctive ("*Not both A and B*") implies an *alternative* ("*Either not-A or not-B*"), it is better to keep the two forms and names distinct.

An *alternative* [1] judgment is one which *asserts* that *one or other* of two (or more) simple judgments is true.

A *remotive* judgment is one which *denies*, or *removes*, or abolishes, the *alternative* altogether, by denying that any single member of the latter is true.

The division of compound judgments into six members is not the only one possible. We find it the most convenient, though it is not at all perfect. The distinctions are much more fundamental in some cases than in others. Besides, the distinction between the first and second member of each pair [(1) and (2)] is a distinction which is based on *quality*; the latter distinction, however, carries with it, in these cases, far weightier consequences than when applied to the simple categorical judgment. It will be noted, further, that the members arrived at on the first three grounds of division (*quality*, *quantity*, *modality*) are illustrated symbolically by *categorical* propositions. It is in their application to categoricals only that we shall examine these distinctions in the present and immediately following chapters.

85. "NECESSARY" AND "CONTINGENT" JUDGMENTS, WITH THEIR SYNONYMS.—We have divided judgments into *modal* and *non-modal*; and the former into "necessary" and "contingent". This latter distinction is the one which really underlies the modal distinction of propositions into "apodeictic" and "problematic," as interpreted below (89). We purpose to examine it here. It arises directly out of the study of the predicables and definition (43-57); and it is not understood in the same way by all. The nomenclature, too, of the members of the division has undergone considerable variation. The Schoolmen spoke indifferently of (*a*) the *propositio* PER SE NOTA and the *propositio* PER ALIUD NOTA; (*b*) the *modus dicendi* (or *propositio* or *enunciatio*) PER SE (*κάθ' αὑτό*), and the *modus dicendi* (or *propositio* or *enunciatio*) PER ACCIDENS (*κατὰ συμβεβηκός*); (*c*) the proposition or judgment *in materia necessaria*, and the proposition or judgment *in materia contingenti*; (*d*) the *metaphysical* and the *physical* judgment; (*e*) the *pure* or *rational*, and the *empiric* or *experimental* judgment; while modern writers speak of (*f*) the *analytic* and the *synthetic* proposition; (*g*) the *a priori* and the *a posteriori* proposition; (*h*) the *verbal* and the *real* proposition; (*i*) the *essential* and the *accidental* proposition; (*j*) the *explicative* and the *ampliative* or *instructive* or *augmentative* proposition.[2]

All those various couples are meant to express—with certain shades of difference—the ame broad distinction; and we may at

[1] Commonly called *disjunctive* also. [2] *Cf.* VENN, *op. cit.*, p. 291.

once state it to be the distinction between judgments or propositions in which the connexion between the subject and predicate is regarded as an *absolutely necessary and immutable connexion* (propositions "in *necessary* matter") and those in which it is regarded as a *contingent or changeable connexion* (propositions "in *contingent* matter"). If the predicate gives (1) *the whole or part of the connotation of the subject*, or (2) anything following *necessarily* from the connotation, as a *property* in the strict sense (47), then the judgment belongs to the former of the two classes; while if it gives an *accidens*, whether separable or inseparable, the judgment belongs to the second of the two classes.[1]

The distinction, as here understood, is best expressed by some one or other of the Scholastic couples, such as "necessary" and "contingent". The use of the terms "analytic" and "synthetic" is somewhat misleading, inasmuch as *every* judgment is in a sense both analytic and synthetic (75). Mr. Joseph suggests[2] that we should call those judgments *verbal* which are true "by convention as to the meaning of words," and those *real* "whose truth does not rest upon the meaning given to words, but which state something about the nature of things". Of the latter, then, some would state what "is seen to be necessary," others what "rests upon mere experience of fact". For these we would suggest the terms *necessary* and *contingent* respectively. He suggests, further, that when the judgment gives us part (or the whole) of the real definition of the subject, we should call it *essential;* otherwise, *accidental. Essential* would then include all verbal and some real judgments. *Accidental*, understood in the sense of Aristotle's "*καθ' αὑτὸ συμβεβηκός*, to include what is demonstrable of a kind, will cover all Kant's 'synthetic' judgments, whether they be grounded on an experience which, so far as we can see, might have been otherwise, or on an insight into a necessary relation of concepts; *i.e.* in Kantian language, whether they are synthetic *a posteriori* or *a priori*" (*cf.* 86). In other words, "accidental" judgments would then include the predication not only of separable and inseparable accidents but of properties in the strict sense. We prefer to call the latter sort of predication "essential," taking this term as synonymous with "necessary" (*cf.* 87).

86. "PROPOSITIONES PER SE NOTAE," AND "MODI DICENDI PER SE".—A proposition of the first class is said to be *per se nota, i.e. knowable by itself*, because from an examination of the terms themselves, from an analysis of the *comprehension* of the notions compared, and without appealing to any independent source of information, we can know that there is a *necessary* connexion between the concepts, that the one *involves* (or *excludes*) the

[1] The same distinction may, of course, be applied to negative judgments, according as the separation or exclusion of predicate from subject is conceived as *necessary*, or as *contingent*.

[2] *Logic*, pp. 189, 190.

other.[1] A proposition of the second class is said to be *per aliud nota; i.e.* we can know that subject and predicate agree (or disagree), not from any mere analysis of the terms or concepts compared—one does not *necessarily* involve (or exclude) the other,—but only by an appeal to facts.

Of course, the predicate of a proposition may *de facto* be *necessarily* connected with the subject without our being aware of the existence of such a necessary connexion. But, provided the necessary connexion is really there, no matter how long and elaborate be the analysis required to make it explicit, Scholastic logicians regard the proposition as *in materia necessaria* and *per se nota* IN SE, *i.e.* knowable *in itself*, even although it be not yet *per se nota* QUOAD NOS, *i.e.* known *by us* to be a necessary proposition. Thus, all the most remote and complex conclusions of the pure mathematical sciences would be no less *necessary* and *per se notae* than the truth that "two straight lines cannot enclose a space," or the truth that "two and two are four".

The point of view from which the Scholastics, following Aristotle, regarded the distinction in question, was, therefore, frankly objective. Hence, they perceived and indicated various ways in which the necessity of the connexion between the terms might make itself manifest. They enumerated various "*per se* modes of predicating," as distinct from "*per accidens* modes of predicating".

The *first* and most manifest "modus dicendi *per se*"—or *essential proposition*, to use one of the more modern expressions—is that in which the predicate gives the whole or part of the connotation or essence of the subject. All definitions belong to this class, as also all propositions whose predicates give the *genus* or the *differentia* of the subject, *e.g.* "*A square is a rectangle*," "*Man is rational*". We may likewise regard as included in this class all *synonymous*, *tautologous*, and *identical* propositions—whether the predicate be a proper name, as "Tully is Cicero," or a connotative name, as, "Veracity is truth"—and also all *purely formal* propositions, such as "A is A," "A either is or is not B," "If all A is B then no not-B is A".[2]

[1] Comprehension, it will be remembered, includes more than connotation: it includes all the attributes which are *de facto* common to all members of the class. It therefore includes all attributes necessarily involved in, or connected with, the connotation.

[2] KEYNES, *op. cit.*, p. 52. An apparently identical proposition is sometimes used to make a real assertion, in which case it is a *real* or *synthetic* proposition; *e.g.*

A *second* and less manifest, though scarcely less important, kind of essential or analytic proposition, or "modus dicendi *per se*," is the one in which analysis of the terms reveals the predicate to be a *property*, necessarily involved in, and connected with, the subject. For example, the predicate of the proposition: "*The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides*"—will be found on analysis to give a necessary property of the subject. Similarly, the proposition "*Every number is either odd or even*," and, finally, all the more remote conclusions of mathematics, will be found to belong to this second class of *per se* propositions.[1]

It may, therefore, require a long and elaborate analysis to determine whether a given proposition is or is not *in materia necessaria*. But it is important to note that this analysis is carried on independently of any appeal to extrinsic sources of information. That is to say, if the proposition is *in materia necessaria*, we can ascertain its truth independently of any additional experience over and above the experience by which we acquired the knowledge we already possess about the system of concepts with which we are concerned:[2] in this sense, and in this sense *only*, have such propositions a right to be called *a priori*, *i.e.* knowable prior to, and independent of, sensible and intellectual experience; for they do and must presuppose *some* experience—that, namely, by which we acquired the concepts in question.

In the case of *accidental* or *synthetic* judgments, on the other hand—as for example, "Napoleon was defeated as Waterloo"—our knowledge of their truth or falsity cannot be derived from

"A man's a man," "Boys will be boys," "War is war". The contradiction of a *purely formal* proposition, might be called a *formal contradiction*, or a contradiction in forms. The contradiction of an ordinary *analytic* proposition is usually called a *contradiction in terms*. Both, of course, involve a contradiction in thought, an incompatibility between the judgments opposed.

[1] "*Per se* duplicitur dicitur," writes St. Thomas. "Uno enim modo dicitur propositio per se, cujus praedicatum cadit in definitione subjecti, sicut ista: Homo est animal; animal enim cadit in definitione hominis. Et quia id quod est in definitione alicujus est aliquo modo causa ejus, in his quae sunt per se, dicuntur praedicata esse causa subjecti. Alio modo dicitur propositio per se, cujus e contrario subjectum ponitur in definitione praedicati; sicut si dicatur: Nasus est simus, vel Numerus est par; simum enim nihil aliud est quam nasus curvus, et par nihil aliud est quam numerus medietatem habens, et in istis subjectum est causa praedicati."—*De Anima*, lib. ii., l. 14. "Ut propositio dicatur per se, sufficit (writes Cajetan) in subjecto includi id quod ponitur in definitione praedicati . . . sufficit subjectum inesse definitioni praedicati, per se vel per aliquid sibi intrinsecum."—CAJETAN, *Comm. in Post Anal.*, chap. iv.

[2] *Cf.* JOSEPH, *Logic*, p. 174, n. 2.

any such analysis of the objects of our thought, but must be sought in extrinsic sources. Hence, also, these may be called with special propriety *a posteriori* judgments, *i.e.* judgments whose truth or falsity we learn *subsequently to* and *dependently on* experience.

87. Is the Distinction Objective or Subjective?—We can now see to what extent the distinction, thus understood, between these two great classes of propositions, is objective or subjective, fixed or variable, independent of, or dependent on, the state of the individual's knowledge in presence of any given proposition.

It does *not* depend on the subjective state of the individual's knowledge—on whether his concepts of the object or reality under examination are poor or rich in their *content* (31). Every judgment implies that in the mind of the person who makes it there is an *analysis* of the subject-matter or raw material of the judgment into distinct aspects embodied in distinct notions, and a subsequent comparison and *synthesis* of these notions [1] (78); and, furthermore, that the mind sees a necessary connexion between the *contents* of the notions thus compared. Hence an analysis of the *content* of either notion must yield the other. In this sense, *every* judgment would be analytic for a given individual except on the first occasion when he consciously grasped or understood it; and a judgment might be analytic for one mind and synthetic for another.[2] But there is nothing to be gained by this departure from the more objective interpretation: it makes the distinction exclusively subjective, and essentially variable.

While not going so far as to make the distinction purely subjective, most modern logicians have introduced a *conventional* element into it, by making it turn on the *connotation* of the terms of the proposition: defining an *analytic* (or *verbal* or *essential* or *explicative*) proposition as *one whose predicate gives the whole or part of the connotation of the subject*, and a *synthetic* (or *real* or *accidental* or *ampliative*) proposition as *one whose predicate gives a property or accident of the subject.* This departure from the Scholastic account is obviously due to the desire to make the

[1] In this sense every judgment is both analytic and synthetic. Hence it is unfortunate that these terms should have been also used to designate necessary and contingent judgments respectively. It has led to a misunderstanding of the latter distinction. *Cf.* Joseph, *Logic*, p. 187; Mellone, *Logic*, pp. 367 *sqq*

[2] This is the manner in which some modern logicians, Bradley (*Principles of Logic*, p. 172) and Veitch (*Institutes of Logic*, p. 237), for example, understand the distinction.

distinction more convenient for logical purposes, and more in keeping with the common teaching on connotation and definition (31, 32, 51). But the distinction is not of any great *logical* importance (though extremely important from the point of view of *metaphysics*), and hence the advantages to be gained by the present interpretation do not seem sufficient to warrant any departure from the view of Aristotle and the Scholastics, *i.e.* the view which bases the distinction on the *necessary* or *contingent* character of the relation established between subject and predicate, and which, consequently, regards the predication of a *property* as constituting an *essential* or *necessary* judgment (*in materia necessaria*).

It is, however, merely a matter of convention and convenience, involving no fundamental difference of view as to the nature of the act of judgment. In either view the distinction is ultimately objective; for the determination of the limits of connotation must be guided, and is in practice always guided, *by a constant appeal to objective facts*, to the *matter* of our thought (32). Hence, the distinction between the two great classes of judgments is both objective and fixed, at least in the same degree as connotation and definition are. A proposition, at first synthetic, may, in process of time and by gradual change in the connotation of the terms, become analytic. But in such cases it is not really the same judgment we are dealing with. "We ought rather to say that the same form of words now expresses a different judgment."[1]

Similarly, in the scholastic view, doubt may arise as to the proper class in which to place a given proposition; and progress in knowledge may clear up the doubt by definitely determining a given predicate to be a *proprium* rather than an *accidens inseparabile* of the subject: but the ultimate test is the objective character of the relation, and not the state of our knowledge (or opinion) as to its character.

For instance, does the judgment "Man is mortal" embody a necessary, analytic truth, or only a contingent, synthetic truth? Are subject and predicate so connected that their separation is inconceivable, impossible? That depends on the intension or meaning of the concepts (31); and this in turn depends, of course, upon the real nature and constitution of the individuals to whom we apply the concepts and the names (32). If "man" is a living being, composed *essentially* of *distinct* active principles whose actual separation or dissociation would mean death; and if "mortal" means merely that these active principles *can be* conceived to be separated: then man would be

[1] KEYNES, *op. cit.*, p. 15.

essentially mortal; "mortality" would be a strict *proprium* of his essence (47); and any individual man would continue to be "mortal" in the sense indicated, even though the possible separation of parts or principles were never actually effected, even though actual death were for ever precluded by the influence of some higher cause. "Man is mortal" would be a metaphysically necessary, analytic judgment.[1]

But if "mortal" means not so much that the living being *can conceivably* "die," or cease to exist, by dissolution of its constitutive parts or principles, but rather that it is exposed to such disintegrating physical agencies as *will* and *must* sooner or later *actually* compass its death; then man would not be *essentially* "mortal," for we could conceive a man exempted perpetually from these disintegrating physical agencies, and should no longer call him "mortal" were he so exempted. We should admit that in the ordinary course of physical or visible nature, as we know it, all men are subject to the physical agencies that culminate in death: that thus "mortality" is a natural *physical* property, or inseparable accident (48), of their condition; but were these natural conditions miraculously changed, whether for some men or for all men, so that these would continue to live indefinitely, we should continue to regard them as "men," as still unchanged in regard to those essential attributes which we consider as constituting the *human* essence or nature, though we should describe such men not as mortal, but as immortal. The judgment "man is mortal" or "all men are mortal," we should thus hold to be a synthetic judgment, *in materia contingenti*, *physically* but not *metaphysically* necessary.

For our assent to the judgment, in this latter sense, as true, we rely on the knowledge we have gathered by experience about the nature of the physical agencies that operate on the life of man, and upon our conviction that these agencies are uniform in their activity throughout all space and time.[2]

Interpreted in the former sense, however,—as *metaphysically* necessary,—the judgment rests on quite other grounds: it asserts a metaphysically necessary connexion between two concepts, and our reason for the assertion is that *an analysis of the concepts* reveals such a relation. This amount of meaning at least, it contains: "If man is a composite, living being, the parts or principles of which he is composed may conceivably be separated"; and the truth of this hypothetical judgment is independent of the truth of its antecedent (132). Whether or not "there are in existence men who are composite living beings," whether or not the judgment "men are composite living beings" is a true judgment, must be ultimately determined not by an analysis of concepts, but by an appeal to experience. This, which is the categorical judgment, referring as it does to the concrete, actual, existing order of things, cannot be metaphysically necessary or analytic: judgments of this latter class refer only to the sphere of *abstract*, *possible* objects of thought.

[1] We may interpret the well-known Scholastic example "Man is risible" ("*Homo est risibilis*") in this way, as meaning that the *capacity* or *faculty* of laughing is a *strict* "proprium" of human nature: the proposition would thus be an *analytic* proposition, and would still be true of a man who *de facto* never laughed, whose faculty *was never reduced to act*. Were we to interpret it, as Father Joyce does (*Logic*, p. 239), as "liable to miraculous frustration" we should regard it as synthetic (as equivalent to the judgment "*Men laugh*").

[2] *Cf.* JOSEPH, *Logic*, p. 491; NEWMAN, *Grammar of Assent*, pp. 280-1.

Now, some of these metaphysically and logically necessary principles are implied in all judgments. There are quite a number of analytical principles —"laws of thought" they are called—*involved* in the mental assertion of even a synthetic judgment; but this fact does not make the judgment itself analytic; nor should it lead us to overlook the real distinction between the two classes of judgments in question.

When, therefore, we are dealing with complex concepts, analysing them and comparing them with one another, two distinct questions arise: *firstly*, how far the various elements in those concepts necessarily involve or necessarily exclude one another—a question which will, in every instance, be answered by an *analytic* or *metaphysically necessary* judgment;—and *secondly*, the question whether these complex concepts themselves represent each an *actually existing* object of thought, or one which is *merely possible*—a question which will always be answered by a *synthetic* judgment, grounded on experience. It is the failure to distinguish between these two questions that has caused such ambiguity and misunderstanding about the proper interpretation of the *Law of Uniformity in Physical Nature*, and the grounds on which we assent to it (223-4). The concepts compared in this principle are complex,[1] *viz.* "a natural cause devoid of freedom" and "a constant, regular, uniform series of effects"; and the more complex the concepts are, the greater the amount of sense experience needed in order to form them; and the greater the amount of sense experience postulated for the knowledge of any truth, the stronger is our tendency to describe the process of acquiring it as "induction".

88. Comparison of the Scholastic with the Kantian View.—It will be noticed that the class of judgments described by the Scholastics as "*necessary* judgments" is very much wider than the class of "*analytic* judgments" in the more modern sense of this term; the former include all the truths—even the remotest conclusions—of the purely deductive sciences, whereas the latter are restricted to the comparatively small group of judgments which constitute definitions, or self-evident axioms and principles.

It will also be apparent that the basing of the distinction on connotation is accountable for the more modern forms of nomenclature for the two classes of propositions. The "verbal" or "explicative" proposition is the one that gives no information beyond what is contained in an intelligent grasp of the connotation or definition of the subject-term; the "real" or "ampliative" or "instructive," the one that gives some new and real information beyond what was had from merely understanding the meaning of that term.

It would, therefore, be a mistake to imagine either that the only

[1] *Cf.* Joyce, *op. cit.*, p. 239.

propositions *in materia necessaria—i.e.* propositions formulating relations which *cannot be conceived to be otherwise than they are*, between the objects of our thought—are those described as *verbal* or *explicative* by some modern writers, or that judgments *in materia necessaria* never give us new and real information. These latter have been called *a priori* judgments in the sense explained above.[1] Leibniz neither very happily nor very correctly described these "necessary" judgments as "analytic," on the assumption that their predicates could be always derived from an analysis of the essence of their subjects, independently of experience; while he called "contingent" judgments "synthetic," "empiric," "*a posteriori*," *i.e.* posterior to, and dependent on, experience. Apart from the fact that *every* judgment involves synthesis (of notions) as well as analysis (78), it is not true that all judgments *in materia necessaria* are reached by finding the predicate in an analysis of the essence of the subject. Kant, who adopted the classification of Leibniz, was not slow to see that only a comparatively small number of such judgments are obtained by an analysis of the subject-term. These merely *verbal* or *explicative* judgments, which give us no new information, he described as "analytic *a priori*" udgments. Another—scientifically unimportant—class of judgments *in materia contingenti*, which, being based *merely* on experience, have not the characteristics of *necessary* and *universal* validity, he called "synthetic *a posteriori*" judgments. There still remained, then, an important and extensive class of judgments *in materia necessaria*—judgments both necessary in character and productive of *real* knowledge—which Kant refused to call "analytic" because they did not verify the narrower definition of that title, and which he refused to call "*a posteriori*" because their *necessary* and *universal* validity—which gave them their scientific value and significance—could never, he believed, have been the outcome of experience, but must have been conferred upon them by the mind itself *prior to*, and independently of, all experience. These he called "synthetic *a priori*" judgments: *synthetic*, because their

[1] The terms "*a priori*" and "*a posteriori*" refer primarily to our reasoning processes; the former denoting those which descend from cause to effect, from what is naturally prior to what is naturally posterior, the latter those which lead us from a knowledge of effects to a knowledge of their causes. True science in the Aristotelean sense of the word (251) is deductive, *a priori*, descends from causes and reasons as antecedents to effects and theorems as consequents; but, very often, before we reach a knowledge of these causes which become the explaining principles and reasons of our science, a long work of *analysis* is needed, to decompose the complex data of experience into its simplest elements.

necessary character sprang from the *synthesis* or union of a purely mental and necessity-producing form of thought in the understanding, with the contingent and ever varying data of sense experience; *a priori*, because this necessity-producing groove of thought—or "category," as he called it—is an innate mental endowment, existing in the mind prior to, and as a necessary condition for, all intelligible mental experience.[1]

Those necessary judgments which Kant calls "synthetic *a priori*" form the very foundation of all scientific knowledge. Such, for example, are the propositions: "Seven and five are twelve," "The straight line is the shortest distance between two points," "Whatever begins to be has a cause".[2] They have undoubtedly this characteristic, that they claim to be *necessarily* and *universally* valid: herein lies their scientific value. But the question as to the nature and ultimate grounds of this characteristic, is not so much a logical as a metaphysical question: it can be, satisfactorily discussed only in psychology and criteriology.

Here it will be sufficient to point out that while positivist philosophers (who endeavour to reduce all human knowledge, with all its characteristics, to sense experience) are logically forced to deny that we have any rational grounds for believing such judgments to be true *universally*, even beyond the limits of our sense experience; Kantist philosophers hold them to be both *universally* and *necessarily* true or valid, *but only within the subjective sphere of the mental forms* or *categories* from which alone their necessary character is derived. That they are true in regard to the nature of extra-mental reality itself, and not merely in regard to reality as revealed to consciousness in and through those subjective forms of thought, the Kantist will either doubt or deny. The Scholastic philosopher, on the other hand, does not derive the necessary and universal validity of judgments *in materia necessaria* from the constitution of the mind exclusively, nor from any supposed subjective forms of thought in the mind, but from the nature or constitution of mind, *together with the nature or constitution of Being itself*.[3] For him the necessary truth of such judgments is based no less upon the constitution of the

[1] The terms "synthetic" and "*a priori*," as applied to the act of judgment, have, therefore, in Kant's philosophy, not quite the same sense as in Scholastic philosophy. The Scholastic "synthesis" is a union or comparison of two intellectual notions or concepts; the Kantian, a union of an innate form of thought with a datum of sense experience. The Kantian "*a priori*" means prior to all mental experience, actual and possible, a prerequisite condition for such experience; the Scholastic "*a priori*" means simply that sense experience does not form the *ground* or *motive* for our belief in the necessary and universal validity of necessary judgments, or for that validity itself, but does not exclude—nay, rather presupposes—the sense experience by which we obtain the data from which to abstract our intellectual concepts.

[2] *Cf.* KANT, *Kritik der reinen Vernunft*,, Einl. iv.

[3] *Cf.* what has been said above (15) about the *necessity* of the laws of thought.

intelligible *object* than upon that of the intelligent *subject*. If you ask a Kantian philosopher why we cannot help thinking that "whatever begins to be has a cause," he will answer: Because the mind is so constituted; because it is endowed with a certain form or category ("causality"), which *compels us to think* things in that way. If you ask a Scholastic philosopher, his answer will simply be: Because things *are* that way, and therefore compel us to think them in that way.

But how can the same being or reality which reveals itself, through our external and internal senses, to our understanding, as existing subject to all the conditions of time and space and change, and as the basis of judgments *in materia contingenti*, be also the basis of judgments that have the opposite characteristics of universal necessity, immutability, and eternity? This ultimate question belongs also to metaphysics. The answer given by Scholastics will explain why they call the latter class of judgments *metaphysical* and the former class *physical*. Physics studies being as revealed to the senses, *i.e.* as subject to *change*, and as existing in the concrete conditions of time and space: *physical* judgments, therefore, are *in materia contingenti*. But the human mind has the power of abstracting from those changing conditions of concrete existence in time and space, and of considering the essences and attributes of things in a purely ideal or possible condition—non-temporal, non-spatial, non-changeable, and absolutely static. It does so in metaphysics;[1] and, manifestly, Being, when considered in that static condition, can and does give rise to those *necessary* judgments, which Scholastics accordingly call *metaphysical*.[2]

89. MODALITY IN CATEGORICAL JUDGMENTS.—A *modal* (categorical) proposition (*propositio "de modo"*) is one which states explicitly not merely that the predicate does or does not agree with the subject, but also *how* it does or does not agree with the latter; whether, namely, it is a *necessity* (or an *impossibility*) or only a *contingency* (or mere *possibility*) that *S* be *P*.

The "*pure*" or *non-modal* proposition (*propositio "de inesse"*), which merely asserts that the predicate does or does not agree with the subject, may be called, as distinguished from the modal, an *assertoric* proposition.[3]

[1] Judgments of *pure mathematics* have the same characteristics, for they, too, abstract from *change*.

[2] *Cf.* Introd., 3-6; and section 249, below.

[3] Some of the authors who distinguish between the pure or non-modal and the assertoric judgment, and who set down the latter as a form of the modal, ascribe to it the function of deliberately *re-asserting* the pure judgment, after the mind has searched for the real ground or cause (*causa essendi*) of the fact stated, without being able to discover such cause: "*Some men detect water with the divining-rod. That is very extraordinary; how do you account for it? I can't, but they detect it*" (JOSEPH, *Logic*, p. 171; *cf.* p. 172). The former of these judgments would be pure or non-modal, the latter an "assertoric modal". There does not seem to be any sufficient reason for such a distinction. Of course, reflection on the grounds we have for asserting something as a fact (*causa cognoscendi*) may lead us to doubt their sufficiency, and

The simple assertion, apart from the mode, was called by the Scholastics the *dictum:* the modal proposition was fully and properly expressed when the *mode* was predicated of the *dictum*, *e.g.* "*That the human soul is immortal* is *necessary—impossible—contingent—possible*" (*propositio modalis* "*de dicto*"). It could be also stated, though less explicitly, by using the *mode* adverbially in making our predication about the original subject or thing (*Res*), *e.g.* "The human soul is *necessarily—impossibly—contingently—possibly* mortal" (*propositio modalis* "*de re*").

The Scholastics also distinguish *material* from *formal* modality. Any adjectival or adverbial mode that qualified the *subject*, or *predicate*, gives rise to a "*materially modal* proposition," *e.g.* "He is *occasionally* angry". This modality is of no logical importance. *Formal* or *logical* modality is produced by a mode which qualifies, not the subject, nor the predicate, but the *copula* of the judgment.

By the *modality* of a judgment we therefore understand, with Aristotle and the Scholastics, the *necessary* or *contingent* character of the relation seen by the mind to exist between subject and predicate. Combined with the distinction based on *quality*, it gives rise to four forms of categoricals: *S must be P*, *S cannot be P*, *S may be P*, and *S need not be P;* but, taken in itself it is twofold: *apodeictic*, which asserts *necessary* agreement or disagreement, and *problematic*, which asserts *possible* agreement or disagreement, of *P with S*. The *apodeictic* form expresses *what must be* or *cannot be*—an affirmative or negative *necessity*—the *problematic*, what *may be* or *need not be*—an affirmative or negative *possibility* or *contingency*.[1]

Understood in this way, the modality of the judgment is *objective*, not *subjective:* whether the relation between subject and predicate be a necessary one or not, depends entirely on the intension of the subject and predicate themselves, the *material* elements of the judgment. In fact, the modal proposition makes explicit the distinction explained above (85-88) between the *essential* and the *accidental* judgment. The *apodeictic* proposition is the modal (and natural) expression of the judgment *in materia necessaria;* and the *problematic* proposition, of the judgment *in materia contingenti*. It would carry us too far into the sphere of metaphysics to discuss in any adequate way the nature and grounds of the distinction between the necessary or apodeictic judgment, the contingent or problematic judgment, and the simple or assertoric judgment.[2] But a few remarks will help to illustrate the matter.

to change our judgment from "S is P" to "S may perhaps be P". On this mode of expressing a doubt, see below, 90.

[1] *Contingency* in reference to modality means simply *non-necessity*. A *contingent* modal is simply a *possible* modal (*S may be P*, *S need not be P*). A simple or non-modal proposition which expresses some *contingent fact; e.g.* "The boys are playing," is sometimes called a *contingent* proposition, because it expresses something that is *de facto*, but might have been otherwise.

[2] *Cf.* the discussions on the problematic judgment in JOSEPH'S *Logic*, pp. 177 *sqq.* The author propounds the *subjective* view mentioned below (90).

The modal proposition interprets the meaning of the judgment in regard to the *intension* of its concepts. The *apodeictic* asserts that there is a necessary relation between the concepts compared: that the attributes connoted by P are necessarily involved in (or excluded from) any reality which embodies the attributes connoted by S. The *problematic* denies that there is any such necessary relation, by asserting that the attributes in question may be found united, or separated, in the reality. Neither form seems to convey as part of its meaning any information as to whether, or in what state, the attributes connoted by S and P *actually exist* in reality (130).

The assertoric proposition, on the other hand, is the natural expression of the judgment *about facts*, the judgment which states simply what *is* or *is not*, without concerning itself with what must be, or what may be, what need not be, or what cannot be. And all such judgments of experience bring into prominence before our minds the denotation or *extension* of their subjects (100).

Of course, the assertoric form of proposition is often used to express—though inadequately—what is really a modal judgment, and may, therefore, be interpreted modally.[1] But the judgment which simply asserts (or denies) a matter of *contingent fact*, is clearly different from the judgment which asserts or denies the existence of a *necessary law*. It is, of course, the aim of scientific progress to pass from the former to the latter in every department—from the study of facts to the discovery of laws.[2] But the immediate ground we have for forming the modal judgment is different from that for the assertoric. In the former case it is an analysis of the concepts compared; in the latter it is experience of actual fact, or inference therefrom.

Whether a judgment of fact, based upon experience, should be expressed as a *necessary* or as a *contingent* modal, will depend upon the nature of the fact in question. Though the facts of our experience lead us to the knowledge of one *Necessary* Fact—the Deity,—they are themselves all contingent. Judgments about such facts will, therefore, be contingent or problematic in their modality so long as they are grounded on experience, and not on the apprehension of some element, in the intension of either concept, which

[1] We can of course immediately infer *possibility* from *actuality*, and *non-actuality* from *impossibility*: *Ab actu ad posse valet consecutio, sed non vice versa; A non-posse ad non-actum valet consecutio, sed non vice versa.*

[2] *Cf.* JOSEPH, *Logic*, p. 170.

necessarily involves the other and thereby renders the predication *necessary*.[1] Though all predication presupposes some experience—that at least by which the concepts in question are acquired,—not all predication is *grounded* on experience. When the comparison of two concepts which represent abstract aspects of reality—irrespective of the existence of the concrete things from which these concepts were derived (88)—shows that they *necessarily* involve each other, the proposition which expresses that judgment, even although it be stated in the merely assertoric or non-modal form, must be interpreted as apodeictic in its modality.

The distinction between the *apodeictic* and the *problematic* modal may be simply expressed by saying that the force of the former is to *affirm* some connexion of concepts to be absolutely necessary, and of the latter to *deny* some connexion to be absolutely necessary. We confine the apodeictic proposition to the expression of an *absolutely* necessary connexion of concepts, *i.e.* to the modal expression of the judgment *in materia necessaria*.

Some logicians embrace under the apodeictic proposition *all* grades of necessity—all judgments which are conceived to be based on "the operation of law".[2] We do not think that the *moral* necessity involved in judgments expressive of *human* laws, nor the *physical* necessity involved in judgments expressive of the uniform activities of *physical phenomena*, should be classed with the absolutely inviolable necessity which is characteristic of abstract, "metaphysical" judgments (88). The *categorical* proposition which expresses a mere *moral* or *physical* necessity, should not, therefore, be interpreted *apodeictically*. All such "operations of law"—physical or human—can, however, be expressed *hypothetically* in such terms as to make the relation in thought between antecedent and consequent an absolutely necessary relation, thus giving rise to a truly *apodeictic* proposition.

90. THE SUBJECTIVE VIEW OF MODALITY.—Many modern logicians, after Kant, take a somewhat different view of modality from that just explained. They mean by the modality of a judgment not the *necessity* or *contingency* of the predication, but the *certitude* or *probability* of our assent to the judgment. For them, distinctions of modality are *subjective*, not *objective*—expressions of the grades of firmness in our belief, not of the kinds of connexion between the objects of our thought. Kant distinguished three degrees of assurance in our attitude towards judgments; and these degrees he held to be reflected in the three forms of proposition, *S must be P* (*apodeictic*), *S is P* (*assertoric*), and *S may be P* (*problematic*).

[1] The assertoric judgment, based on experience, contains, however, an implication of the real existence of the objects compared, while this implication is absent from the modal. The importance of this we shall see later on (130).

[2] KEYNES, *op. cit.*, pp. 88, 89.

This is unsatisfactory. There is, of course, a plain connexion between it and the objective interpretation of modality—in so far as our assent is due to intrinsic evidence springing from an analysis of the essence or comprehension of the objects of our concepts. The firmness of our subjective assent should be in proportion to the objective evidence for the judgment. Where the judgment is seen to be apodeictic in the *objective* sense, *i.e.* where there is seen to be an absolutely necessary connexion (of agreement or disagreement) between S and P, the evidence will be cogent and the assent will be most firm. But the modal form (*must; must not*) expresses rather *the necessity of the relation*, not the *certitude of the mental assent.* Absolute certitude of assent can also be had *on grounds of experience* for *assertoric* judgments, of the form S *is* P, irrespective of the necessity or contingency of the relation between the attributes S and P. The distinction between the forms, S *must* (or *must not*) *be* P, and S *is* (or *is not*) P, is useless as a means of measuring the degree of our mental assent. The estimation of the latter must be left to the *Logic of Probability*, where it will find more exact and adequate expression than Kant's threefold distinction can give it. Ordinary logic, in regarding the assertoric categorical proposition, S *is* P, or S *is not* P, as the assertion of an *objective truth* (80), assumes that it *can* be assented to with certitude, that its truth can be verified; but does not concern itself with the attitude of this or that individual's mind towards the judgment which is the mental expression of that truth.

Moreover, on *no* view of modality can the *assertoric* categorical judgment be classed as one of the members of the modal division. If its equivalent be sought in the *objective* modal scheme, it will be apodeictic or problematic according to the *necessary* or *contingent* character of the relation between the attributes of S and P; and on the *subjective* view of modality, it will, presumably, be apodeictic if we are certain of it and problematic if we are not.

But neither is the form of the *problematic*[1] modal proposition a satisfactory expression of mental *doubt.* The modal form, S *may* (or *need not*) *be* P, is ambiguous. It may be interpreted to mean, and indeed very often does mean, and express, simply and

[1] The terms "problematic" and "apodeictic" do, indeed, suggest doubt and certitude respectively, and have probably arisen in connexion with the present view. But, without adopting the latter, we may retain the names as synonymous with the older terms "necessary" (or "impossible") and "contingent" (or "possible").

solely mental doubt, or uncertainty, about the judgments *S is P* and *S is not P*; but if this be taken as the import of the form in question, it expresses *no logical judgment at all* about *S* and *P*, but merely the absence of a judgment about them. It reveals indeed the actual state of the thinker's mind, and may involve or suggest the existence of a judgment in his mind about the strength of the evidence[1]; but it *expresses no mental assertion of an objective truth* (80) about *S* in terms of *P*. Interpreted, therefore, as the mere expression of a subjective uncertainty, it is not in itself the expression of any distinct judgment—of which ordinary logic, at all events, can take cognizance. It must, therefore, be interpreted—if a *meaning* is to be given to it at all—as denying the objective truth of the corresponding apodeictic judgment, *S must* (or *cannot*) *be P*, *i.e.* as asserting that there is no absolutely necessary relation between *S* and *P*.[2] In addition to the information it gives us about its subject, *S*, we take it to be no part of its *meaning* to give us any information about the degree of assent demanded from us by the judgment *S is* (or *is not*) *P*.[3] It may demand *for itself* full assent. Logic, in so far as it *assumes* judgments to be true, and analyses what further implications are contained in them, tacitly assumes at the same time that every true judgment is held by the mind with certitude. Or rather, we may say, it abstracts from this question. If, therefore, the form "*S may* (or *need not*) *be P*" is to be regarded as a distinct proposition, expressing in itself an objective truth, it must be interpreted *objectively*, as referring to the *objective relation* between *S* and *P*, not *subjectively*, as referring to *our mental attitude* of doubt (or otherwise) towards that relation.[4]

JOYCE, *Logic*, 51-54, 58-61. KEYNES, *Logic*, pp. 49-56. WELTON, *Logic*, i., pp. 160, 161. JOSEPH, *Logic*, pp. 185-190. VENN, *Empirical Logic*, pp. 291 *sqq*. MELLONE, *Introd. Text-Book of Logic*, pp. 367 *sqq*.

[1] *Cf.* n. 4.

[2] *Cf.* SIGWART, *Logic* (*tr.* DENDY), i., pp. 178 *sqq*.

[3] *Cf.* n. 4.

[4] This latter is the judgment it *expresses* for those who take the subjective view of modality: "In every judgment I intend to assert truth, but not necessarily about the particular reality that my judgment refers to; the truth I assert may be that I am unable to discover the truth about this reality" (JOSEPH, *op. cit.*, p. 182). The proposition "S may (or need not) be P" would then mean "I am uncertain whether the judgment 'S is P' is true or not". "If I find the content of a suggested judgment involved in conditions about which I am uncertain, I assert it to be possible; such a judgment is called problematic and expressed in the form 'X may (*or* may not) be Y'" (*ibid.*).

CHAPTER III.

QUANTITY AND QUALITY OF CATEGORICAL JUDGMENTS AND PROPOSITIONS.

91. The Traditional Fourfold Scheme of Propositions: Distribution of Terms.—We have divided propositions into *Affirmative* and *Negative* in quality, and into *Universal* and *Particular* in quantity. This broad, traditional division will serve as starting-point for dealing with quality and quantity; and it will be found convenient to treat of both these properties of the judgment together. Both are sometimes said to constitute the *form* of the proposition—as distinct from its *material* elements, the subject and predicate. The quality affects the *copula*, being expressed by "*is* (are)," "*is* (are) *not*"; and the division of propositions into affirmative and negative is sometimes said to be the "formal" division, the division *ratione formae*. The various signs of quantity constitute a second formal element, affecting the *subject* of the proposition. The latter will be universal or particular according as the predication made in the judgment refers explicitly to the *whole* denotation, or only to an *indefinite portion* of the denotation, of the *subject*; or, to express the same distinction more briefly, according as the *subject* is *distributed* or *undistributed*. The predicate, too, of a proposition may, if its denotation is considered, be distributed or undistributed, as we shall see presently; but this does not affect the quantity of the proposition: the latter depends on the quantity of the subject alone, not of the predicate.

Combining quantity with quality, we have the four propositions of the traditional logical scheme: the *Universal Affirmative*, the *Particular Affirmative*, the *Universal Negative* and the *Particular Negative*. These are represented by the first two vowels of the words *affirmo* and *nego* respectively, *A*, *E*, *I*, and *O*; and the corresponding propositions may be thus expressed symbolically:—

Universal Affirmative (*A*) . . *S a P* . (Every S is P)
Particular Affirmative (*I*) . . *S i P* . (Some S's are P)
Universal Negative (*E*) . . . *S e P* . (No S is P)
Particular Negative (*O*) . . . *S o P* . (Some S's are not P)

Although it is the *connotation* of the *predicate* that is more usually thought of in the act of judgment (100), still the predicate has its denotation as well; and this denotation may possibly be thought of in the act of judgment. Hence we may inquire whether or when the predicate is *distributed*, *i.e.* taken in its whole denotation, in our judgments. We shall find that the answer to this question depends on the *quality* of the judgment: that *negative judgments distribute their predicates*, *while affirmative judgments do not.* A little reflection, aided by a few simple examples, will make this quite clear.

The *affirmative* proposition asserts that some subject (*S*) possesses a certain attribute or group of attributes (*P*), but it does not by any means assert that there are not, or cannot be, any other things which also possess that same attribute (or group).[1] There may be many other things besides the *S*'s, which possess *P*, and to which, therefore, the class name *P* may be applied, since they fall within its denotation. When we say that "All men are animals" or that "Some men live exclusively on vegetables," our propositions do not exhaust the class of "animals" or the class of "beings that live exclusively on vegetables"; they do not state that there are not, or may not be, other animals besides "all men," or other beings that live on vegetables besides the "some men" in question; *i.e.* the propositions *do not refer to the whole denotation* of their predicates: they *do not distribute* their predicates.[2]

Negative propositions, on the contrary, *do distribute* their predicates. The force of the negative proposition is to exclude a certain attribute (or group of attributes) from a certain subject (or group of subjects). But, evidently, it will not succeed in doing this unless it totally separates the *whole class* of things (*P*'s) possessing that attribute (or group), *P*, from the subject, *S*; *i.e.* unless

[1] It may, of course, happen to distribute its predicate—*e.g.* if it be a definition or give a *proprium* (in the strict sense) of the subject, as "All men are (all) rational animals"—but this is not by reason of the *form* of the proposition: it arises from the *matter* of objects dealt with. Such *reciprocal* universal propositions are called sometimes "*U*" propositions (*cf.* 105).

[2] If the extension of the predicate is not explicitly before the mind at all, the predicate is also said to be "undistributed". *Cf.* JOSEPH, *op. cit.*, pp. 195, 196.

it totally excludes the *S*'s referred to in the proposition, from *any place in the denotation of P:* to do which, it must, of course, refer to *the whole denotation* of *P*, or, in other words, *distribute P*. For example, the proposition "No men are angels" means that no men possess that nature which is found in any and every being in the class of "angels," and in virtue of which these beings are each and all called angels; in other words, it means that "no men are *any* angels" (and not merely that "no men are *some* angels, although they *might* perhaps be, or be identical with, *other* angels"); that is to say, the *whole denotation* of the term "angels" is explicitly referred to, and explicitly excluded from, the subject "all men": the predicate is *distributed.* Similarly, if we say "These men are not Americans," we mean that they are not *any* Americans, that the *whole class*, "Americans," does not contain anywhere within its denotation the men referred to: the predicate is *distributed.*

Summing up the results reached in this way, we see that

Universals [*A and E*] *distribute their* SUBJECTS.
Particulars [*I and O*] *do not.*
Negatives [*E and O*] *distribute their* PREDICATES.
Affirmatives [*A and I*] *do not.*

Or, in other words,

E distributes *both subject and predicate.*
A „ *subject only.*
O „ *predicate only.*
I „ *neither subject nor predicate.*

92. UNIVERSAL PROPOSITIONS.—A *Universal Proposition* is *one in which the predication is made about the whole denotation of the subject.* The subject may be a class, expressed by a general term; or it may be a single individual, expressed by a singular term. Hence arise two sub-classes of the universal proposition, *viz.* the *General* and the *Singular.*

(*a*) *General Propositions.* A *General Proposition* is *one in which the predication is made distributively about each and every member of a class.* The usual quantitative signs of the general proposition are *All*, *Each*, *Every*, *Any*: *e.g.* "*All* men are mortal," "*Each* member was shown to a seat," "*Every* man has his own peculiarities," "*Any* coin of the realm will suffice," "*Any* person may have this picture," "*Any* house is a shelter in a storm".

All, as a sign of the general proposition, must be used *distributively* (*omnes*): not *collectively* (*cuncti*) in the sense of "all *together*". When it is used in the latter sense, it is not regarded as an ordinary sign of quantity, attached *extrinsically* to the subject to distribute it, but as incorporated with the latter to constitute it, and as equivalent to "*a* (or *the*, or *some*, or *any*) COLLECTION of . . . ," being merely used as a substitute for a collective name. Whether, then, such a collective proposition will be general, singular, or particular, will depend altogther on whether the predication is made about the whole class of such "*alls*" or "collections," or about some definite individual collection, or about an indefinite number of such collections. For example, the collective proposition "All the angles of a triangle are equal to two right angles" is a *general* proposition, because the predication is made of any and every such collection of angles—"All the angles of *a* (*i.e. any* or *every*) triangle . . . ". The collective proposition "All the books in the British Museum would fit in Westminster Abbey" is *singular*, because the predication is made about the whole denotation of the singular subject—the said denotation being *unity*. So, too, is the collective proposition, "The Romans conquered Gaul," because the predication is made about that one individual people or nation collectively. But the collective proposition "All the books of some public libraries would not weigh a ton" is a *particular* proposition, because the predication is made *indefinitely*, about "some" collections of books. A collective proposition, therefore, may be either general, singular, or particular, whether its subject be a collective term—like army, navy, etc.—or a collection formed by a distributive class name qualified by "all" in the sense of "all together".

Owing to the ambiguity arising from this double use of "all" it would be preferable to use "every" as the logical sign of the general proposition. However, long usage is not easily discontinued. "All" is to be interpreted *distributively* wherever the collective use is not obvious from the context, or otherwise expressly indicated. The symbolic form "All S's are P" is free from the ambiguity; not so the form "All S is P".

Neither is the sign "*any*" free from ambiguity. It is usually equivalent to "every," "all" (distributively); for we cannot predicate *P* about any unit taken at random from the denotation of *S*, unless we can predicate it of all the *S*'s, of each and every *S*. But "any" sometimes introduces an indefinite singular proposition which should be classified as a *particular* [see (*b*)]. For example, "Anyone who wins this race will receive a silver cup" is *particular*, because it refers to *some* as yet undetermined individual "winner of the

race". "Any" also seems to have the force of "some"—and so to introduce a *particular*, not a *universal*,—"(*a*) in the principal clause of an interrogative sentence, *e.g.* 'Are any subscribers dissatisfied because some non-subscribers were admitted?' (*b*) in the subordinate clause of a negative sentence, *e.g.* 'Some people do not think that any men are perfect,' (*c*) in the antecedent of a pure hypothetical, *e.g.* 'If any men are perfect some men are mistaken'." [1]

Although all general propositions are expressed by the aid of the same signs of quantity, yet they admit of division into two classes of vastly different degrees of importance.

(i) There are, firstly, those general propositions which make predications about (all the members of) *concrete*, *definite*, *limited* classes or collections of things; classes that have been formed by enumeration, or by actual experience, or by some tentative enlargement or generalization from actual experience. Their characteristic feature is that, the knowledge of them having been reached by actual experience merely, they claim to have no further force than that of asserting *what actually is*: they are simply assertoric: from the point of view of *modality* they would be *problematic*, that is, they would merely deny, or at least fall short of asserting, the existence of any *necessary* relation—affirmative or negative—between subject and predicate (89). A few examples will suffice to illustrate this: "All the books on this shelf are histories"; "All the Apostles were present at the Last Supper"; "All the days of the week are named after pagan deities"; "All the planets move around the sun in elliptical orbits"; "All ruminants are cloven-footed"; "No scarlet flowers are sweet-scented"; "All lions are tawny". Even where the "all" of such propositions carries us somewhat beyond the range of actual experience, and expresses what, in inductive logic, is called an empirical generalization (247), as in the latter few of the examples just given—even then the proposition contains no certain expression of anything like a *necessity*, a *law*, a "*must*," but simply states what *is*, *de facto*. They are no less concrete than such singular judgments of fact as that "The Romans conquered Gaul," or that "Dublin is the capital of Ireland". They are universal, inasmuch as the predication is true of *all* the members of the class or collection; but it holds good only within the limits of time and space to which the identity of the class or collection is obviously subject.

(ii) Quite distinct from all such *concrete*, *collective*, *enumerative*,

[1] KEYNES, *Formal Logic*, 3rd edit., p. 68, note.

or *empirical* universals, as we may call them,[1] and of much greater importance for the scientific worth of human knowledge, is what we may call the *necessary*, *abstract*, or *generic*, universal judgment, wherein the connexion between subject and predicate is asserted to be not merely universally verified *in fact*, but to be an absolutely necessary, inviolable connexion, altogether beyond exception and entirely independent of any change in conditions of time or space. Examples of such judgments are: "Each of the interior angles of any triangle is less than two right angles"; "No pair of parallel lines enclose a space"; "All men desire happiness"; "Every virtue is to be esteemed"; "Every event has a cause"; "No pair of contradictory judgments are either both true together or both false together". Now, the characteristic of the class of universal judgments illustrated by these examples is this, that their universality is known not by an actual enumeration of all the instances, actual or possible, not by any concrete experience of, or any appeal to, the whole denotation of the class of things about which the predication is made, but by an appeal to the *connotation* of both subject and predicate, by such an analysis of the ideas compared as will show us that the predicate in each case is *necessarily* connected with (or excluded from) the nature of the subject, and may therefore be predicated (affirmatively or negatively) of every conceivable instance—whether actual or possible—of this subject. In their *modality*, such judgments are *apodeictic* (89). The sign of universality ("all," "every," etc.) is here independent of all conditions of time and space; and the judgments are free from all possibility of exception or change.

Although the distinction between these two classes of universal judgments[2] is of sufficiently far-reaching importance for any general philosophical theory of human knowledge, yet, for our present formal treatment of propositions, it need not be—and indeed could not conveniently be—maintained. It is what we

[1] To this class belongs the *singular* judgment [*cf.* (*b*) below]. The distinction between the *abstract* universal on the one hand, and the *concrete* universal, the singular, and the particular, on the other hand, is indeed not properly a *quantitative* distinction; while the distinction between the three latter is often purely quantitative. *Cf.* JOSEPH, *Logic*, pp. 155, 157. For the distinction between the *definite* and the *indefinite* singular, see below (*b*).

[2] The student can scarcely fail to notice that this distinction is already familiar to him, as that between judgments *in materia necessaria* and judgments *in materia contingenti* (85-7).

may call the *modal* or *connotative* expression[1] of the latter class of judgments that gives us an insight into their nature and the grounds of our assent to them; whereas these latter points are rather concealed by the *denotative* or *quantitative* form of expression, which would rather lead us to believe (erroneously) that such judgments are reached, like concrete singular and general judgments, through experience and enumeration of instances. If a formal distinction in expression be sought for the two classes, the concrete universals might, perhaps, be appropriately expressed by "All *S*'s are *P*," the abstract by "*S is P*," or "*S as such is P*,"—with the corresponding negatives.

One point in the expression of the universal negative calls for special notice. The proper form for expressing judgments of this class is "No *S*'s are *P*" or "No *S* is *P*". The form "All (or Every) *S* is not *P*" (or "All *S*'s are not *P*") is ambiguous. Although it could, absolutely speaking, be interpreted as expressing the universal negative,—the contrary of "All (or Every) *S* is *P*,"—nevertheless the "*not*" is usually interpreted as qualifying the "*all*" instead of the copula, "*is*" or "*are*"; so that the form is the same as "Not all (or 'Not every') *S* is *P*," which really means "Some *S* (or *S*'s) is (or are) not *P*". That is to say, such propositions are really *particular* negative (or *O*) propositions; *e.g.* "All is not gold that glitters" means "*Some* glittering things are not gold"; "All men are not saints" means "*Some* men are not saints". The universal negative cannot, therefore, be safely expressed in this form. Only when "*All*" is used *in a singular collective*, can the form "All *S* is not *P*" be appropriately used to deny the form "All *S* is *P*": "All the books in the British Museum would not fit in a small room" is the denial of "All the books in the British Museum would fit in a small room". But in the case of the *general* collective, the use of the form remains ambiguous; "All the interior angles of a triangle are not equal to two right angles" might be (erroneously) interpreted as a particular negative, meaning "Some triangles have not all their interior angles equal to two right angles".

(*b*) *Singular Propositions* are usually regarded in logic as a sub-class of universal propositions (*A* or *E*), on the ground that the subject is a single individual and that the predication is made *definitely* about that one individual, about the whole denotation of the subject—which is *unity*. Some authors, however, classify them as particulars, on the ground that the predication is made about a portion—namely, the least possible portion—of the denotation of the subject class.[2] As a matter of fact, some singular propositions must be classed as universal, others as particular. It is

[1] "Virtue ought to be esteemed"; "An event must have a cause"; "Man necessarily desires happiness"; etc.

[2] *Cf.* CLARKE, *Logic*, p. 274.

possible to distinguish between two clearly different kinds of singular proposition, the *indefinite* and the *definite*.

(i) The *indefinite* singular is one in which the reference is to some *undetermined* member of a class, to the "*individuum vagum*," as it is called. The quantitative sign of this is the indefinite article "*a*" meaning "some *one*" (not, however, when "*a*" is equivalent to *any* or *every*, in which case it introduces a general proposition), or better still, "a certain," "*quoddam*," "*aliquod*"; *e.g.* "A certain man had two sons," "A man was killed at the meeting". Propositions, of this kind are properly classified as *particular*, because the reference in the predication is to an *indefinite portion* of the denotation of the subject-class (*man* in the examples given)—*indefinite* inasmuch as, although limited to one individual, it leaves that individual undetermined, unidentified.

(ii) The *definite* singular proposition is one in which the subject is explicitly determined or pointed out, so to speak. It may be a proper name, as in the proposition "Maynooth is a town in the county of Kildare". Or it may be a significant individual term (28), *i.e.* a general term limited to one definite individual by some qualifying word,[1] *e.g.* "*This* man is old"; "*That* man is young";[2] "*The* chimney is on fire" (where circumstances make the reference unmistakable); "*The first* Pope came from Galilee"; "*The tenth* General Persecution of the early Christians took place under the Emperor Diocletian"; "*The last* Queen of England reigned for over half a century". Propositions of this class are most conveniently treated as universals, because they give us *definite* information, whereas the characteristic of the particular proposition is that the reference of *its* predication is left *indefinite*.

But—it may be objected—in the examples given, is not the predication made about a *portion only*, of the denotation of the subject, not about *the whole* denotation? This depends on whether we take the "subject" of these propositions to be *the class name alone*—regarding "this," "that," "the," "the first," etc., as extrinsic signs of quantity, *formally* affecting it, like "every," "none," "some," etc.—or rather to be the *whole combination of class name and individualizing epithet*—regarding the latter as *constituting* the singular subject, just as the collective "all" constitutes the individual collection [see (3) above]. It is better to interpret the subject in the latter way, and to regard such propositions as universal, inasmuch as the predication is then both *definite* and *applied to the whole denotation* of the subject—which, in these cases, is *unity*.

[1] *Cf.* VENN, *Empirical Logic*, pp. 163-169.

[2] "*These*" and "*those*" are signs of the *concrete general* proposition.

93. PARTICULAR (OR INDEFINITE) PROPOSITIONS. — The *Particular Proposition is one in which the predication is made about an indefinite portion of the denotation of the subject.* This indefiniteness of reference is its essential characteristic. Its recognized logical sign is "some," [1] and its usual symbolic expression is "Some *S* [or *S*'s] *is* [*are, not*] P". We must, however, make clear the meaning traditionally attached in logic to the word "*some*". In ordinary language it means *at least one;* in this direction it is exclusive of "*none*". But in the opposite direction it does not quite so clearly exclude "*all*". As a rule, no doubt, it does exclude "all,"—meaning, therefore, "some *but not all*". We may even say that as a rule it excludes *most, more than half*, the *majority*, and means *a comparatively small number*, the *minority, less than half:* increasing portions above a comparatively small number would be usually expressed, not by "*some*," but by "*a good many*," "the *majority*," "*most*," "*nearly all*". Nevertheless, the fact remains that sometimes the word "*some*" in ordinary speech is compatible with "*all*," meaning "some, *perhaps all*".[2] Now logicians have found it convenient to fix this latter meaning on "*some*"; and in logic we must always, therefore, interpret it—unless the context clearly forbids us—as meaning "some, *possibly all*". Used in this sense, it brings out the indefiniteness of predication which is characteristic of the particular proposition. The proposition "Some *S*'s are [not] *P*" merely assures us that the reference of the predication is to the denotation of *S:* whether to *one single S*, or to *all S*'s, or to some number intermediate between *one* and *all*, it does not inform us.

The particular proposition, by asserting or denying something about an indefinite portion of the denotation of a subject, conveys of its very nature *imperfect, incomplete* knowledge: and to express such knowledge is the particular function of the particular proposition. *In itself*, the particular proposition is of no great scientific importance, but *as marking a stage* in knowledge it must not be neglected. It expresses judgments based on observation and "referring to individuals not enumerated, or to

[1] Less frequently, "certain": "Certain men were arrested on suspicion".

[2] The thought, whether or not it includes "all," is often entirely absent from the mind. For example, when I say "Some friends of yours were in town yesterday," my statement does not imply any knowledge on my part as to whether the "some" referred to are or are not "all".

an universal not fully determined";[1] judgments which, therefore, approximate either to the *collective* or to the *abstract* universal.

Examples of each would be "Some women have ruled kingdoms," and "Some pigments fade". The former is a statement "about unnamed individuals";[2] the *extension* of the subject is thought of; the judgment is *assertoric*; and it has been described as an *historical* judgment. The latter asserts the compatibility or separability of attributes; the *intension* of the subject is thought of; the judgment is really a contingent or problematic *modal;* and it has been described as a *scientific* judgment.

When the particular proposition results from observation of instances of a class, it serves to *suggest* the corresponding universal as a proposition that may possibly be true, *i.e.* as a scientific *hypothesis*;[3] and it thus marks a stage in our progress towards some *universal* or *scientific* truth:[4] either towards the proposition "*S as such* is *P*"; or towards the proposition that "All the *S's* of a certain kind, with a certain limitation, are *P*"; that *e.g.* "All S's that are *M* are P," or "*SM as such* is *P*"; and if this latter sub-class of *S*'s be scientifically important a special name will soon be found for it. It is *universal, necessary* truths—the expression of laws, metaphysical, physical, and moral—that are of importance as embodying scientific knowledge.

Then, as we shall see later, the particular proposition is a more appropriate form than the universal for asserting the *existence* of certain things. "If, for example, we say that 'some engines can drag a train at a mile a minute for a long distance,' our object is primarily to affirm that there *are* such engines; and this would not be so clearly expressed in the universal proposition of which the particular is said to be the incomplete and imperfect expression".[5] It is, perhaps, this *implication of the existence of S*'s in the universe of discourse, that mainly marks the distinction between the *denotative* or *assertoric* particular "Some *S*'s are (not) *P*'s," and the corresponding modal form, the *problematic* modal, "*S* may be (need not be) *P*" (130). Anyhow, the *incompleteness* of the knowledge expressed by "Some *S*'s are *P*" does not lie in doubt as to whether the enumeration of *S*'s is complete, but rather in *uncertainty about the nature of S*—about its *connotation* rather than its *denotation*,—uncertainty as to whether it *necessarily* involves *P:* which uncertainty is more clearly expressed by the modal "*S* may be *P*" (90).

Finally, perhaps the most important function of the particular proposition is to *contradict* the *universal of opposite Quality*, thus furnishing us with an exceedingly useful check upon false or hasty generalizations.

94. PLURATIVE AND NUMERICAL PROPOSITIONS. MULTIPLE QUANTIFICATION: COMPLEX PROPOSITIONS.—A *plurative* proposition is one to whose subject are prefixed the quantitative signs

[1] JOSEPH, *op. cit.*, p. 158. [2] *ibid.*, p. 218. [3] *Cf.* book iv., chap. v., *infra*.
[4] *Cf.* VENN, *op. cit.*, p. 168. [5] KEYNES, *Formal Logic*, p. 102.

"*most*," or "*few*". Each of these signs is open to two interpretations.

"*Most.*" Strictly speaking, "most" should mean a majority, at least one more than half, but not all. However, it *may* be interpreted, like "some," as consistent with "all," and *is* so interpreted in logic. For example, "Most of the people present wore badges," need not necessarily be held to imply that some did not wear badges. "Most," therefore, in logic, will be taken to mean "more than half, possibly all". It therefore introduces a *particular* proposition, and may in most logical processes be replaced by "some". Not always, however: we shall see later that from two particular propositions, "*Some M's are P; Some S's are M*," no conclusion can be drawn; while from the two plurative propositions, "*Most M's are P; Most S's are M*," we may draw the conclusion that "*Some S's are P*".

"*Few.*"[1] "Few *S*'s are *P*" is really a negative proposition: its force is obviously negative: what it means, and equivalently asserts, is that "Most S's are not P". And this latter proposition, according to the usual logical interpretation of "most," is equivalent to "More than half of the S's are not P, and possibly none of them are P". It is, therefore, a *particular* negative (or *O*) proposition, represented—though inadequately—in the traditional fourfold scheme by the form "Some *S*'s are not *P*". And this is in keeping with the meaning often attached in ordinary language to the form "Few *S*'s are *P*": the meaning "Less than half of the *S's if any, and possibly none of them at all*, are *P*".

Of course, the form "Few *S*'s are *P*" can be interpreted strictly (after the analogy of "Most *S*'s are *P*") to mean that "The majority of the *S*'s are not *P*, but some are *P*—at least some *one*". Interpreted in this stricter way, each of these plurative propositions ("Most *S*'s are *P*," and "Few *S*'s are *P*") would be resolvable into two simple propositions, one affirmative and one negative. Propositions resolvable in this way are called *exponible* propositions (95).

"*A few.*" Care should be taken to distinguish between the force of "few," and "a few". The latter has not the negative signification of the former. "A few" is for the most part logically equivalent to "some," and introduces a *particular* proposition. Sometimes it has a *collective* force, meaning a small number or collection, as in the proposition, "A few Greeks

[1] "Hardly any," "Scarcely any," are synonymous with "Few".

defended the Pass at Thermopylæ". This belongs to the class of *indefinite singulars*, which we have seen to be a sub-class of *particulars* [92 (*b*)].

Numerically definite propositions, such as "Two-thirds of the S's are *P*" are ambiguous; for they may be interpreted in the *stricter* sense to signify that, for example, "Two-thirds *exactly* of the S's are P, and the remaining third are not *P*," in which case they are *exponibles;* or to mean that "Two-thirds at least, and *perhaps more*, of the S's are P". In ordinary language, sometimes the former is meant, sometimes the latter. The latter form gives the judgment the lesser amount of meaning. Hence, in so far as logic takes account of such propositions, it must, in obedience to the *Law of Parsimony*, which we have applied more than once already, interpret them as containing the lesser. In the present case, therefore, we interpret as "Some *S*'s are *P*". In virtue of the law just referred to, when a form of statement, as it stands in its context, is capable of two or more interpretations, which ascribe to it various amounts of meaning, we are not at liberty to select, in logic, any other interpretation than that which ascribes to the statement the minimum amount of meaning: the reason being, of course, that we have no warrant to ascribe to it in the particular case any further meaning than the minimum it can bear, and we might therefore err in doing so.

Multiple Quantification. The predicate of a proposition may be applied to the whole, or to an indefinite portion, of the denotation of the subject, *not simply and absolutely, but with certain limitations as to time, space, or other such conditions.* For example, "All schoolboys feel unhappy sometimes"; "In some schools all the pupils are diligent"; "In all schools all the pupils are at times diligent". Such modifications of the primary quantity of a proposition are instances of *multiple quantification.*

Complex Propositions. Are plurative and numerically definite propositions, and propositions with multiple quantification, to be regarded as *simple*, or *complex*, or *compound* propositions (84)? Can they, on account of their *import*, or of their *implications*, be strictly interpreted (as *exponibles*), and resolved, at least with the aid of immediate inference (82), into two or more simple categorical propositions? The ambiguity in classifying these and certain other similar propositions as *simple*, *complex*, or *compound*, arises from diversity of usage in regard to these latter names.

The logical term which stands as subject or as predicate of a categorical proposition, may be either a single-worded or a many-

worded term (22). The presence of the latter does not take away the *simple* character of a simple proposition. Thus,

> "The castled crags of Drachenfels
> Frowns o'er the wide and winding Rhine,
> Whose breast of waters broadly swells
> Between the banks which bear the vine;"

—is a simple categorical proposition [78 (*f*)]. The subject or predicate of such a proposition may consist of a *principal* term, qualified in various ways by a number of *incident* terms, or even of incident phrases or clauses introduced by a relative or its equivalent. Those clauses invariably belong to one or other of two classes. They are either *explicative* or *limitative* (*determinative*): explicative if they refer to the whole denotation of the principal term, explaining it or giving some further information about it; limitative (or determinative) if they restrict the denotation of the principal term. In the former case the incident clause may contain a statement which may be co-ordinated with the main proposition by substituting "and" with the principal subject (or the personal pronoun), for the relative. For example, "The speakers, who were all very eloquent, discussed the question fully" may be stated: "The speakers . . . discussed . . ., and they were all very eloquent". No such substitution can take place when the subordinate clause is *limitative*: the proposition, "All the inhabitants who take part in the riots will be punished," cannot be dealt with like the previous example.

Some logicians would describe the many-worded terms which enter into the above propositions as *complex* terms.[1] Others regard them as *simple* terms, though many-worded: defining a *complex term* as the conjunctive or alternative combination of two distinct simple terms, and a *complex proposition* as one having a complex term either for its subject or for its predicate.[2]

Thus "tall and courageous" would be a complex term (conjunctive); so also "red or white" would be a complex term (alternative). There seems to be this ground, at least, for the latter usage, that the complex term would express *two distinct concepts*, held as distinct in the mind; while the simple term, even though many-worded, would express *only one concept*, composed, undoubtedly, of a number of notes or attributes, but yet conceived as one mental whole.

95. EXPONIBLE PROPOSITIONS.—There is a similar uncertainty of usage regarding the expression: "*compound* proposition" (84).

[1] *Cf.* WELTON, *Logic*, i., pp. 176, 177.
[2] KEYNES, *Formal Logic*, pp. 468, 478.

The mere grammatical combination of two or more categorical propositions in one and the same sentence does not give rise to a compound proposition, in the sense in which we have used this term (*ibid.*): to have a *strictly* "*compound*" proposition, the combination of the simple ones must give rise to *a new judgment*, distinct from each and all of the simple components. However, the grammatical statement which merely combines two or more simple categorical propositions, without thereby expressing any further judgment as arising from their combination, has been usually called a "*compound categorical* proposition"; particularly when, as is usually the case, the grammatical combination of them leads to a more condensed form of expression than they could receive if expressed separately.

Bearing in mind this wider acceptation of the term "compound proposition," we are in a position to deal with a form of proposition of which we have already had a few examples, the *Exponible Proposition.* This may be defined as a *categorical proposition which, though apparently simple, is really capable of being resolved into two or more simple propositions.* Hence the name "exponible". As we have seen already, *plurative* and *numerically definite* propositions are, when *strictly* interpreted, exponibles. The two most important classes, however, of exponibles are (*a*) the *exclusive* proposition, and (*b*) the *exceptive* proposition; to which we may also add (*c*) *inceptive* and *desitive* propositions.

(*a*) The *Exclusive Proposition* is introduced by "alone," "only," "none but," "none except," "none (no) . . . who are (is) not". For example, "The virtuous alone are happy," "Only graduates are eligible (or, Graduates are the only eligible people)," "None but the brave deserve the fair". Logicians are not agreed as to whether such propositions are "compound" or "simple".[1] The question turns on the distinction between *import* and *implications* (82). There can be no doubt about what is at all events the principal meaning of such propositions. What is primarily meant (1) is: "No non-virtuous are happy," "No non-graduates are eligible," etc.: in general "No *non-S* is *P*". They are, therefore, universal negatives. The further propositions, (2) "Some *S*'s (if there be any) are *P* ["Some graduates (if there be any) are eligible"], and (3) "All P's (if there be any) are S" ["All eligible people (if there be any such) are graduates"], will

[1] *Cf.* WELTON, *Logic*, i., p. 179. KEYNES, *Formal Logic*, pp. 104 n., 205. MELLONE, *Introd. Text-book of Logic*, p. 61.

be seen (118, 120) to be *implied* in the primary form, No *non-S* is *P*, being "immediate inferences"—(2) the *inverse*, and (3) the *obverted converse*—from this primary form.

(*b*) The *Exceptive Proposition* is one which withholds the predicate from some portion of the denotation of the subject, by such words as "except," "unless," "but". For example, "All members, except those over seventy years of age, are bound to be present," "All the passengers, but two, escaped uninjured," "No one is admitted, unless on business and by appointment".

These examples will show that the exceptive and exclusive propositions are merely two different ways of expressing the same meaning, that the exceptive may always be changed into the exclusive *by making the excepted portion the subject of the new proposition, and changing the quality:* the examples just given may be expressed, "Only members over seventy years of age are not bound to be present," "Two passengers alone did not escape uninjured," "Only people who come on business and by appointment are admitted".

(*c*) Yet another form of exponible proposition is the *Inceptive*, or the *Desitive* proposition, *i.e.* the proposition which asserts something as *beginning* or *ending*. Such a proposition may be resolved into two, the one showing the state of affairs before, the other after, the change. For example, "Dirigible airships came into use during the first decade of the twentieth century". This is equivalent to two propositions, one stating that such airships were not in use previously, the other that they were in use subsequently, to the time referred to.

96. INDESIGNATE PROPOSITIONS.—If the subject of a proposition is affected by any of the signs of quantity discussed in the preceding paragraphs, there will be comparatively little difficulty in determining whether the proposition is universal or particular. But not all propositions in ordinary discourse have signs of quantity attached to their subjects. A proposition which contains no such sign to indicate its quantity is called an *Indesignate Proposition.* It might also be appropriately described as *Indefinite*—inasmuch as its quantity is left undefined;—were it not that the term "*indefinite*" is often used as synonymous with "*particular*". Hence, we had better describe simply as *indesignate*, propositions which have no sign of quantity expressed. And in regard to these, we have to inquire how we are to determine their quantity—whether they are universal or particular.

For, the *judgment* in the mind is conceived either as universal, or as particular, and its quantity is tacitly attached to the proposition, although not expressed in the formulation of the latter.

Whenever we meet such a proposition we must decide its quantity by examining its meaning in the context in which we find it. We have seen already that the *abstract* or *generic* universal judgment finds its more natural expression in the non-quantified or *indesignate* form "*S* is *P*," or "*S as such* is *P*": "Evil-doing is deserving of punishment," "Man is rational," "The lark is a species of singing-bird," "The triangle has the sum of its interior angles equal to two right angles." Furthermore, we have met with a class of propositions in which the connexion between the predicate and the subject is a necessary one—propositions *in materia necessaria* (85). Such propositions are the expression of *necessary* judgments: and of these we can see at once that the predicate must be affirmed (or denied) about *every single member* in the denotation of the subject. Hence our first rule:—

(*a*) If an indesignate proposition be *in materia necessaria, i.e.* if the predicate be the *genus*, *species*, *differentia*, or a *proprium*, of the subject (or, in a *negative*, anything incompatible with any of these), the proposition is to be interpreted as *universal*.

Propositions in which the predicate gives an *accidens* of the subject are said to be *in materia contingenti*. If the predicate gives a *separable accident*, the proposition, when indesignate, must evidently be interpreted as particular; for it gives us no guarantee that the predication is made about the whole denotation of the subject. Even when the predicate gives an *inseparable accident*, we have no guarantee, no ground in the nature of subject or predicate, to regard the connexion as a *strictly* invariable, and therefore universal, connexion.[1] Hence, although such propositions would be regarded as in the ordinary, looser sense, universals, and are in fact called universals—that is, of a sort: *physical* and *moral* universals,—nevertheless they are not *absolutely* universal, and must be logically classified as particular. Hence our second rule:—

[1] Of course, if the context limits the subject to an actual, concrete class, about every member of which we know that the predication is *de facto* true, we have then the concrete or collective universal (92). If, for example, the statements "Ruminants are cloven-footed" or "Crows are black" be understood to refer to these classes only in so far as the latter are known, and known to have no exceptions, the propositions are concrete universals.

(*b*) If an indesignate proposition be *in materia contingenti*, *i.e.* if the predicate be merely an *accidens* of the subject, the proposition must be interpreted as *particular*.

The propositions "Parents love their children," "Virtuous people are happy," "Dogs are faithful creatures," "Oranges do not grow in Ireland," "Cretans are liars," "Women are talkative," "Seaside resorts are agreeable," may be taken as examples of indesignates which, though "morally" universal, must be regarded in logic as particular propositions.

97. AFFIRMATION AND NEGATION.—Propositions have been divided on the basis of *Quality* into *Affirmative* and *Negative*. This distinction is fundamental; although some philosophers have attempted to reduce negation to affirmation. It is also an exhaustive division of predication; although Kant endeavoured to find place for a third type of predication, which he claimed to be distinct from both the affirmative and the negative, namely, *the affirmative predication of a negative term* or notion, giving rise to what he called the *Limitative* or *Infinite* judgment, the expression of which has been called a *Propositio Infinita*, "*S* is *not-P*" (77). But this is not really a distinct type of predication. As far as *form* is concerned, the propositions "*S* is not *P*" and "*S* is *not-P*" are really equivalent; while, in its *meaning*, the proposition "S is *not-P*" is really a negative—on the assumption that *P* is a positive term. We know that "*S* IS *not-P*"—*i.e.* that it is included in the denotation, or includes the connotation, of "*not-P*"—only *by knowing already* that "*S* IS NOT *P*," *i.e.* that *S* is excluded from the denotation, or excludes the (whole collective) connotation of *P*. So that whenever we thus affirmatively predicate a negative term, we do so only by having first *negatively* predicated the corresponding positive term. The form of negation belongs naturally to the *copula* and not to the predicate—even though we artificially transfer it to the latter (59). Thus we see that we cannot, by means of negative predicates, get rid of negative predication, or reduce it to affirmative.

It may conveniently be noted here, that in the *affirmative* proposition, each and every single constituent element of the connotation of the predicate is affirmed of the subject. The truth of the proposition "Man is an animal," demands that he be a substance, corporeal, organic, living: that *each* of these *distributively*, *and all collectively*, be affirmable of him. We have, so to speak, an implicit *intensive* distribution of the predicate. Now take

the *negative* proposition, *e.g.* "No square is a triangle," "No bird is a quadruped," "Some dogs are not collies". A moment's consideration of these examples will show us that what the negative proposition excludes from its subject is the connotation of the predicate *taken as a whole, collectively;* but that it does not at all exclude from its subject each and every constituent element or portion of the connotation of the predicate. On the contrary, some portion of the connotation of the predicate agrees with, or is found in, the subject, in the examples given: in all of them the subject has something—more or less—in common with the predicate. Now, it may be asked, Is this the case in all negative predication? In a certain sense it is, and must be, the case. In order to have an intelligible comparison of two ideas at all, it would appear that they must have *something* in common. And, as a matter of fact, all objects of our thought have something in common: at least the notion or concept of *being*, or *thing*, or *reality*, in its most general sense.

Further than this, however, it is widely contended that in order to have intelligible negative predication at all, the subject and predicate must have in common not merely the notion of *being*, but the notion of a comparatively proximate genus of the subject of the judgment: that there is, for example, no real predication in the proposition "Virtue is not blue" or "Virtue is not-blue". We have already discussed this question at some length (39), and pointed out that the Principle of Excluded Middle cannot be held to be *universally* applicable unless we recognize as intelligible such an alternative proposition as "Virtue either is or is not blue". Negative judgments of the kind referred to are *intelligible* when made; but they never need to be made in real life: they are superfluous, because *the real function of the negative judgment is to ward off the error that would be contained in the affirmative which it denies*, and people do not fall into such obvious errors as the assertion that "Virtue is blue" would be.[1] But this function of the negative judgment, and the relation of denial to affirmation generally, need to be examined a little more in detail.

98. NATURE OF SIGNIFICANT DENIAL: ITS RELATION TO AFFIRMATION: ITS GROUNDS.—In all judgment there is a reference to reality (80); therefore, also in the negative judgment. As an interpretation of reality, the negative judgment, "*S* is not *P*," must have its ground in that reality; and that ground—the logical ground or reason, we may call it—must be *something*, in our experience of the reality, *whereby we are enabled to exclude P from S.*

[1] *Cf.* KEYNES, *Formal Logic*, p. 120.

Again, the only occasion we have for forming the negative judgment, "*S* is not *P*," seems to be the presence in our mind of a suggestion that "*S* is *P*"—an error which we wish to remove. The presence of such a suggestion we may call the psychological cause or motive for proceeding to judge about *S*. Of course, it is not necessary that before we formulate the negative judgment "*S* is not *P*," we *ourselves* should have *asserted* the affirmative "*S* is *P*". The negative does not involve the affirmative in this sense. The latter may be suggested to our minds in various ways: by the assertion or question of another, as one of a number of alternatives, etc. But the affirmation, "*S* is *P*," *by ourselves*, is not a prerequisite to our forming the negative judgment "*S* is not *P*". All that is necessary is that we should have *thought* of the affirmative judgment "*S* is *P*," that it should be present to our minds: and in this sense every negative judgment presupposes an affirmative.

In another way, also, the negative judgment "*S* is not *P*" *may* (though not necessarily) presuppose an affirmative judgment. In this way: Our appreciation of the logical ground for our denial may involve an affirmation. We may deny that *S* is *P* because we see that there is in *S* something (*M*, let us say) which separates it from *P*: in other words, because we see and *judge* that "*S is M*". "If I assert of a distant object that 'it is not red," I do so because I think the question of its being red has been or may be raised [psychological motive], and also because I think that *it is some other colour* which is incompatible with red [logical ground or reason]."[1] Hence, we may say that every negative judgment presupposes an affirmative; all negation presupposes affirmation.[2]

But does affirmation, though the more fundamental, involve negation? We may admit that in a certain sense it does: in this sense, that in order to form any positive ideas, such as *S* and *P*, to keep them distinct from each other, and to compare them in judgment, we must have made each of them *definite* by *limiting* it, *i.e.* by marking it off from all that is not itself; and this process of *determining* each of our concepts, by setting it over against all others, involves the idea of *distinction*, *difference*, *other-*

[1] MELLONE, *Introd. Text-book of Logic*, p. 373.

[2] "There is," says Aristotle, "one primary assertive λόγος, affirmation; then there is denial"; "affirmation is prior in thought to denial" (*De Int.*, c. 5, *An. Post.*, i. 25; *cf. Poetics*, c. 20;—*apud* MELLONE, *op. cit.*, p. 373).

ness, and the judgment that each of them is *not* any other. Thus affirmation and negation are correlative, reciprocal, complementary, and inseparable from each other.

But this process is *abstraction* (2) rather than negation. We form a definite concept of any mode of being or reality, by abstracting mentally from every other mode. Nor must we forget that this mental separation of a concept from all that is not itself, does not necessarily involve the *incompatibility* of the mode of being represented by it, with other modes of being in the real world: if it did, affirmative judgments would be impossible. Our concepts represent each only an abstract, partial, and limited aspect, of the real. Some of these are mutually exclusive: others are not, though they are often found *de facto* separated in the actual world. Hence, we must not confound the *contingent* negative judgment with the *necessary* one: what *is not* with what *cannot be;* or imagine that the ground of every negative judgment, "*S* is not *P*," must be something positive in *S*, that is *incompatible with P*.[1]

The negative judgment must always rest on *positive* grounds, *i.e.* on a positive examination of *S* in relation to *P;* but it is important to distinguish between the content of the judgment, *i.e.* its import or meaning, the information directly conveyed in the proposition that expresses it (82), and the grounds on which it is based.[2] The same judgment may be entertained by different people on different grounds; and a true judgment may be entertained on erroneous grounds. Logic deals with the judgment as the expression of an objective truth, not with the subjective or mental attitude of this or that individual who holds it; although, in the individual's mind, the judgment is inseparable from the grounds on which he bases it.

[1] Thus, Professor WELTON writes: "Negation is due to *S* possessing an attribute incompatible with the proposed *P*, and this is implicit in the negative judgment" (*Logic*, i., p. 162). This is not universally true. The attributes that constitute *S* must be conceived as *different* from *P*. This is needed for all intelligent predication, whether affirmative or negative. But, for intelligent denial, *S* need not contain anything *incompatible* with *P*. Mr. JOSEPH writes: "The reciprocal exclusiveness of certain attributes and modes of being is the real truth underlying negation" (*Logic*, p. 162). But, apparently, he means by exclusiveness not necessarily incompatibility, but either the latter or the actual separation of *S* and *P*, in addition, of course, to the mental discrimination of the concepts, *S* and *P*, from each other. "But for that [exclusiveness]," he continues, "everything would be everything else; that is, as positive as these several modes of being themselves." Which means, we take it, that the fact that one mode of being is found to exclude, whether necessarily or actually, other modes, and to be conceived by us as different from other modes, is as positive a characteristic of each mode of reality (being the limit of the latter) as the content of the mode itself. But, how far are the various modes in which we conceive and judge of reality—mutual limitation and discrimination of concepts, affirmation, negation, etc.—how far are they modes of *being* (*entia realia*) or only modes of *thought* about being (*entia rationis*)? This question would carry us too far into metaphysics.

[2] For example, the relation between *S* and *P* may be revealed to us on the word of another, and we may take his word as sufficient. In this case the ground—*viz.* authority—is extrinsic to the judgment itself.

Further than this, there is an undeniable natural tendency to give expression to the grounds, in or with the expression of the judgment itself. In the negative judgment, therefore, whose function is to *deny* or *contradict* a suggested affirmative, we rarely, if ever, rest content with the *mere contradictory*, the *bare denial* itself: we tend to go beyond this and to assert, if we can, in opposition to the suggested affirmative, something positively *incompatible* with, or *contrary* to, that affirmative (40, 41). Some authors contend that this is always necessary: that the *bare denial in itself, apart from its grounds, has no significance*, conveys no information.[1] This is going too far: for the concepts compared in a bare denial are each intelligible, and their mutual exclusion by a bare denial does convey information. The bare denial "*S* is not *P*," carries on the face of it the information that *S* has been examined in relation to *P*, and has been found without *P*. Nothing else need necessarily be known about *S*, as a result of examining it, than the absence of *P* from it, and in such a case this absence of *P* is what has been positively discovered in *S*, and what is accepted as a ground for the denial "*S* is not *P*". It is not necessary for an intelligible denial of the proposition "*S* is *P*," that we discover in *S* some *positive* attribute *exclusive* of *P*. The "something in *S* which excludes *P*," the "*M*" referred to above, may be simply the absence of *P*, the negative attribute "*not-P*"; and, in such a case, even the assertion of the ground for the negative judgment "*S* is not *P*," would not yield an antecedent affirmative judgment. Dr. Keynes,[2] following Sigwart,[3] expresses all this both briefly and clearly by saying that "the ground of a denial may be either (*a*) a deficiency or (*b*) an opposition"; and he adds this apt illustration: "I may deny that a man travelled by a certain train either (*a*) because I searched the train through just before it started and found he was not there, or (*b*) because I knew he was elsewhere when the train started—I may, for instance, have seen him leave the station at the same moment in another train in an opposite direction."

It is not true, therefore, that we *always* reach *mere contradictory* or *denying* judgments through the affirmative predication of attributes *contrary to*, or *incompatible with*, the subject in question.

WELTON, *Logic*, i., pp. 161-80. JOSEPH, *Logic*, chap. ix., pp. 161-3. KEYNES, *Formal Logic*, pp. 91-107, 119 *sqq*.

[1] *Cf.* BOSANQUET, *Logic*, pp. 305, 383;—*apud* KEYNES, *Formal Logic*, p. 122; WELTON, *Logic*, i., p. 162: "Pure negation has no existence in fact and cannot be really thought."

[2] *op. cit.*, p. 121. [3] *Logic*, i., p. 313.

CHAPTER IV.

EXTENSION AND INTENSION IN CATEGORICAL JUDGMENTS AND PROPOSITIONS.

99. Possibility of Various Meanings in the Propositions.—Every categorical judgment announces a certain sort of agreement or disagreement between subject and predicate, according as it is affirmative or negative in *quality*. But what sort of agreement or disagreement? We know that most general concepts or terms have two kinds of meaning, intensive and extensive (30); that the extension of the subject determines what is known as the *quantity* of the proposition (91); and that the predicate of the latter can have its extension, as well as the subject (*ibid.*). Consequently, if we take an affirmative categorical proposition in which both terms are general, as *e.g.* "All men are mortal," there is evidently room to ask what exactly is the nature of the relation affirmed to exist between the subject "men" and the predicate "mortal". Since each term has both intension and extension, the propositional form may possibly be open to a variety of *meanings* (82). When from among these alternatives we select some one as *the* meaning to be attached to a given propositional form, we do not thereby deny that the form in question can be made to bear any other alternative meaning (82); but still, we should undoubtedly be guided by *what people commonly mean*, in fixing the import of propositional forms for ordinary logical treatment.

100. "Predicative" or "Attributive" Interpretation. —*Subject read in Denotation and Predicate in Connotation.*—Taking, then, the judgment expressed by the proposition "*All men are mortal*," we shall probably be going nearest to what people ordinarily have in their minds, by saying that the relation is here asserted between the *denotation of the subject* and the *connotation of the predicate;* that is to say, between the members of the *class* "men" and the *attribute* "mortal". The *attribute* "mortal" is

asserted to belong to, or to be found in, or to be possessed by, *subjects* of a certain class, namely "men". This is called the *predicative* or *attributive* mode of interpreting the meaning of the categorical proposition. It gives us, as the meaning of the judgment, what is usually most prominent in the mind. Its correctness is borne out by the fact that the subjects of our judgments are usually substantives—names of objects and classes of objects, —while their predicates are usually adjectives—names of attributes. Hence, too, we regard the quantity of our judgments as determined by the quantity of the term whose denotation is most prominent, the subject. And, finally, the traditional fourfold scheme of propositions (91) is based upon this mode of interpretation.

101. REVERSE OF PREDICATIVE INTERPRETATION.—*Subject read in Connotation, Predicate in Denotation.*—It is rarely that in forming a judgment we think of the *connotation* of the subject in connexion with the class *denoted* by the predicate. Yet examples will occasionally be met with.

The familiar proverb, "All is not gold that glitters," is instanced by Dr. Keynes:[1] "Taking the subject in connotation, and the predicate in denotation, we have, he says, *The attribute of glitter does not always indicate the presence of a gold object;* and it will be found that this reading of the proverb serves to bring out its meaning really better than any of the . . . other readings. . . ." Another example, from the same source, and typical of a fairly common class of judgments, is this: *No plants with opposite leaves are orchids.*

102. CONNOTATIVE INTERPRETATION.—*Subject and Predicate read in Connotation.*—Although, however, the subject is most usually thought of as a class name, although it brings before the mind a class of objects, yet we know that it does so *only in virtue of certain attributes possessed in common* by those objects, *i.e.* only in virtue of something more fundamental than denotation, *viz. connotation* (32, 33). And we know, furthermore, that the reason or ground for affirming or denying the predicate of the class-subject is often because the *connotation* of the latter has been found to involve, or admit, or exclude the former, as the case may be (97, 98). We do not say that this is *always* the case. It raises the question about the grounds of predication, and hence about the distinction between *necessary* and *contingent* judgments, and *modality* (85-90). In necessary judgments, at all events, the

[1] *Formal Logic*, p. 186.

reason of the predication brings the connotation of the subject so prominently before the mind that this may fairly be claimed to be equally prominent with, and more important than, the denotation—even when the subject is expressed denotatively or quantitatively. Propositions *in materia necessaria* may, therefore, be taken to express, as their *meaning*, the *necessary accompaniment* or *non-accompaniment of the connotation of their respective terms.* For example, the proposition "*Man* is *mortal*" would mean "The attributes connoted by *man* are necessarily accompanied by the attributes connoted by *mortal*".

In the case of *contingent* judgments, whether universal [92, (*a*) (i)] or particular (93), it is possible similarly to regard the connexion asserted or denied as one of *actual* accompaniment or non-accompaniment of the connotations of the respective terms. Thus, "Some men are learned" would mean that "the attributes which constitute the connotation of *man* are *sometimes* actually found to be accompanied by the attribute of *learning*"; "All ruminants are cloven-footed" would mean that "the attributes which constitute a *ruminant* are *always de facto* accompanied by the attribute *cloven-footed*".

We have already called attention to the fact (97) that in a negative proposition the whole connotation of the predicate is taken *collectively*, and denied, *as a whole*, of the subject. In the present, or connotative, interpretation of the judgment, the same is true of the subject in regard to the predicate. A term read in connotation is not quantified; its quantity is not explicitly thought of;[1] but, as in the examples given above, the element of denotation is restored to the proposition, by the words "always" and "sometimes". Mill, who advocated this mode of interpretation, was right in emphasizing the greater and more fundamental importance of *connotation* in the subject of the judgment. It is the more important side of the meaning of terms; and in *necessary* judgments, where the predication is based not on enumeration of instances, but on analysis of intension (92), the connotative interpretation approaches nearer to what is actually in the mind than the predicative interpretation. Not so, however, in the case of *contingent* judgments. Mill was wrong in claiming that the connotative is the more appropriate interpretation for these. And he was also wrong in endeavouring to separate the connotative side, altogether from the denotative side, of the proposition, and to substitute, accordingly, the *Nota notae* for the *Dictum de omni*, in his doctrine on the basis of syllogistic reasoning (153).

103. COMPREHENSIVE INTERPRETATION—*Subject and Predicate in Comprehension.* Comprehension is the sum-total of the attributes (known and unknown) actually common to all the members of a class. Can we interpret the categorical judgment

[1] It is in this sense "undistributed". *Cf. supra*, (91). A term is sometimes described as "undistributed" in a proposition when its extension is not explicitly thought of at all in the judgment. *Cf.* JOSEPH, *op. cit.*, pp. 195-6.

as expressing a relation between the comprehension of the subject and that of the predicate?

Sir William Hamilton taught that as all terms have comprehension and extension, so also all judgments may be interpreted in two distinct ways: in comprehension and in extension. With the latter way, in which S and P are regarded as classes, we shall deal in the next section. According to the comprehensive interpretation, such a judgment as "All men are mortal" would mean that the attributes which make up the comprehension of "man" *contain* among them those that make up the comprehension of "mortal".

As in the case of the *Connotative* interpretation, so also here, the comprehension of each term must always be taken in its totality: so that we cannot work out a scheme which would enable us to refer to all the notes distributively, or to an indefinite portion of the notes, which go to make up the *comprehension* of subject or predicate, as we work out the scheme for referring to the whole, or to an indefinite portion, of the extension of those terms.

The present interpretation differs from the connotative merely in this, that instead of the *connotation* of S being understood to *accompany* that of P, here the *comprehension* of S is understood to *include* that of P. It is sufficiently appropriate for *necessary* or *analytic* propositions; but as applied to *contingent* or synthetic propositions, which are supposed to give some new information, not derived from our previous knowledge of S and P, the comprehensive interpretation, "S contains P," can only mean that "P is now transferred by me for the first time from the unknown to the known part of the comprehension of S—thereby becoming for me a part of the *Subjective Intension* of S" (31).

The predicative view alone, of those discussed so far, seems appropriate for judgments that are particular, collectively or concretely universal, based on observation and experience, synthetic, contingent—as opposed to strictly universal, necessary, abstract, or generic judgments.

104. Extensive or Class-inclusion Interpretation.—*Subject and Predicate in Denotation.* According to this interpretation, the meaning of the proposition is to assert a relation of mutual *inclusion* or *exclusion* between two classes. "*All men are mortal*" would mean that "All the objects denoted by the term *man* are included among the objects denoted by the term *mortal*": "Men are mortals".

There has been much dispute among logicians as to the advisability, and even as to the correctness, of interpreting the categorical judgment in this manner. Against the extreme advocacy of this interpretation by Sir William Hamilton, there has been an equally extreme reaction.

It may be granted at once that in most of our judgments we think of the intension of the predicate rather than of its extension, and that, accordingly, the predicative interpretation gives a more faithful expression of the psychological facts than the present view gives. But it cannot be denied that in many of our judgments it is the extension, and not the intension, of the predicate that is uppermost in our minds. In all those sciences and departments of knowledge in which the *classification* of the things considered is prominent, our judgments bring into consciousness *class relations.* For example, *Irishmen are Celts; Whales are Mammals; Palms are Endogens; None but solid bodies are crystals.* Of course, the intension of the predicate is in all cases *more fundamental* than its extension, even when the class-inclusion interpretation is adopted: but the same is true of the subject when it is read in extension in the predicative view.

Then, again, it cannot be denied that in many logical processes which will come up for discussion—in the conversion of propositions, and in syllogistic reasoning, for example—we attend explicitly to the denotation of the predicate. We have already considered rules for the distribution of the latter (91).

Finally, the proposition, interpreted in this way, lends itself to diagrammatic representation. It is only the class-relations of terms that can be instructively illustrated by diagrams. The possible relations of two objectively determined classes will be seen to be neither more nor less than the following five:

(1)

All S is all P;

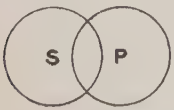
(2)

All S is some P;

(3)

Some S is all P;

(4)



Some S is some P;

(5)

No S is any P.

Here the classes are represented by circles—called "Euler's circles," from their first application by Euler, a Swiss logician of the eighteenth century. It will be seen that they suppose "Some" to mean "a definitely known portion less than all". That is, *they express all the relations that are objectively possible* between two classes of things; *not our possibly imperfect and indefinite knowledge of those relations.* Whether or not the class-inclusion view necessarily leads to the above fivefold scheme of propositions, we shall see presently. Were we, in the *predicative* view, to interpret "some" as above, instead of indefinitely, we should have a *threefold* instead of the traditional *fourfold* scheme [1]; for, either particular, I or O, would then involve the other and convey the knowledge that "Some S is, and some S is not, P". Any attempt to use combinations of the above five figures for the purpose of expressing the four traditional propositions, A, E, I, O, will bring out the contrast between the full and definite knowledge implied in the judgments represented by each of Euler's diagrams, and the imperfect knowledge contained in the A, E, I, O, judgments.

105. QUANTIFICATION OF THE PREDICATE: HAMILTON'S EIGHTFOLD SCHEME.—While recognizing, therefore, certain uses and advantages in the class-inclusion view, we must, nevertheless, reject the claims put forward by Sir William Hamilton on its behalf. He enlarged the traditional fourfold, to an eightfold scheme of propositions, by explicitly quantifying the predicate and thus making its distribution or non-distribution in every case independent of the quality of the proposition (91). This gave rise to the following scheme:—

From A	*All S is all P*	*afa*	U	*S u P*
	All S is some P	*afi*	A	*S a P*
From I	*Some S is all P*	*ifa*	Y	*S y P*
	Some S is some P	*ifi*	I	*S i P*
From E	*No S is any P*	*ana*	E	*S e P*
	No S is some P	*ani*	η	*S η P*
From O	*Some S is not any P*	*ina*	O	*S o P*
	Some S is not some P	*ini*	ω	*S ω P*

The symbols used by Hamilton himself were *afa*, etc., *f* meaning affirmation, *n* negation, *a* distribution, *i* non-distribution. The symbols more commonly employed are U, A, Y, etc., introduced by Archbishop Thomson.[2]

106. HAMILTON'S POSTULATE.—Hamilton justified this interpretation of the judgment by an appeal to the postulate of logic, that we "be allowed to state explicitly in language all that is implicitly contained in the thought". The meaning of the postulate, thus stated, is not clear. If it demands the right to make mere verbal changes that will not alter the *meaning* of a proposition (82), it is unimpeachable. If it refers to the *inference* or *drawing out* of *implications*, latent in the *meaning* of a judgment or judgments, it indicates rather what it is the function of logic to teach us how to do correctly. But in its application to the present subject-matter it is evidently intended to mean rather that we have a right to state in language what is *actually and explicitly* in our thought. We certainly have; but Hamilton abuses the right by

[1] KEYNES, *op. cit.*, p. 183.
[2] *An Outline of the Laws of Thought*, p. 137.

claiming as explicitly thought in the act of judgment what is not really so. He quantifies the predicate explicitly in all propositions because he contends that, psychologically, in all our judgments we think of the quantity or extension of the predicate: "in thought the predicate is always quantified". This is notoriously untrue. A simple appeal to consciousness will assure us that far more frequently we think of the intension of the predicate in our judgments, not of its extension. When we are told that "All A's are B's" so far are we from spontaneously thinking the meaning to be "All A's are some B's," that at first it is only by an effort we realize this is so, even when we are assured that it is. When the learner is told that the proposition "All A's are B's" cannot be simply converted to "All B's are A's," but only to "some B's are A's," he is told a thing that he may never before have explicitly adverted to; and which, perhaps, he will not grasp fully until he works on some familiar example like "All men are animals": "Some animals are men".

107. General Discussion of the Quantification Doctrine.—Notwithstanding the unsoundness of its psychological basis, "Quantification of the Predicate" was regarded by many as a means of bringing about quite a wonderful simplification of logical processes. It was expounded and applied by Dr. Baynes in his *New Analytic of Logical Forms*. It promised great things; but it was found to simplify practically nothing. Baynes contended, in defence of it, that we cannot have intelligible predication without quantifying the predicate; and, that, furthermore, the quantitative relation established must in all cases be *determinate*. "If this relation," he writes, "were indeterminate—if we were uncertain whether it was of part, or whole, or none—there could be no predication."[1] "This is perfectly true," replies Dr. Keynes,[2] "so long as we are left with all three of these alternatives: but we may have predication which involves the elimination of only one of them, so that there is still indeterminateness as regards the other two." This latter, in fact, is the predication which expresses "the very common state of doubt, when we know that every S is P but do not know whether or not any other objects are P as well";[3] and when we do express such a judgment in the explicitly quantified form, "All S's are some P's," the word "some" bears the traditional indefinite meaning, "some, possibly all". But the Hamiltonian scheme gives no form for expressing this very common state of doubt, or partial knowledge, about a class, if it expresses only quantitative relations between *definite* or *determinate* classes:[4] for in doing this latter we must take the word "some" to mean "a definite, determined portion, not all."

And then, furthermore, if we do take "some" in this latter sense—so that in all cases we must have a definite knowledge of the relative extent of two classes, before comparing them in judgment—we ought to reach a *fivefold*, rather than an *eightfold* scheme of propositions: there being only five alternatives in the actual quantitative relations of two classes (104).

Thus we see that the scheme is at once defective and redundant. There is, moreover, a wide divergence of view among Hamilton's disciples as to the proper interpretation to be given to "some": and a scarcely less

[1] *apud* Keynes, *op. cit.*, p. 197.
[2] *ibid.*, p. 199.
[3] Welton, *Logic*, p. 200.
[4] *Cf.* Keynes, *op. cit.*, p. 203.

hopeless confusion of different interpretations in Hamilton's own writings. Let us see a few of the chief results to which the various interpretations of "some" will lead us.

108. Various Alternatives and their Unsatisfactory Results. —(*a*) *If "some" means "some only,"* the scheme is redundant; for each affirmative proposition containing "some" involves a negative, and *vice versa.* Thus A and η mutually involve each other, and similarly Y and O,[1] while ω is actually equivalent to U—incredible as this may seem at first sight. "*Some* (*but not all*) *S's are not some* (*but not all*) *P's*" asserts that certain *S*'s are not to be found in a certain section of the *P*'s but are to be found elsewhere among the *P*'s, while the remaining *S*'s are to be found in the former section of the *P*'s, and thus *All the S's are P's;* and in precisely the same way it is seen that all the *P's are S's.* That is, *All S is all P;* or ω is equivalent to U.

We are thus left with five forms instead of eight, *viz.*, U (or ω), A (or η), Y (or O), I, E—expressing the five alternative relations of two actual classes (104). Each of these propositions is incompatible with each of the others; but it is by no means a "simple" process to find the *contradictory* of any one of these. We can contradict U, for example, only by affirming an alternative between A, Y, I, and E. The traditional fourfold scheme is simplicity itself compared to this.

If it is an essential of any scheme of formulating propositions that the forms be simple and irreducible, we see that the present interpretation is very defective indeed, containing, as it does, forms that are exponible, and forms that are ambiguous. This indeed is the necessary outcome of interpreting "some" to mean "some only," instead of giving it the simpler and more fundamental meaning of "some at least". "Some only" implies the formation of two judgments; that "some are . . ." and that "some are not . . ." and presupposes, or rather fails to take account of, that prior and more indefinite stage of knowledge at which we know that "some are . . .," but do not know anything about the remainder of the class.

(*b*) "*Some*" *interpreted as* "*some at least,*" *in two of the eight forms.* —Hamilton himself does not keep consistently to either view of "some": when it enters into both terms of the judgment, *i.e.* in I and ω, he inclines to retain the traditional meaning, "some at least"; but he does not adhere to this view in the confusing applications to which it would lead.[2] It does not remove the difficulty that A is equivalent to η, and Y to O. Its combination, in one and the same scheme, with the meaning "some only," is very confusing; and we shall see presently that its application to I and ω does not lead to any useful results.

(*c*) "*Some*" *interpreted as* "*some at least, possibly all*".—Returning to this, which is the traditional logical interpretation, adopted in the predicative scheme, let us apply it to each form of the present scheme in turn.

[1] And besides this, η and O may each be interpreted as equivalent to I—so that these forms do not succeed in giving definite information, but are ambiguous. The passage from A to η and from Y to O is called "Integration": because, given a part, it "integrates" the whole by introducing the other part (Bowen, *Logic*, pp. 169, 170).

[2] *Cf.* Keynes, *op. cit.*, p. 201.

The U *proposition* is equivalent to the combination of the two predicative forms *S a P* and *P a S*. The form "*All S's are all P's*" is rarely met with; but there are forms equivalent to it: (i) propositions which state explicitly that two classes are coextensive, *e.g.* "*Equilateral triangles and equiangular triangles are coextensive*"; "*Christianity and Civilization are coextensive*"; "*Bluestone is the same as sulphate of copper*"; "*Sunday, Monday . . . Saturday are all the days of the week*". A sub-class of those would be propositions with singular subjects and singular predicates, *e.g.* "*The present Pope is Pius X*". (ii) All *definitions* may, of course, be interpreted as U propositions if the denotation of the terms be attended to; for the subject and predicate, being identical in connotation, are coextensive in denotation. But this reading is quite subsidiary to the primary one, which, in the case of definitions, is always the connotative reading—relation of attributes being thought of in the first place.[1] The U proposition is, therefore, not a simple but an exponible form, if the quantification of its predicate is to be interpreted as in the predicative scheme. It is better, therefore, to use its two equivalents, for logical purposes, than to retain the U proposition itself.

The A *proposition* is the same in both schemes.

The Y *proposition* expresses what we have described as exclusive and exceptive propositions in the predicative view. "*Graduates alone are eligible*" yields, as we have seen (95), the proposition *Some graduates are all eligible people:* which is the Y form. This, no doubt, preserves the original subject and predicate, but it is not a simple predicative form, any more than the original. Its converse, however, *All eligible people are graduates*, is a simple predicative form (A), and conveys the same information about the compared classes as Y does, *viz.* that one class is contained in an indefinite portion of the other. It is better, then, to use the A form for general logical processes than the Y form.

The I *proposition* will be the same as in the predicative scheme.

The E *proposition* will also be the same in both schemes.

The η *proposition* is never met with; but a form that is said to be equivalent[2] occasionally occurs: the form "Not *S* alone is *P*," *provided this is not taken to convey that any S is necessarily P.* For instance, to the boy's generous wish "I should like to be a millionaire in order to be a great public benefactor," the philosopher may reply "It is not millionaires alone who are great public benefactors": which asserts that "some great public benefactors are not millionaires" (or that "some non-millionaires are great public benefactors") and *does not assert* that any great millionaires *are* great public benefactors. This example might be expressed, in accordance with the η form, to mean that "the whole class of millionaires is excluded from an indefinite portion (possibly the whole) of the class of great public benefactors," or, equally, *by the O form of the predicative scheme*—"Some P's are not (any) S's"—"Some great public benefactors are not millionaires". The form η is thus seen to be equivalent to O. It is, moreover, the contradictory of Y: a relation which will be more easily realized if Y be written "S alone is P," and η "Not S alone is P". "*The virtuous alone are happy*" is contradicted by "*Not the virtuous alone are happy*".

[1] *Cf.* JOSEPH, *op. cit.*, p. 200 n.

[2] *Cf.* KEYNES, *op. cit.*, p. 206.

The O *proposition* is the same in both schemes.

The ω *proposition* is really a sort of logical monstrosity. We have seen how it involves U when "some" means "some only". Taking "some" now in the more indefinite sense, the ω proposition will be found to assert nothing and to deny nothing. It denies nothing, for it is compatible with all the affirmative forms, even with U. Although, for example, "all equiangular triangles are all equilateral triangles," yet it is still true that "any one (indefinite) equiangular triangle is not any other (indefinite) equilateral triangle," and this is all that the ω form asserts. But this assertion is no assertion where "some" is an indefinite class term, for it can be made about any two members of a class: conveying only what we know already—that in any class of things no one member is identical with another.

Of course, where S and P are both singular terms, and both names of the same individual, the truth of ω is excluded; but in this case ω is inappropriate, for there is no place for the indefinite reference of "some" in such a proposition.

Dr. Keynes points to some interesting results[1] that might be obtained by supplementing the traditional fourfold scheme of propositions by the two mutually contradictory forms, U and η,—with "some" interpreted in the indefinite sense. But for ordinary purposes it is better to replace these forms, as well as the U form, by their respective predicative equivalents.

109. EQUATIONAL READINGS OF THE LOGICAL PROPOSITION.—We may now draw attention to the question whether it is possible, or in what sense it is possible, to regard a logical proposition as an *equation*—and, more especially, whether the eight Hamiltonian forms are really equational forms.

When we introduce the sign of equality into logic and write the proposition in the form "$S = P$": "*Equilateral triangles* = *equiangular triangles*," "*men* = *mortal men*": what sort of identity is it that we endeavour to express? It is an identity analogous to mathematical equality, to numerical identity. It is, therefore, not an identity of connotation in the terms compared, but an identity of denotation. It asserts that the class denoted by the subject-term is coextensive and identical with the class denoted by the predicate-term; and it implies that this class can be reached or determined in either of two different ways—through the connotation of the subject, or through the connotation of the predicate. Every equational scheme of interpreting propositions is, therefore, an "extensive" or "class" scheme, and labours under all the defects of this latter.

An equational reading must necessarily, of course, possess this one advantage—the value of which, however, seems to have been greatly overestimated—that, since the logical copula "*is*"

[1] *op. cit.*, p. 207.

will express identity, the distinction between subject and predicate will be merely a distinction of place, the proposition may be read equally well backward and forward, will be perfectly symmetrical, and may always be simply converted (118).

In regard to the propriety of expressing ordinary judgments as logical equations—as announcing the identity of the two classes—we may say at once that it is inappropriate, that it does not express what we are really thinking of, except in the rare cases, already referred to, of propositions like "Civilization and Christianity are coextensive".

In order to have an equational proposition, the subject and predicate must be taken *collectively*, as names of definite classes, as two single collections, which are either coextensive, and accordingly identical, or are not. It has been claimed that the eight Hamiltonian propositions are equational. But how is this so in the propositions containing "some?" If "some" be taken indefinitely as "some at least" the proposition cannot be equational; for an indefinite portion of a class is itself a class indefinite in its denotation, and there is no meaning in making an indefinite class "identical" with any other class, definite or indefinite.

"Some" therefore, must be taken as expressing a definite portion—not all—of a larger class; *i.e.* as "some only". Nor will it even now yield a strictly equational proposition: except on the further understanding that it indicates *a specified group;* and this reservation does not appear in the form of the proposition. In order that the A proposition—*All S is some P*—be simply convertible—to *Some P is all S*—it is necessary that the "Some P" be the same particular group in both propositions, and that it be one special, singular group, not any group taken at random. The converse of "All men are some animals"—*viz.* "Some animals are all men"—is not true except of the one particular group of animals referred to in the former proposition, and there is nothing in the form of that proposition to show such limitation.

Attempts have been made to find equational forms corresponding to the four forms of the traditional scheme (91).[1] Jevons expresses the U and A forms by the sign of equality, writing the former "$S = P$" and the latter "$S = SP$". The form $S = P$ he calls a *simple identity.* It expresses the U proposition *All S's are all P's.* But, clearly, we cannot write the proposition *All S's are P's* in the form $S = P$, for S may include more than P. But it can be written $S = SP$, where SP denotes the S's that are P's. For instance, *All men are mortal* may be written *Men = mortal men.*

Now, since particulars contradict universals (112), if we express the latter as equalities we should express the former as inequalities—by the signs $>$ and $<$. Thus $S > SP$ would imply that the class S includes more than SP, *i.e.* that *Some S's are not P's.* "If we further introduce the symbol o as expressing nonentity, *No S is P* may be written $SP = o$, and its contradictory, *i.e. Some S is P*, may be written $SP > o$. We shall then have the following scheme (where p = *not-P*):—

[1] *Cf.* KEYNES, *op. cit.*, p. 193: whose treatment is here followed.

All S is P expressed by $S = SP$ or by $S\not{p} = 0$;
Some S is not P „ „ $S > SP$ „ $S\not{p} > 0$;
No S is P „ „ $SP = 0$ „ $S = S\not{p}$;
Some S is P „ „ $SP > 0$ „ $S > S\not{p}$."[1]

This scheme, as Dr. Keynes remarks, is based on the assumption that particulars imply the existence of their subjects, and that universals do not. This question of the existential import of propositions, and the scheme for expressing it, will be discussed in a subsequent chapter.

JOSEPH, *Logic*, pp. 198 *sqq*. KEYNES, *Logic*, pt. ii., chap. vi. WELTON, *Logic*, bk. ii., chap. ii.

[1] *Cf*. KEYNES, *op. cit*., p. 193.

CHAPTER V.

OPPOSITION OF CATEGORICAL JUDGMENTS AND PROPOSITIONS.

110. MEANING OF LOGICAL "OPPOSITION": THE SQUARE OF OPPOSITION.—We have now sufficiently examined the *quality* and *quantity* of propositions, and the various interpretations arising from the *intension* and *extension* of the terms contained in them, to inquire next into all the possible implications of truth and falsity derivable from any given categorical judgment. These implications are made explicit, partly by way of "opposition," and partly by way of "eduction". With the latter we shall deal in the next chapter, with "opposition" in the present one.

By the *Logical Opposition* of propositions we mean *the relation, in respect of quantity and quality, between any two propositions which have the same subject and predicate;* or, the *relation between two propositions identical in matter and different in form* (91). Thus, two propositions are said to be logically "opposed" to each other when they have the same subject and predicate, but differ in quantity, or quality, or both. Evidently, two such propositions need not be incompatible with each other; for instance, A and I (*SaP* and *SiP*) are not incompatible, although they are, according to our definition, "opposed" to each other. This purely technical use of the word "opposition" is a little disconcerting because it is so unusual; but we need some such word to express the general set of relations referred to, and, if this purely logical use of the word "opposition" be noted and remembered, no confusion can arise.

We saw in a previous section (91) that, taking the ordinary predicative view of the judgment, the latter either *affirms* or *denies* something about either the *whole* or an *indefinite portion of* something else: thus giving us the four propositional forms, *SaP*, *SeP*, *SiP*, *SoP*. The mutual relations between the pairs will, therefore, be :—

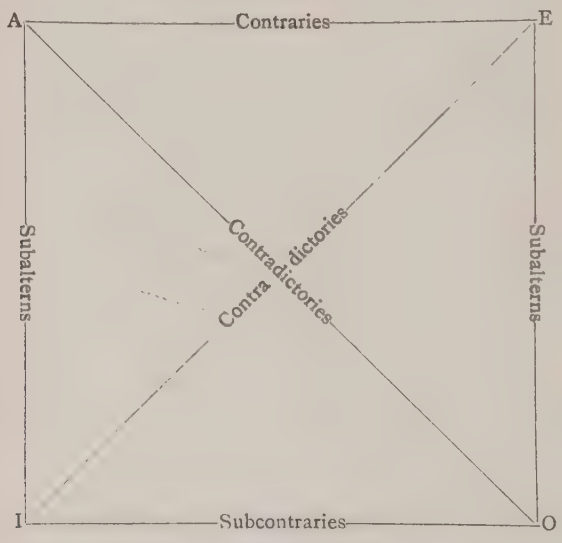
(1) Between a universal and the particular of the same quality (A and I ; E and O)—called SUBALTERNATION.

(2) Between a universal and the particular of different quality (A and O ; E and I)—called CONTRADICTION.

(3) Between a universal and the universal of different quality (A and E)—called CONTRARIETY.

(4) Between a particular and the particular of different quality (I and O)—called SUBCONTRARIETY.

These relations between the various pairs of the four traditional propositions are clearly set forth in what is known as the "square of opposition" : a device which will help to stamp the whole theory of opposition clearly upon the mind :—



Here the universals are (naturally) higher than the particulars, and the contraries than the subcontraries ; the affirmatives are (naturally) prior, in their lines, to the negatives ; the diagonals, being longest, mark the logically perfect kind of opposition, that between affirmation and denial—*viz.* contradiction ; propositions on the same horizontal line are equal in quantity, differing only in quality ; those on the same vertical line differ only in quantity, agreeing in quality ; the vertical line appropriately suggests subalternation ; while the diagonal suggests diversity both in quantity and in quality. Now for the different *kinds* of opposition in detail.

111. SUBALTERN OPPOSITION *is that which exists between a universal and the particular of the same quality.* The universal is called the *subalternant ;* the particular the *subaltern,* or *sub-*

alternate: from the Latin terms *subalternans* and *subalternata*, respectively. An inference from universal to particular (of same quality) is called a *conclusio ad subalternatam;* an inference from particular to universal (of same quality) is called a *conclusio ad subalternantem;* furthermore, to suppose a proposition *true* is to "posit" it, to suppose it *false* is to "sublate" it. Given either a universal, or the particular of the same quality, as true, or as false, what can we infer about the truth or falsity of the other?

The answer is given in the following two laws:—

(1) *The truth of the particular follows from the truth of the universal*[1] *of like quality, but not vice versa.*

(2) *The falsity of the universal follows from the falsity of the particular of like quality, but not vice versa.*

These rules are immediate applications of the *Principle of Identity* (12). Thus, in regard to rule (1), if we assert that "all men are mortal" we may assert that "*some* men are mortal," or if we assert that "*no men are angels*," we may assert that "*some*[2] *men are not angels*"; but if we assert that "*some men are fools*" we have no right to infer that "*all men are fools*," nor if we assert that "*some men are not learned*" have we a right to assert that "*no men are learned*".

Similarly, in regard to rule (2), by denying the truth of "*all men are fools*" we merely deny that folly can be predicated of *all* men, and cannot thence deny that "*some men are fools*"; and by denying the truth of "*no men are wise*" we merely deny that wisdom can be excluded from *all*, and do not deny that it may be excluded from some, or that the proposition "*some men are not wise*" is true. But if we deny the truth of "*some men are wise*" we *a fortiori* deny the truth of "*all men are wise*," just as by denying the truth of "*some men are not mortal*" we *a fortiori* deny the truth of the assertion that "*no men are mortal*".

Whenever we meet two propositions, however they be formulated, so related to each other that one of them may be formally inferred from the other but not *vice versa*, we may extend our present meaning of the term *subalternation* to such propositions, always describing such a relation as *subalternation.*

112. CONTRADICTORY OPPOSITION may be defined as *that*

[1] This and other statements in the present chapter may have to be modified by what will be said below (chap. viii.) on the *existential import* of propositions.

[2] It must be borne in mind that "some" does not mean "some only," but "some, possibly all".

which exists between two propositions which have the same subject and predicate but differ both in quantity and in quality. This definition is specially adapted to the propositions of the traditional square of opposition. It coincides with the more fundamental conception of contradiction as *the opposition between simple affirmation and simple negation*, or the opposition *between two propositions one of which must be true and the other of which must be false.* This latter is the real test of contradictory opposition between any two propositions: that *they can neither be true together, nor false together with a mean of truth between them, but one must be true and the other false.* Hence the two laws:—

(1) *Contradictories cannot be true together*;
(2) *Contradictories cannot be false together.*

Let us illustrate these laws by applying them to the four propositions A, E, I, O.

(1) If it be true to affirm P of all the S's (SaP) the *Principle of Contradiction* (13) forbids us to deny P of any of them, *i.e.* to say some S is not P; hence SoP is false. Conversely, if it be true to deny P of some S (SoP), it cannot be true to affirm P of them all; hence, SaP is false. If it be true to deny P of all the S's (SeP), it must be false to affirm P of any of them; hence SiP is false. Conversely, if it be true to affirm P of some of the S's (SiP), it cannot be true to exclude P from all of them; hence SeP is false.

(2) If it be false to affirm P of all the S's (SaP), it must be true, by the *Principle of Excluded Middle* (14), that at least one of them is not P; hence SoP is true. Conversely, if it be false to deny P of any—even one—S (SoP), it must be true that all the S's are P; hence SaP is true. If it be false to deny P of all the S's (SeP), it must be true that some one at least of the S's is P; hence SiP is true. Conversely, if it be false that any—even one—S is P (SiP), then it must be true that none of them are P; hence SeP is true.

From this we see that contradictories are incompatible both as regards their truth and as regards their falsity; that the truth of either is inferable from the falsity of the other, and the falsity of either from the truth of the other; and that they are thus perfectly correlative. These characteristics we shall find in no other species of logical opposition; hence *Contradictory Opposition is the most perfect of all forms of opposition.*

Since contradiction is the relation between affirmation and denial, to deny the truth of a proposition is the same as to assert the truth of its contradictory, and to assert the truth of a proposition is to deny the truth of its contradictory. Every affirmative judgment, therefore, asserted as true, implies the falsity of a negative judgment suggested by it; and similarly, every negative judgment asserted as true, implies the falsity of a suggested affirmative judgment (98).

Every judgment, therefore, has a contradictory. Have any judgments more than one contradictory? Simple judgments have not. But it has been stated sometimes by logicians that *compound* judgments *may be contradicted in various ways*, or, *have more than one contradictory*. This is not true, if we take the terms "*contradict*," "*contradictory*," in the strict sense. What is true rather is this, that a compound judgment yields two or more simple judgments, each of which may have a (simple) contradictory, whose truth is *incompatible* with the truth of the (compound) original, though its falsity may not be incompatible with the falsity of the original. But these are not real contradictories of that original. The compound judgment has really *only one contradictory*, which *will be also a compound judgment*. For instance, the compound "*All S is P and all P is S*" is contradicted by the compound "*Either some S is not P or some P is not S*". The simple judgment "*Some S is not P*" is *incompatible* with the original; but it is not the contradictory of the original: for even were it false we could not thence infer the truth of the compound original.

A little reflection on this example will show that in order to *contradict*[1] a proposition we *must assert only the minimum which is necessary and sufficient to break down the truth of that proposition*. Therefore it is that there is no mean between a proposition and its contradictory. But if we pass beyond this minimum, necessary to break down the truth of the former, we leave place for a mean of truth between the original and our new proposition. Hence, from the falsity of this latter we cannot infer the truth of the original: both may be false together. Our new form is something more than the mere contradictory of the original: it has

[1] We may of course *refute*, or show the falsity of, a proposition, by going further than merely contradicting it, *e.g.* by establishing the truth of its *contrary* (see 113).

set up a positive counter-assertion in the direction of *contrary* opposition.

Two contradictories must affirm and deny the same thing about the same thing under the same respect. One of them must, therefore, be false by the *Principle of Contradiction* (13). One of them must be true by the *Principle of Excluded Middle* (14).

Any pair of propositions which cannot be true together, or false together, may rightly be called contradictories, even though they be not logically formulated according to the A, E, I, O scheme, or even though they have not exactly the same terms as subject and predicate, *e.g.* "*All virtuous people are happy*" and "*Some non-happy people are virtuous*".

113. CONTRARY OPPOSITION *is that which exists between two universals of opposite quality:* between *All S's are P* and *No S's are P.* Contrary propositions thus differ in quality only, while contradictories differ in quality and quantity. The rules of contrary opposition are :—

(1) *Contraries cannot be true together;*

(2) *Contraries may be false together.*

(1) We prove the first rule by reference to the square of opposition, employing the rules we have already laid down for subalternation and contradiction. Thus: If A is true I is true (by *subalternation*); but if I is true E must be false (by *contradiction*); therefore if A is true E is false. Similarly, we prove that if E is true A is false. Thus, we see that if contraries were true together, contradictories would also be true together—which is impossible. The contradictory of a true proposition must be false; but the contrary is still further removed from the original than the contradictory is. Therefore, *a fortiori*, the contrary of a true proposition must be false. Hence, *contraries cannot be true together.*

(2) The second rule conveys that if one of any pair of contraries be given as false we can infer nothing about the other: this latter may be false or it may be true: we cannot tell. Thus, by sublating A we posit O, but by positing O we cannot either posit or sublate E: from the truth of the particular we cannot infer the truth of the universal—nor its falsity. Similarly, by sublating E we posit I, but cannot thence infer anything about A. Therefore *contraries may be false together.*

Or, to put the proof in another way, one of any pair of contraries does much more than merely *deny the entire truth* of the

other: it *asserts the entire falsity* of the latter in all its parts, by setting up a counter-assertion as far removed as possible from the latter (112). But, evidently, in doing this it may go too far. It may pass the mean in which the truth may lie, and fall itself into the falsity of the other extreme.

Comparing contradictory with contrary opposition, we see that the latter is less perfect than the former, inasmuch as contraries are incompatible only as regards their truth, not as regards their falsity, and are not mutually inferable as contradictories are.

We can disprove, or deny, or break down the truth of, a universal proposition by establishing the truth either of its contradictory or of its contrary. The former is obviously the easier and the safer way. It is the *easier* way, because any *one single exceptional instance*, expressed by a *particular* proposition, suffices to break down the truth of a universal rule; whereas complete knowledge is necessary in order to establish the contrary, *which is a universal*. It is the *safer* way, because while one single exception might overthrow the contrary that has been set up in disproof of a given universal, it would take a universal proposition to overthrow the contradiction set up in disproof of that universal. "One would deny that 'all men are liars' with much greater strength of conviction than one would assert that 'no men are liars'". [1]

Evidently, it is only between propositions *farthest removed from each other on some scale*—as the universal affirmative and negative are—that *contrary* opposition in the strict sense can exist. And between the strict contradictory and the strict contrary of a given extreme, or universal proposition, we may conceive an indefinite number of propositions all incompatible with the latter, and increasing in their divergence from it, towards the contrary. Any one of these will form, with the original proposition, *a pair of contraries in the wider sense:* propositions which, although not farthest removed from each other, are incompatible as regards their truth, but may both be false, admitting a mean of truth between them.

In giving the laws of the various kinds of opposition so far explained, we have confined our attention to the *formal* opposition which arises from an examination of quality and quantity. If we take into account a knowledge of the *matter* (85)—whether, namely, the propositions are *in materia necessaria* or *in materia contingenti*—we can make further inferences. If, for instance, the propositions in question be *in materia necessaria*, we can infer equally from *subalternant* to *subalternate*, or *vice versa*, both as to truth and as to falsity; and also that two contraries *in materia necessaria* cannot be false together.

[1] WELTON, *Logic*, i., p. 236.

114. SUBCONTRARY OPPOSITION *is the opposition which exists between two particular propositions of opposite quality*: between I and O. The laws of subcontrary opposition are:

(1) *Subcontraries cannot be false together;*

(2) *Subcontraries may be true together.*

(1) They cannot be false together, for if they were, their respective subalternants also, A and E, would each be false (by laws of subalternation), and hence contradictories would be false together, which is impossible. Or thus: if either subcontrary be false its contradictory must be true, and hence the subaltern of that contradictory must be true; but this latter is the other subcontrary. Hence *subcontraries cannot be false together.* This proof is an appeal to the *Principle of Excluded Middle*, and may also be stated in this way: since "some" is entirely indefinite, and has the same sense of "some, possibly all" in each subcontrary, we can see at once that there can be no mean between the statement "some are . . . " and "some are not . . . " Either must be true.

It has been argued, against this law, that I and O may be false together: "A and E may both be false. Therefore I and O being possibly equivalent to them, may both be false also."[1] This is a fallacious argument, an instance of the fallacy *a sensu diviso ad sensum compositum.* It argues that because I is sometimes equivalent to A, and O sometimes to E, therefore the two equivalences can exist at the same time; but it is precisely when A and E are both false that I and O cannot be simultaneously equivalent to A and E respectively.

(2) Subcontraries may, however, be true together; so that, given the truth of one, we cannot infer the falsity of the other; (unless in the case of judgments *in materia necessaria*, for the expression of which subcontraries are inappropriate).

This law may be verified by assuming I as true, for example: A then remains doubtful; E is false; but from this we cannot infer that I is false: it may be true or false. This proof simply shows that the *Principle of Contradiction* does not apply to subcontraries, and that therefore they may be true together, since either does not go sufficiently far to give even a bare denial of the other.

Of course, whenever both *are* true together, the actual "some" referred to in each proposition is different, though there is nothing in the form to tell us this. In such a case, the "some" *actually means* "some, not all"; but we are not supposed to know this

[1] STOCK, *Deductive Logic*, p. 139.

owing to our indefinite interpretation of "some"; were we to interpret "some" as meaning "some, not all" we could lay down as the law of subcontraries that they *must* be either true together or false together.

Subcontraries might, then, be also generally defined as *propositions one of which* MUST *be true and both of which* MAY *be true.*

115. OPPOSITION IN SINGULAR AND IN MODAL CATEGORICALS: SUMMARY OF RESULTS.—(*a*) *Singular propositions* have no *formal* contraries, but only contradictories. "*Socrates is wise*" is contradicted by the simple denial, "*Socrates is not wise*". This opposition of singulars is called *secondary opposition.*

Of course, if we introduce a secondary quantification into a singular (94) by such qualifications as "always," "in all respects," etc., we can complete the square of opposition; but the proposition so treated ceases to be singular in the proper and strict sense of the word. If such quantification is implicit in a statement about a singular subject, it should be made explicit in order to avoid fallacies in opposition.

We may, of course, also find a *material* contrary for a singular proposition, by means of a "contrary" predicate if we can find such. A pair of such contraries would be "Socrates is happy" and "Socrates is miserable".

(*b*) *Modal categoricals* yield, of course, all the inferences of the ordinary square of opposition for assertoric propositions; the modal forms being respectively A, *S must be P;* E, *S cannot be P;* I, *S may be P;* O, *S need not be P.* We have dealt already with the import of these forms (89, 90).

(*c*) The following table summarizes all the inferences obtainable by the Square of Opposition:—

(1) *If* A *is true:* I is true, E is false, O is false;
(2) *If* A *is false:* O is true, E *is doubtful*, I *is doubtful;*
(3) *If* E *is true:* O is true, A is false, I is false;
(4) *If* E *is false:* I is true, A *is doubtful*, O *is doubtful;*
(5) *If* I *is true:* E is false, A *is doubtful*, O *is doubtful;*
(6) *If* I *is false:* O is true, A is false, E is true;
(7) *If* O *is true:* A is false, E *is doubtful*, I *is doubtful;*
(8) *If* O *is false:* I is true, E is false, A is true.

From this table it will be noticed that (*a*) positing a universal is the same as sublating the contradictory particular, and yields the

largest number of inferences [three]; (*b*) that positing a particular is the same as sublating the contradictory universal, and yields the smallest number of inferences [one only].

WELTON, *Logic*, i., pp. 228-48. KEYNES, *Logic*, pp. 109-24. JOSEPH, *Logic*, pp. 205-8. MELLONE, *Introd. Text-book of Logic*, pp. 70-78. JOYCE, *Logic*, pp. 84-90.

CHAPTER VI.

EDUCTIONS FROM CATEGORICAL JUDGMENTS AND PROPOSITIONS.

116. IMMEDIATE INFERENCE: KINDS OF EDUCTIONS.—We have distinguished between the *meaning* and the *implications* of a proposition (82). *Meaning* is that amount of knowledge understood to be directly and explicitly conveyed by the proposition. The further items of knowledge involved in, and derivable from this, are called *implications. The process of making explicit, by analysis, the various implications contained in any single proposition, is called* IMMEDIATE INFERENCE: to distinguish it from *Mediate Inference,* or *Reasoning* proper, which will be dealt with in Book III. The propositions which express those further items of knowledge are also called *immediate inferences* from the given proposition. Some of these inferences we have already reached and dealt with in examining the *Opposition of Propositions* (Chap. V.).

It will be noticed that propositions thus opposed had the same subject and predicate, differing only in quality, or quantity, or both. But, from any proposition given as true, we can in other ways —now to be explained—derive a much larger number of true propositions not having for subject and predicate exactly the same terms as the original, and in the same order; but those terms *or their negatives,* and in various orders. It is these new propositions especially that are commonly called "immediate inferences," rather than the forms reached by the square of opposition. But it is better to use the term "immediate inference" to cover the forms reached in the preceding as well as in the present chapter, and to use a special name for the inferences reached otherwise than by opposition. These have been appropriately called *Eductions* by some recent writers. The term has come into currency and we shall make use of it.

EDUCTION may therefore be defined as *the process by which, from*

any proposition, taken as true, we derive others implied in it, though differing from it in subject or predicate or both.

Since each of the two terms in a categorical proposition, S and P, has a conceivable negative, *non-S* and *non-P* (usually written $\overline{S}$ and $\overline{P}$), every categorical proposition suggests four terms, S, P, $\overline{S}$, and $\overline{P}$. We may, therefore, inquire how many legitimate predications may be made about S or $\overline{S}$ in terms of P or $\overline{P}$, and, *vice versa*, how many about P or $\overline{P}$ in terms of S or $\overline{S}$.

Supposing the subject of the original proposition to be S, we may conceive three other derived propositions whose respective subjects would be P, $\overline{P}$ and $\overline{S}$. And each of these four terms can have either of two alternative predicates: S and $\overline{S}$ may have each P or $\overline{P}$ for predicate; and P and $\overline{P}$ may have each S or $\overline{S}$ for predicate.

Thus, starting with the form

(1) S—P, we get
(2) its *obverse* . . . S—$\overline{P}$, by a process called *Obversion;*
(3) its *converse* . . P—S, „ „ „ *Conversion;*
(4) its *obverted converse* P—$\overline{S}$, „ obverting the converse;
(5) its *contrapositive* . $\overline{P}$—S, „ a process called *Contraposition;*
(6) its *obverted contrapositive* $\overline{P}$—$\overline{S}$, „ obverting the contrapositive;
(7) its *inverse* $\overline{S}$—P, „ a process called *Inversion;*
(8) its *obverted inverse* $\overline{S}$—$\overline{P}$, „ obverting the inverse.

It will be noted that the *converse*, *contrapositive*, and *inverse* have *positive* predicates; and the obverted form of each—as well as the obverse of the original—*negative* predicates.

We have, then, to examine four forms of Eduction, *viz. Obversion*, *Conversion*, *Contraposition*, and *Inversion:* and it will be found that the two latter depend on, and are only repeated applications of, the two former.

117. OBVERSION *is that process of immediate inference by which we infer from a given proposition another having for its subject the former subject and for its predicate the contradictory of the former predicate.*

The original proposition is called the *obvertend*, the inferred proposition the *obverse*.

This inference is always legitimate, provided we *change the quality* of the proposition in the process. Hence the *Law of Obversion: Negative the predicate and change the quality of the proposition, leaving its quantity unaltered.*

Hence A obverts to E; E to A; I to O; O to I.

We can pass backward from the obverse to the obvertend: they are *equivalent* propositions.

In applying the process of obversion to concrete examples, care must be taken not to use the material negative, or privative term (39, 41), instead of the formal contradictory, unless in cases when the former is equivalent to the latter. Thus, instead of *non-mortal*, *non-equal*, *non-material*, we may use the terms *immortal*, *unequal*, *immaterial*. But instead of the terms *non-happy*, *non-rich*, *non-holy*, *non-kind*, we may not use the terms unhappy, poor, unholy, unkind.

It may be asked whether obversion is really an *inference* at all, and not merely a *verbal change* (82, 106). We are commonly understood to have drawn an inference when we have made some *distinct step or advance in thought*, from our first judgment; when we have *made explicit* something that was not explicit, something that was *not part of the meaning* of our first proposition; when our second proposition can be said to express *a new truth*, or *a new judgment*, and not merely the original judgment in different words. This test is usually accepted in theory, but the application of it is often difficult: owing to the fact that the *same* judgment may be formulated in *different* terms; and often, conversely, *different* judgments may be expressed in *one and the same* (ambiguous) propositional form.[1] Judged by the principle in question, obversion can scarcely be said to be a real inference,—at least if the negative term be understood as purely negative (39). It is rather a verbal change. There is, in the process, no distinct, conscious advance of thought. Mere denial and affirmation are too closely allied to each other, too intimately involved in each other, to leave room for any real progress of thought in passing from the one to the other. If P and *not-P* are formal contradictories, then, by the *Principle of Contradiction*, we can pass from "S is P" to "S is not $\bar{P}$," from "Twice two and the half of eight are equal" to "Twice two and the half of eight are not unequal": where the latter proposition denies no positive alternative to the former, but merely denies the denial of the former; and, by the *Principle of Excluded Middle*, we can pass from "S is not P" to "S is $\bar{P}$," from "Steam is not visible" to "Steam is invisible": where the latter predicate gives no positive alternative to the former.

But if, in any judgment, we give *not-P* a *positive* connotation, by limiting P and *not-P* to a restricted universe, if, *e.g.*, we take P and *not-P* to mean *blue* and *some colour other than blue*, then we cannot pass from the proposition "Noble blood is not blue" to "Noble blood is not-blue" (meaning

[1] *Cf.* JOSEPH, *op. cit.*, p. 218.

"some colour other than blue"), unless on the tacit assumption that "Noble blood is either blue or *of some other colour*". Here, then, we have not obversion at all, but a real inference which is rather mediate than immediate, being in reality a mixed disjunctive or alternative syllogism (181)—an inference by which we "pass from a determinate positive predicate to another predicate less determinate but still positive".[1]

Similarly, if, in passing from "*S* is *P*" to "*S* is not *not-P*," we were to understand *not-P* as giving us something more than the mere denial of *P*, as giving us some *positive alternative* to *P*, we must know that this *not-P* is incompatible with *P*, before we may, by affirming one of the alternatives (*P*), infer the denial of the other (*not-P*). It is only when alternatives are mutually exclusive that we can, by positing one, sublate the others (182): "because we can predicate of a goose that it hisses, we are not precluded from applying any predicate but hissing".[2] And when we are precluded our reasoning is not obversion, but disjunctive inference based upon a suppressed alternative premiss. "This ink is black; therefore it is not not-black, *i.e.*, any colour other than black, *e.g.* red." This is a mediate inference, implying the premiss "The same thing cannot be both black and red".

We have already referred (97, 98) to the inconvenience of recognizing these obverse forms with "indefinite" or "infinite" terms for predicates, as independent types of proposition. But they are not devoid of meaning; and they are valuable as steps towards more important inferences.

Obverse forms are not alone—among the forms arrived at by the processes investigated in this chapter—in being unusual, strained, and violent modes of expressing truths that admit of straight and simple statement. But even such forms, though not in common use, are valuable as aids to understanding the full import and implications of the ordinary proposition.

The process of *Obversion* has been called by many other names: *Permutation* (Fowler, Ray, Stock, Joseph); *Æquipollence* (Ueberweg, Bowen, Ray, the Scholastics); *Infinitation* (Bowen); *Immediate Inference by Privative Conception* (Jevons); *Contraversion* (De Morgan); *Contraposition* (Spalding).

Professor Bain, who gave currency to the term *obversion*, distinguished between *formal* obversion (described above) and a process which he called *material* obversion. He gives as instances of the latter: "Warmth is agreeable, therefore, Cold is disagreeable"; "Knowledge is good, therefore, Ignorance is bad," etc. It is incorrect to call these *inferences*. The second proposition of each pair is suggested indeed by the first, but is by no means inferred from it. It is guaranteed only by a quite independent examination of the facts. Even if they were legitimate inferences, they would resemble *inversions* more closely than *obversions*.

118. CONVERSION *is that process of immediate inference by which from a given proposition we infer another having for its subject the former predicate and for its predicate the former subject.*

The original proposition is called the *convertend*, the inferred proposition the *converse*.

[1] JOSEPH, *op. cit.*, p. 221. [2] *ibid.*, p. 222.

Every proposition reached by the transposition of the terms of another may be called *in a wide sense* the converse of the latter. Thus "*All P's are S's*" would be, in this wider sense, the converse of "*All S's are P's*". This is also called the *geometrical* converse: because the propositions of geometry are, as a rule, thus *reciprocal*. But, taken in this wider sense, the converse of a true proposition need not be itself true: or, if true, its truth must be known independently of the convertend. It is only when the process of conversion is so conducted that the converse is *necessarily* involved in the convertend, that we have *illative* conversion, or *logical conversion* in the technical sense. And for the legitimacy of this process the two following rules must be observed:—

RULE OF QUALITY: *The quality of the proposition must remain unchanged;*

RULE OF QUANTITY: *No term may be distributed in the converse which was not distributed in the convertend.*

Applying these rules to the four traditional predicative forms, A, E, I, O, we obtain the following results:—

All S is P converts to *Some P is S.*
Some S is P „ „ *Some P is S.*
No S is P „ „ *No P is S.*
Some S is not P—has no converse.

In other words A converts to I; I converts to I; E converts to E; O does not convert at all.

About the *Rule of Quality* there can be no difficulty; for, since the same two terms are compared in convertend and in converse, it is clear that in order that the latter be true it must affirm or deny the connexion according as this was affirmed or denied in the convertend.

The reason for the *Rule of Quantity* is no less obvious. While unfolding what was implicit in the original proposition, we are obviously not at liberty to connect, in the converse, *any more* of the denotation of either term, *any greater portion* of either term, with the other, than was connected with this latter in the convertend. This is exactly what the *Rule of Quantity* lays down. If we bear in mind that "Universals distribute their subjects and particulars do not; while negatives distribute their predicates and affirmatives do not" (91),—we shall have no difficulty in working out the results set down above.

But the rule of quantity brings us face to face with the fact that, in the process of conversion, by making the predicate of the convertend the subject of the converse, we must turn from the *connotation* of this term to its *denotation;* from regarding it as the name of an *attribute* (or group of attributes), we must pass to regarding it as *the name of a class of objects;* from the *predicative or attributive* reading of the proposition, we must pass to its *extensive or class-inclusion* reading. It is this *change of standpoint* in reference to the predicate that involves *a distinct movement of thought*, and makes the conversion of the traditional predicative judgments, A, E, and I, rank as *a real process of interpretative inference.*

In the *equational* scheme of propositions (109), conversion is, of course, not an inference at all. But the process analogous to the contraposition of the predicative proposition—the process by which we pass from $S = SP$ to $\overline{P} = \overline{P}S$—is inference.

In the existential schedule, *SaP* is represented by $S\overline{P} = O$. Here the conversion of *SaP* to *PiS* is represented by the passage from "$S\overline{P} = O$," to "Either $SP > O$ or $S = O$"; and this is a process of inference. But the contraposition of *SaP* (to $\overline{P}a\overline{S}$)—which is a process of inference—is represented by the passage from "$S\overline{P} = O$" to "$\overline{P}S = O$," which process is only a verbal change: "Conversion, but not contraposition, now appears as a process of inference. It follows that there is inference when we pass to this schedule from either of the others, or *vice versa*." [1]

We have seen that obversion gives us what are really equivalent or equipollent propositional forms, rather than any new judgments deserving of the name of inferences. Some logicians contend that this is equally true of conversion; [2] and, consequently, of contraposition and inversion: which are nothing more than repeated applications of the two former processes.

Mr. Joseph's treatment of conversion [3] is instructive and deserving of notice. He contends that the conversion of A to I, if both propositions be understood as concrete or *historical* propositions, referring to actual individuals or groups of individuals (92, 93), or if both be understood as *scientific* or modal propositions, is not inference: inasmuch as we recognized and intended from the outset in the A proposition what is stated in the converse. That we *intended* in A what is stated in the converse, I, might perhaps be denied, if we draw a clear distinction between import and implication.

He admits that there is inference if one of the two propositions be understood as historical and the other as scientific, but denies that the inference is immediate: "Suppose the proposition 'All X is Y' to be understood historically, and the converse 'Some Y is X' scientifically; then there is inference. If in fact all the ruminants do part the hoof, then generally rumination is

[1] KEYNES, *op. cit.*, p. 423.
[2] *Cf.* J. S. MILL, *Logic*, II., i. 2.
[3] *op. cit.*, pp. 218 *sqq.*

compatible with a cloven foot. Set out in full, the argument would be that cows, and stags, and camels, and so forth, which ruminate, part the hoof, and therefore an animal that parts the hoof may ruminate. But the inference is no longer immediate. It is really in the third figure of syllogism."

"Similarly, if the convertend is understood scientifically, and the converse historically; because whatever ruminates parts the hoof, therefore any given animals which ruminate [if there be any such [1]] will do so, and they will be animals which exhibit both characters, so that some cloven-footed animals ruminate. This is also inference, though not immediate, for we are applying a general principle to particulars which fall under it, as in the first figure of syllogism."

He treats the conversion of I in a similar way. If convertend and converse be both historical, or both scientific, there is no inference. If the convertend be scientific and the converse historical, the passage from the former to the latter is not permissible [From "X may be Y" we cannot infer that "Some Y's are X's"]. If the convertend be historical and the converse scientific, there is inference, but it is through the third figure of syllogism [the "some" that are X being identical, as middle term, with the "some" that are Y].

The conversion of E he regards as involving real inference, whether convertend and converse be understood both scientifically, or both historically: "'No X is Y ∴ No Y is X,' understood scientifically, means 'If anything is X it is not Y ∴ If anything is Y it is not X'. This inference is the same as . . . [is] . . . found in the contraposition of A [119], and . . . in hypothetical reasoning. . . . But if the convertend be intended historically, we cannot infer the converse in its scientific intention. . . . On the other hand, let the convertend be understood scientifically, and the converse historically, and there will be inference . . . again, however, the convertend, as understood scientifically, fails to assert the existence of any actual cases".[2]

"The general result of our investigation is, that from the symbolic form of these processes [obversion, conversion, contraposition, etc.] it cannot be determined whether they contain any real inference or not; that where there is real inference it is either, as in the conversion of E and the contraposition of A, of the kind that we shall study in dealing with hypothetical arguments: or, as in the permutation of E and O, of the kind that we shall study in dealing with disjunctive arguments: or, as in the conversion of A and I, and that of O by negation, it involves suppressed syllogism. Immediate inferences, therefore, so far as they are inferences, are not a distinct kind of inference; so far as they seem distinct and specially unquestionable, it is because they merely bring out another aspect of what we have already intended in a proposition, without any fresh step in thought. This result may throw some doubt upon the appropriateness of the name by which they have come to be known".[3]

Is the legitimacy of the process of conversion self-evident, or does it need

[1] The *form* of the proposition "Whatever is X is Y" does not guarantee the actual existence or occurrence of X or Y in the universe of discourse. *Cf.* the statement of the first law of motion, *infra*, 128.

[2] *ibid.*, p. 220. [3] *ibid.*, p. 223.

proof? Its legitimacy is self-evident. It is the immediate application of the Principle of Identity [1] to the concepts of which our judgments are composed. These concepts, each with its two aspects of meaning, intensive and extensive, afford us different views of the same thing in the process of judgment, and thus constitute that diversity within which alone we can discern identity (12). What justifies the process of conversion is the consciousness that we are looking throughout at *the same reality* under different manifestations.

Appeals have been made to Euler's diagrams [2] to illustrate and justify conversion and the other forms of immediate inference.

Aristotle's attempt to prove, or rather, to illustrate indirectly, the process of conversion, by showing the absurdity that would result from denying its legitimacy—succeeds rather in showing that the legitimacy of the process cannot be proved without assuming it in the proof; in other words, that the validity of the process is too evident to need proof.

He argues thus: *No S is P*, therefore *No P is S;* for if not, *Some individual P, say Q, is S;* and hence *Q is both S and P*; but this is inconsistent with the original proposition.[3] That is quite true; but it assumes the equivalence of the propositions "*S is Q*" and "*Q is S*"—which, after all, is the point to be proved.

"*Conversio per Accidens*" (of A). We cannot pass legitimately from "*All S's are P*" to "*All P's are S*," because the former proposition gives us information only about an indefinite "some" of the *P*'s, and, therefore, does not empower us to make any assertion about *all the P's* but only about the indefinite "some". Our inference, therefore, must be about "some *P*'s". In other words, from a universal affirmative (A) we can reach by conversion only a particular affirmative (I)—"*Some P's are S's*—a proposition which is not *equivalent* to the convertend, and from which we cannot get back by conversion to the convertend: thus showing that we have lost some of the meaning of the convertend in our passage from this to the converse.

Conversion of this kind—in which we infer only a particular from a universal, in which the quantity of the convertend is not retained but depressed, in which there is a loss of import in the passage from convertend to converse—is called *Partitive Conversion; Conversion by Limitation* (κατὰ μέρος); or, again, *Conversio per accidens* (κατὰ συμβεβηκός).

It is called "partitive" or "by limitation" because we can predicate *S* only of an indefinite *portion* of *P*. It is called

[1] Those who maintain that whatever of real inference there is in conversion is syllogistic, or hypothetical, would hold that its legitimacy is embodied in the self-evident axioms in which such reasonings may be formulated (*cf.* 151, 152).

[2] *Cf.* KEYNES, *op. cit.*, pp. 131, 157 *sqq.* [3] *ibid.*, p. 130.

"accidental," "*per accidens*," because "the relation of a predicate to its subject may be either accidental or essential. It must at least be accidental, and therefore from its bare form we are entitled to convert an A proposition as if *P* were an accident of *S*; but we are not entitled to do more . . . if *P* is an accident of *S*, *i.e.* coincides in the same individual subject with *S*, then *S* is predicable of a subject which *P* characterizes, and we may say that some *P* is *S*."[1]

The conversion of an A proposition without limitation or depression of the quantity, is one of the commonest of all fallacies; and it is particularly liable to be incurred in dealing with indesignate propositions (96). Thus, it is wrongly inferred that because clever people have large brains all who have large brains are clever; that because idlers are commonly out of work all unemployed people are idlers; that because pious people go to church all church-going people are pious; that because all beautiful things are agreeable all agreeable things are beautiful.[2]

Of course, in some cases the *universal* (*i.e.* simple) converse of an A proposition may be true: when the latter is a *reciprocal* proposition, *i.e.* one in which *S* and *P* agree in connotation and in denotation; but we cannot know or infer this from the information given us by the *form* of the convertend: we must know it otherwise. The proposition "All equilateral triangles are equiangular," though true, cannot be *formally* inferred from "All equiangular triangles are equilateral". Were we given the *U* proposition (105, 108), "All equiangular triangles are *all* equilateral triangles," we should have the proposition "All equi-

[1] JOSEPH, *op. cit.*, p. 212 (we have changed the symbols *X* and *Y*, used by the author). He adds the following important and instructive note: "Even when the predicate is known to be of the essence of the subject, we must convert *per accidens* if the predicate is the genus: *e.g.* "all men are animals"—"some animals are men". We cannot call animal an accident of man, but we may say that it is an accident that an animal should be a man, in this sense, that the conditions necessary to the generation of an animal must *coincide* with the conditions necessary for the generation of a man, if the animal is to be a man. The expression *coincide* is not strictly suitable (nor therefore can the relation of man to animal be strictly called accidental), because it is only in thought that the conditions necessary to the generation of an animal can be separated from the special conditions necessary for the generation of some particular species: there is no coincidence of *independent* series, as when one series of events brings a train to a point whither another series has brought a flood and washed away the metals, and the result is a railway accident." But the usage is analogous in both cases. Their consideration will recur in connexion with the concept of chance (264) and with the fallacy of the *Accident* (274, *d*).

[2] WELTON, *op. cit.*, p. 257; MELONE, *op. cit.*, p. 83.

lateral triangles are equiangular" contained in it without any inference.

Simple Conversion (of E *and* I). *Simple Conversion*, as opposed to *Conversion by Limitation*, is that in which *the converse retains the quantity as well as the quality of the convertend.* The *simple* converse is therefore *equivalent* to the convertend: we may pass back from converse to convertend: no part of the import is lost in either process.

The E proposition can be converted *simply:* for it asserts that the attributes connoted by *P* are to be found in none of the objects denoted by *S*, and that therefore none of the objects in which *P* is found, none of the objects denoted by *P*, possess the attributes connoted by *S*. In other words it distributes its predicate, *P*, thus totally separating the things that possess the attribute *S* from those that possess the attribute *P*. If "*No squares are circular*," then "*No squares are any circles*": and hence "*No circles are square*," or "*No circles are* (*any*) *squares*". Thus *SeP* converts *simply* to *PeS*.[1]

Similarly, the particular affirmative, *SiP*, which distributes neither of its terms, may be *simply* converted to *PiS*, which also distributes neither of its terms. Both propositions are *equivalent:* we may pass from either to the other without any loss of meaning: the process is reciprocal.

The conversion of O is impossible. *SoP*, if converted, must convert to a negative (by the *Rule of Quality*): but the negative will distribute its predicate, which will be *S:* this, however, is impossible (by the *Rule of Quantity*), seeing that *S* was not distributed in the convertend, *SoP*.

The student should never attempt to convert a proposition without first (1) reducing it to logical form, determining (2) the quantity of the subject, and (3) the quantity of the predicate. And he should remember that the logical copula can be only *is* (not), or *are* (not).

It is easier to convert propositions which have substantives for predicates than those which have adjectives. In converting the latter we must supply the appropriate substantive—the next highest genus of the subject—or change the adjective into a substantive. Thus: *All men are mortal—Some mortals are men.* Verb-predicates must be resolved into the copula and a participle,

[1] *Cf. infra*, chap. vii., for influence of suppositions regarding existential import on the doctrines of the present chapter.

or substantive with relative clauses, before converting. Thus "*Americans travel much*"—"*Some who travel much are Americans*," or "*Some great travellers are Americans*".

Owing to the difficulty of formulating some sorts of judgments in the *subject-copula-predicate* form, and to the further difficulty of quantifying their terms when thus formulated, the best results we can arrive at by conversion are often awkward, artificial, and cumbersome. But the exercise is an extremely useful one for teaching us how to *interpret* judgments and propositions; and, moreover, we can avoid much uncouthness of expression by making use of the right that logic allows us to *vary the form of expression as much as we please provided we retain the meaning intact.*

At the same time, it must be remembered that some judgments can be forced into the traditional fourfold scheme only with considerable difficulty, and that even then they absolutely resist conversion. That is to say, their converses are unnatural judgments. This will be obvious if we try to convert such judgments as: "Some men have not the courage to appear as good as they are," "We cannot all command success," "It is raining," "Ireland is an island".

The last of these examples is an instance of the singular proposition with a general term for predicate:[1] such a proposition may of course be converted *per accidens*, as an A proposition, but the converse, "Some island is Ireland," is an unmeaning form, for we cannot predicate a singular term, "Ireland," of a general term, "island". Properly speaking, we cannot predicate a singular term at all—even of another singular term. If there is real predication in such a proposition as "The elder Pitt is Chatham" (or its converse), what we mean to assert is that the individual denoted by the subject-name *possesses the attribute* of being also *designated by* the predicate-name: that "The elder Pitt is *called* or *named* Chatham".

Again, it is *only men* who are thought of in the other examples as having or not having the courage to appear good, of being able or unable to command success; and it is therefore unmeaning to say that some who possess or do not possess such attributes "are men": "we do not predicate of an attribute *partially* the subject presupposed by it," as Mr. Joseph[2] observes in reference to the converse of "Some men are Christians".

These difficulties, in the application of such formal processes as conversion to our actual judgments, arise from the simple fact that in the latter the matter is really inseparable from the form, and this matter refuses the new forms which these inferential processes would try to force upon it. "It would

[1] Singular propositions with *singular predicates* convert *simply*—for the good reason that they cannot be converted in any other way;—though for general purposes they are classified as A propositions. "Maynooth College is the largest ecclesiastical college in the world" converts to "The largest ecclesiastical college in the world is Maynooth College".

[2] *op. cit.*, p. 214.

be absurd to say that because conversion is a strictly formal process, we must therefore convert propositions by its rules, according to their forms as A, E, or I. Logic investigates the actual nature and procedure of our thought; and when we find that our thought is not governed by the bare form of a judgment irrespective of its contents, it is no use to pretend otherwise. . . . This is said not in order to discredit the abstract and formal treatment of conversion, which is sound within its limits; but in order to emphasize the fact that the form and matter (or the form and content) of thought are not capable of separate consideration, like the mould and the pudding: what from one point of view is form is from another matter, and the same form in different kinds of content is not altogether the same, any more than is the same genus in different species. The importance of this fact must excuse the reiteration of it; meanwhile in a text-book of Logic, as of any other science, we must consider typical cases, with a general *caveat* that the subject is thereby artificially simplified." [1]

The following examples will help to illustrate the foregoing paragraphs, and to disclose some possible pitfalls to the beginner: "All water contains air," does not convert to "All air contains water," or to "Some air contains water," or to "Some air is contained by water," but to "Something that contains air is water". "He jests at scars who never felt a wound" does not convert to "Some scars are jested at by him who," etc., or to "Some scars are jests to him who," etc., or to "Some jests at scars are made by him who," etc., or to "Some scars jest at him who (!)," etc., but to "Some who jest at scars are people who have never felt a wound".

"John struck James" does not convert to "James struck John," but to "Some person who struck James is John". "The poor have few friends" converts to "Some who have few friends are (the) poor". "Great is Diana of the Ephesians" converts to "Some great being is Diana of the Ephesians". "A stitch in time saves nine" does not convert to "Nine stitches save a stitch in time (!)," or even to "Nine stitches are saved by a stitch in time," but only to "Some (one) way of saving (*or* something that saves) nine stitches is a stitch in time". "Whatever is immortal is not material" converts to "Whatever is material is not immortal". "*Non omnis moriar*" means "Some part of me is immortal," and converts to "Something immortal is part of me". "All men have not faith" means "Some men have not faith," and is therefore inconvertible. Similarly "All is not gold that glitters" means "Some glittering things are not gold," and is inconvertible. "Not everyone that saith unto me, Lord, Lord, shall enter into the kingdom of heaven" means "Some who say unto me, Lord, Lord, shall not enter the kingdom of heaven," and is inconvertible. But these O propositions all yield *contrapositives*, as we shall see presently (119). We can, for example, infer from the last-mentioned proposition "Some who shall not enter into the kingdom of heaven are people who say unto me Lord, Lord".

"Life every man holds dear" does not convert to "Some life is held dear by," etc., or to "No man holds death dear" (!), but to "Something (*or* one of the things) held dear by every man is life".

"No triangle has one side equal to the other two" converts to "No plane figure having one side equal to the other two is a triangle".

[1] *op. cit.*, p. 214.

"No woman is now hanged for theft in England" converts to "No one now hanged for theft in England is a woman".

These last two examples are peculiar: they suggest questions about the sphere of reference of the judgment, and about the existence of the things spoken of, in that sphere. These questions will recur later on (Chap. VII.).

Exclusive and *exceptive* propositions, such as "None but the brave deserve the fair," when resolved into their constituents, yield (1) an I proposition, "Some brave deserve the fair," which may be converted simply; and (2) an E proposition, "No non-brave deserve the fair," which also converts *simply* to "None who deserve the fair are non-brave". This latter obverts to "All who deserve the fair are brave," which gives (1) as its converse, and which is itself the simple predicative form that represents most fully the force of the original exponible.

119. CONTRAPOSITION *is that process of immediate inference by which from a given proposition we infer another having for its subject the contradictory of the original predicate.* This definition leaves it an open question whether the *predicate* of the inferred proposition will be the original subject or its contradictory. It may be either: for each of these forms [$(\bar{P}—S)$ and $(\bar{P}—\bar{S})$] is the obverse of the other.

We arrive at both forms indirectly—by a successive application of *obversion* and *conversion*. Each form makes an assertion about the *contradictory of the original predicate*. This contradictory term can be reached only by obverting the original proposition; and it can be transposed into the place of the subject if this obverse proposition can be converted. Thus SaP obverts to $Se\bar{P}$: and this latter converts to $\bar{P}eS$—which is the first contrapositive form given above. By obverting this again we get $\bar{P}a\bar{S}$, the second form given above.

Some logicians call the former of these two the *contrapositive*, seeing that it is reached first; and the latter the *obverted contrapositive*. Others, however, looking upon contraposition as a species of conversion—calling it *conversion by contraposition*, or *conversion by negation*—and seeing that ordinary conversion *does not change the quality* of the original proposition, fix upon the second and more symmetrical form $(\bar{P}—\bar{S})$ as the contrapositive. Modifying somewhat a suggestion of Dr. Keynes,[1] we will call the former $(\bar{P}—S)$ the *partial* or *simple* contrapositive, and the latter $(\bar{P}—\bar{S})$ the *full* or *obverted* contrapositive.

Hence the following rule:—

[1] *op. cit.*, p. 135.

For the partial *or* simple *contrapositive, first obvert, then convert*; *for the* full *or* obverted *contrapositive, obvert again.*

Applying this process to A, E, I, O, respectively, we have the following table :—

Original proposition	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
Obverse	$S\ e\ \overline{P}$	$S\ a\ \overline{P}$	$S\ o\ \overline{P}$	$S\ i\ \overline{P}$
Partial contrapositive	$\overline{P}\ e\ S$	$\overline{P}\ i\ S$	None	$\overline{P}\ i\ S$
Full contrapositive	$\overline{P}\ a\ \overline{S}$	$\overline{P}\ o\ \overline{S}$	None	$\overline{P}\ o\ \overline{S}$

We saw that the O proposition could not be converted : we now note the interesting fact that it *can be contraposed*, whereas the I proposition, which can be converted, *cannot be contraposed.*

We note, secondly, that the contraposition of A and of O is *simple, i.e.* unaccompanied by a loss of quantity ; whereas E, losing its quantity, may be said to suffer contraposition *per accidens* or *by limitation.*

The comparative scarcity of formally negative terms makes the expression of the contrapositive somewhat awkward in the case of categorical propositions. We simplify matters somewhat by substituting for "not-P". "Whatever (whoever) is (are) not P". Thus, "All S's are P" : "Whatever is not P is not S". This mode of inference is of very common occurrence ; but it is more usually made in the hypothetical (139) than in the categorical form. The strict categorical universal "All S is P," involves that "If anything is S it is P," from which we infer that "If anything is not P it is not S," which is another and better way of stating that "All not-P's are not-S," or "No not-P's are S". From "All ruminants are cloven-footed," it follows by contraposition that "If an animal is not cloven-footed it is not a ruminant". The corresponding inference from the E proposition, "No S is P" ("If anything is S it is not P"), would be "If anything is not P it (may be or) need not be S," which is the modal hypothetical equivalent of "Some not-P is not S". From "No clergymen are members of Parliament," it would follow by contraposition merely that "If anyone is not a member of Parliament he (may be or) need not be a clergyman". The O proposition gives a similar contrapositive.

The recognition that "*Whatever is S is P*" formally involves "*Whatever is not P is not S*"—independently of the matter or meaning of the terms of these propositions—"renders unnecessary the special proofs that Euclid gives of certain of his theorems. . . . It will be found that taking Euclid's first book, proposition 6 is obtainable by contraposition from proposition 18, and 19 from 5 and 18 combined ; or that 5 can be obtained by contraposition from 19, and 18 from 6 and 19. Similar relations subsist between propositions 4, 8, 24, and 25, and again, between axiom 12 and propositions 16, 28, and 29."[1]

In physical induction, in which we seek to establish general physical

[1] KEYNES, *op. cit.*, p. 136.

laws from analysis of particular facts, the contrapositive of the universal proposition is of importance. For, in order to establish the truth of the ideally scientific or reciprocal universal (92, 138), we must know that P is an *inseparable natural property of* S *alone*, so that not merely all the S's are P's ($S\ a\ P$), but that furthermore all the things possessing the property (or properties), P, are S's ($P\ a\ S$). Hence, after having satisfied ourselves by *positive observations and experiments* that "*All S's are P*" ($S\ a\ P$), we may aim at proving that "*All P's are S*" ($P\ a\ S$). Now this latter is most easily proved by *proving its contrapositive*, namely, that *No non-S's are P*, or, *None but S's are P* ($\overline{S}\ e\ P$). This is done by instituting a series of *observations and experiments on negative instances*, to show that wherever S is absent P is absent (*cf.* 221-2 ; 229-30).

120. INVERSION *is that process of immediate inference by which from a given proposition we infer another having for its subject the contradictory of the original subject.*

The original proposition is called the *Invertend*, the inferred proposition the *Inverse.*

Here, as in contraposition, we may arrive at two forms, one the obverse of the other : the one with P for predicate we will call the partial inverse ; the one with $\overline{P}$ for predicate the full inverse.

Here, too, as in contraposition, the forms we seek can be reached only by a repeated, combined application of obversion and conversion. For, since the only way to get the contradictory of a term in a proposition is to make it predicate and then obvert the proposition, it is plain that, starting with S as subject, we must make it predicate by conversion, then make it $\overline{S}$ by obversion, and finally transfer this latter to the position of subject by another conversion.

Let us take the four traditional propositions in turn and see what results we shall reach, first by commencing (*a*) with conversion, then by commencing (*b*) with obversion, and applying each alternately. The ineffectual attempts to reach an inverse will be enclosed in brackets.

(1) *Inversion of* A. [(*a*) $S\ a\ P$ converts to $P\ i\ S$ which obverts to $P\ o\ \overline{S}$—*which cannot be converted.*] (*b*) $S\ a\ P$ obverts to $S\ e\ \overline{P}$, which converts to $\overline{P}\ e\ S$, which obverts to $\overline{P}\ a\ \overline{S}$ *which converts to* $\overline{S}\ i\ \overline{P}$, *which obverts to* $\overline{S}\ o\ P$ *:* thus giving the two desired inverses, $\overline{S}\ i\ \overline{P}$ and $\overline{S}\ o\ P$.

(2) *Inversion of* E. (*a*) $S\ e\ P$ converts to $P\ e\ S$, which obverts to $P\ a\ \overline{S}$, *which converts to* $\overline{S}\ i\ P$, *which obverts to* $\overline{S}\ o\ \overline{P}$

—the two desired inverses. [(*b*) $S\ e\ P$ obverts to $S\ a\ \overline{P}$, which converts to $\overline{P}\ i\ S$ which obverts to $\overline{P}\ o\ \overline{S}$—*which cannot be converted.*]

(3) *Inversion of* I. [(*a*) $S\ i\ P$ converts to $P\ i\ S$, which obverts to $P\ o\ \overline{S}$—*which cannot be converted.*] [(*b*) $S\ i\ P$ obverts to $S\ o\ \overline{P}$—*which cannot be converted.*]

(4) *Inversion of* O. [(*a*) $S\ o\ P$ *cannot be converted.*] [(*b*) $S\ o\ P$ obverts to $S\ i\ \overline{P}$, which converts to $\overline{P}\ i\ S$, which obverts to $\overline{P}\ o\ \overline{S}$—*which cannot be converted.*]

Hence we see, firstly, that *only universal propositions yield inverses: particulars do not.* We see, secondly, that in order to invert A we must begin by *obverting;* that in order to invert E we must begin by *converting;* that in all other cases we are arrested by the appearance of an O proposition for conversion.

Hence the rules for inversion:—

(1) *Convert the obverted contrapositive of* A.

(2) *Convert the obverted converse of* E.

We see, thirdly, that the full inverse comes first in inverting A, that it comes after the partial inverse in inverting E; that in both cases it is *of the same quality* as the original proposition; and that all inversion involves a *depression of quantity* from universal to particular.

In inversion, as in contraposition, the passage of thought is often through hypothetical rather than categorical judgments: from "If anything is S it is P" to "If anything is not S it may be or need not be P" (137, 139).

Though not a very common form of inference, inversion puts us on our guard against the not infrequent fallacy of inferring from "*Whatever is S is P*" that "*Whatever is not S is not P*". We are entitled to infer only that "*Something that is not S is not P*". "If all triangles are plane figures, what information, if any, does this proposition give us concerning things that are not triangles?"[1] This question simply asks for the inverse of the proposition "*All triangles are plane figures*". The answer therefore is: "*Some things that are not triangles are not plane figures*".

A glance at this example may suggest a doubt about the validity of the process of inversion. How can we reach a proposition in which the term "plane figures" is distributed, from one in which it is undistributed? Or, generally, how can we validly pass from "*All S is P*," in which P is undistributed, to "*Some non-S is not P*" in which P is distributed? And, moreover, how do we know from the original proposition that there are in the universe of discourse any *non-S's—e.g.* any "things that are not

[1] Jevons, *Studies in Deductive Logic*, p. ix.; *cf. ibid.*, p. 48.

triangles"? The answer is that unless we assume this latter as given with the original proposition our inference should be stated *hypothetically*, "Some non-triangles, *if there be any such things*, are," etc.; and, furthermore, that unless we assume as also given that *there are things that are not P*, inversion is not a valid process.[1] It is from the original proposition *understood as accompanied by this assumption*, that the inverse is derived; this tacitly assumed proposition, "*Some things are not P*," with *P* distributed, justifies the distributed *P* in the inverse. The validity of the inversion of A is therefore contingent on the existence of $\overline{S}$ and $\overline{P}$ in the universe of discourse. It will be found, similarly, that the validity of the *inversion* of E is dependent on the existence of $\overline{S}$ and *P*; that of its *conversion* on the existence of *P*; that of the *contraposition* of A on the existence of $\overline{P}$.

Thus we see that the inferences we have reached in the present chapter are based on the assumption that *S*, *P*, $\overline{S}$ and $\overline{P}$ represent classes actually existing in the universe referred to by the original propositional form, *S*—*P*. How these results will be modified by other assumptions we shall see in Chap. VII.

121. SUMMARY OF RESULTS: TABLE OF EDUCTIONS.—It will be noticed from the subjoined table that universal propositions yield seven eductions each, and particulars three each; that we can infer from the former in terms of *S*, $\overline{S}$, *P* and $\overline{P}$, from the latter in terms of *S* and of either *P* or $\overline{P}$; that the converses of A and I are the same, and the contrapositives of E and O the same.

		A	E	I	O
1	Original proposition	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
2	Obverse	$S\,e\,\overline{P}$	$S\,a\,\overline{P}$	$S\,o\,\overline{P}$	$S\,i\,\overline{P}$
3	Converse	$P\,i\,S$	$P\,e\,S$	$P\,i\,S$	
4	Obverted converse	$P\,o\,\overline{S}$	$P\,a\,\overline{S}$	$P\,o\,\overline{S}$	
5	Partial contrapositive	$\overline{P}\,e\,S$	$\overline{P}\,i\,S$		$\overline{P}\,i\,S$
6	Full contrapositive	$\overline{P}\,a\,\overline{S}$	$\overline{P}\,o\,\overline{S}$		$\overline{P}\,o\,\overline{S}$
7	Partial inverse	$\overline{S}\,o\,P$	$\overline{S}\,i\,P$		
8	Full inverse	$\overline{S}\,i\,\overline{P}$	$\overline{S}\,o\,\overline{P}$		

122. MATERIAL EDUCTIONS.—The inferences we have been discussing so far depend largely on the *formal* elements—the quantity and quality—of the proposition, and have, therefore,

[1] For example, the inverse of the proposition "All future free acts are foreseen by the Deity" would be "Some things that are not future free acts are not foreseen by the Deity". But this is valid only on an implied assumption which is false, *viz.* that God does not foresee all things.

when properly made, the same absolute cogency as the fundamental laws of thought on which they are based. There are, however, some other less important classes of inferences whose validity will depend not upon the form of the propositions given, but rather upon the meaning we attach to their terms. In the case of such inferences, it is not easy to preserve throughout the process the same force or meaning in the terms used, and so to avoid falling into fallacies.

(*a*) INFERENCE BY ADDED DETERMINANTS *is the process by which we infer from a given proposition another proposition of narrower application by limiting the terms of the original proposition in an identical manner.* This is done by adding to the subject and predicate of the original proposition some qualification which *does not belong to these in their whole denotation*, and which, therefore, *limits*, or *determines*, or *narrows*, their application in an identical manner. The added qualification is called a *Determinant.*

Thus, from "All negroes are men" we infer that "All *honest* negroes are *honest* men"; from "Poetry is good for the imagination" that "*Good* poetry is *good* food for the imagination"; from "Wrongdoers are deserving of punishment" that "*Female* wrongdoers are *females* deserving of punishment": and, in general, from "*All P is Q*" to "*All AP is AQ*".

But the *added determinant* must retain exactly the same meaning in subject and in predicate: it must be *really* and not only *apparently* the same term, the same "A," throughout. Now it is precisely because this cannot always be secured that inferences of the class we are considering are so often fallacious. Terms *involving a comparison* are more especially liable to change their force according to the nature of the class of things they qualify.

Hence, we cannot infer from "An elephant is an animal" that "A *small* elephant is a *small* animal," but only that it is *small for an elephant;* nor from "An ant is an animal" that "A *large* ant is a *large* animal," but only that it is *large for an ant;* nor from "A bass singer is a man" that "A *bad* bass singer is a *bad* man," but only that he is *bad as a bass singer;* nor from "The army is worn out with fatigue" that "Half the army is half worn out with fatigue" but that it is *half of the body worn out*, etc. (for the determinant qualifies the *denotation*, not the *connotation*, of the terms to which it is added).

Sometimes the subject and predicate of one proposition serve as equivalent determinants of the subject and predicate of another proposition. When this is the case a third proposition may be inferred from the combination of the two former. Thus, from "Theft is deserving of punishment" and "Unemployed workmen are poor," we may infer that "Unemployed workmen who steal are poor men who deserve punishment".[1]

(*b*) INFERENCE BY COMPLEX CONCEPTION *is the process by which we combine the subject and predicate of a given judgment with some third concept in order to form a new judgment with the complex concepts thus obtained.* In the previous kind of inference the subject and predicate of the original proposition were determined by the third term; here they rather determine the third term.

For example, from "Arsenic is poison" we infer that "A dose of arsenic is a dose of poison"; from "A dog is a quadruped" that "The head of a dog is the head of a quadruped"; from "Poverty is a temptation to crime" that "The removal of poverty is the removal of a temptation to crime". But here, again, we cannot infer from "Judges are lawyers" that "A majority of judges are a majority of lawyers"; nor from "A sheep is not a dog" to "The owner of a sheep is not the owner of a dog".

(*c*) IMMEDIATE INFERENCE BY CONVERSE RELATION[2] *is the process by which we infer from any relation between one object and another the corresponding relation between the latter and the former.* For example, "*A* is greater than *B*, therefore *B* is less than *A*"; "Alexander is the son of Philip, therefore Philip is the father of Alexander"; "Belfast is north of Dublin, therefore Dublin is south of Belfast"; "John arrived before James, therefore James arrived after John".

These relations belong to the "Logic of Relatives," and are not analysed in ordinary logic, which is supposed to confine itself to such relations, between objects of thought, as can be expressed by the logical copula *is* (*not*), *are* (*not*).

WELTON, *Logic*, i., pp. 248 *sqq.* KEYNES, *Formal Logic*, pp. 126 *sqq.* pp. 420-423. JOSEPH, *Logic*, pp. 209 *sqq.* JOYCE, *Logic*, pp. 92 *sqq.*

[1] WELTON, *op. cit.*, p. 269. [2] KEYNES, *op. cit.*, pp. 149-51.

CHAPTER VII.

EXISTENTIAL IMPORT OF CATEGORICAL JUDGMENTS AND PROPOSITIONS.

123. REFERENCE OF TERMS AND PROPOSITIONS TO A SPHERE OF OBJECTIVE REALITY: THE POSSIBLE AND THE ACTUAL.—We have already made passing references to the question whether or how far the categorical judgment or proposition implies the actual existence of the objects denoted by its subject or predicate in the realm to which the judgment refers. At this stage an investigation of the question will reflect some light upon certain interpretations of the judgment (80, 109), and upon some processes of Immediate Inference (111 n. 1; 120); while it will serve as a natural transition to the treatment of *Hypothetical* and *Disjunctive* judgments.

It is important to understand clearly the meaning of the present inquiry. We have already seen that every judgment must refer to some *objective sphere* of reality, *i.e.* to some realm over and above the subjective, passing thought of the individual thinker's mind, to some universe of discourse in which the claim of the judgment to truth may be checked and guaranteed (80). Every such sphere is called *objective reality*, in the sense of being a something beyond the *subjective thought* of the individual thinker.[1] In this sense every judgment must *refer* us to some portion or other of *objective reality;* and this latter might thus be rightly said to constitute the *ultimate subject* of all our judgments. We have referred also (80) to a certain definition of the act of judgment which would even make all reality the *logical* subject of every judgment. This merely exaggerates the truth that every judgment makes a predication *which* REFERS US *to some objective sphere or other which is a portion, at least, of all*

[1] Of course if the individual, by a process of psychological reflection, thinks and judges about the present current of his own thoughts, the "objective sphere" will be the sphere of his own thoughts considered as objects; but even then, these will be "objective" to his acts of reflex judgment.

conceivable reality. In this sense every judgment implies the existence or reality of its *ultimate* subject.[1]

Indeed, not only does every judgment thus refer us to an objective sphere, but the intelligible use of any *logical term* whatever, in human discourse, refers speaker and hearer alike to some such sphere, called the *appropriate realm* (33), in which the objects or attributes denoted or connoted by the term may be supposed to occur or not to occur—or to some restricted portion of such a realm. The sphere of *actual* reference, whether thus restricted or not, is called, as we have seen, the *universe of discourse* (*ibid.*).

Now, in the case of any term used in a judgment, this universe may be (1) the *actual* visible universe of things past, present, and future, that can come directly into our sense experience: the universe from which we get the raw materials (1, 2, 4) of *all* our intellectual concepts. Or it may be (2) the spiritual universe of suprasensible realities which most people believe to be *actual*, but which for unbelievers resolves itself into a sphere of *actual* beliefs (*viz.* of the believers). Or it may be (3) some universe that is *actual* in the sense of being actually invented or produced by the mental activity of man, such as the plays of Shakespeare, or the literature of heraldry, or the collections of oral or written traditions or beliefs which constitute the folklores of the various nations.[2]

Those various spheres are all portions of *actual* reality; and in order that the terms used in a judgment refer to any one of them, the objects or attributes signified by those terms must be at least *capable* of existing in some one or other of those actual spheres. In other words, the objects and

[1] Hence the special appropriateness of the verb of existence, the substantive verb "to be," for expressing the function of predication in the act of judgment, seeing that this act does in some true sense always assert existence or reality. *Cf.* JOSEPH, *Logic* (p. 147): "the case seems to be thus: that every judgment does imply existence, but not necessarily the existence of the subject of the sentence. The distinguishing characteristic of a judgment is, as we have seen, that it is true. . . . All judgments besides affirming or denying a predicate of a subject, affirm themselves as true. But a judgment which affirms itself as true claims to express, as far as it goes, the nature of things, the facts, or the reality of the universe. In doing this it may be said to imply existence, not of its grammatical subject, but of the whole matter of fact expressed in it."

[2] "The universe of the Greek mythology does not consist of gods, heroes centaurs, etc., but of accounts of such beings currently accepted in ancient Greece, and handed down to us in Homer and other authors. . . . The universe of folklore does not consist of fairies, elves, etc., but of descriptions of them based on popular beliefs, and conventionally accepted when such beings are referred to. Of course for anyone who really believed in the existence of fairies . . . the universe of discourse would be different."—KEYNES, *op. cit.*, pp. 213-14.

attributes of which we think must be *possible*, *conceivable*, *thinkable*, free from logical contradiction. *They must belong to the sphere of* OBJECTIVELY POSSIBLE *things*. Here is a kind of "existence," namely "logical existence" or "conceivability," which is inseparably bound up with every term, and therefore implied in every judgment.

Every judgment, therefore, implies the actual existence of the objects or attributes denoted or connoted by its terms, *in the sphere of logical conceivability*. Nor do even such judgments as "*Round squares are impossible*" form a real exception to this statement; for they do not deny the existence of the objects or attributes "squares, circles, round," etc., but only of *attempted combinations* of the latter, within the sphere of logical possibility. In order that a number of attributes be synthesized by the individual mind so as to form one object of thought, and so exist *as one object* in the sphere of objective possibility, they must be mutually compatible, free from mutual contradiction. Hence, although all our concepts, without exception, refer us to some portion or other of the actual, physical universe—whence we derived them through our senses in the first instance (1, 2, 4)—still, when used in judgments, they sometimes refer us more directly, for the verification of the latter, to the sphere of *objective possibility*. For example, the concepts "matter," "spirit," "square," "circle," "quadruped," "two-legged," "twice," "four," "ten," are all derived from our experience of the *actual* physical universe in which we live; but it is directly to the sphere of *what is possible* that we are referred by the judgments "matter and spirit are identical," "no square can be circular," "some quadrupeds are two-legged" "twice four are ten," for the grounds of their truth or falsity.

This sphere—of the *purely possible*—is conceived by us to comprehend the ideal types (whether known or unknown to us) of every thought-object that is capable of existing in any department of *actual* reality. It is a *real* sphere, in the sense that it is not a mere creation of our mental activity. Possible things are possible, not because we conceive them in our thought; but, rather, we can conceive them in our thought because they are possible. Why they are possible, what makes them possible, what is the ultimate ground of their possibility—is not for logic, but for metaphysics, to decide. At all events, what guides us in determining what is intrinsically possible or conceivable, and what not, is our acquaintance with *actual* reality. And since this is so, we must hold that all judgments—even those which can be verified by a reference to what is contained in the compared concepts themselves, *i.e.* by a reference to the sphere of *possibility* or *conceivability*—refer us beyond this to some portion or department of *actual* reality.

124. MEANING OF INQUIRY AS TO EXISTENTIAL IMPORT.—The question, therefore, which we have now to propose is this: whether every categorical judgment, besides *referring* us to some such sphere of *actual* reality, *implies* also, as part of its import, that the objects compared in the judgment actually exist or occur in that department—in addition to their being present in the sphere of the logically possible or thinkable.

It is one thing that the objects with which we deal in our

use of terms and propositions should, besides having *an existence as objects of thought in the sphere of the thinkable*, also *refer us* to some sphere or realm of the *actual*, in which the truth of our judgments about these objects may be tested; it is another question altogether whether the various categorical judgments which we make about those objects *imply, or do not imply, that those objects actually exist in those realms.* This latter is the question with which we are at present concerned. We cannot, of course, use a term intelligibly except *in reference* to some universe of discourse, *i.e.* some sphere of *actual* reality; but this does not oblige us to suppose that the object it represents *actually exists* in this, or indeed in any other, universe of discourse.[1] For instance, the proposition, *There are no such things as unicorns*, while referring us to the visible, material universe, expressly denies the existence of its subject in that universe: although unicorns do exist in another universe—that of heraldry. In discussing the existential import of a judgment our first duty, therefore, is to determine with certainty to what universe of discourse the judgment refers us. No general rule can be laid down for determining this, but there will be scarcely ever any doubt or ambiguity about it. It will easily be gathered from the context. Where subject and predicate would naturally suggest different spheres, the probability is that it is in reference to the sphere of the *subject* the assertion is made. In inference, a tacit change of the sphere of reference of the judgment would cause fallacy. For example, in converting "*Some witches are old women*" to "*Some old women are witches*" both propositions must be referred to the same sphere. Where there is any ambiguity it must be removed; for unless we know the universe to which the judgment refers us, we cannot be said to *understand* the judgment.

And when we ask the question whether a given judgment implies that the objects denoted by its terms *exist* in the universe referred to, it must not be thought that we are raising any question about the ultimate nature either of "existence" in general, or of the universe in question. For, although there may be, and always has been, much controversy among philosophers, and indeed among all thinking men, about the nature of these various objective spheres: whether, for instance, the sphere of "spiritual" beings, as distinct from visible, material things, is a mere creation of human beliefs like

[1] If the direct reference of the term or judgment is to the sphere of the purely possible or conceivable (in which case there is always the further reference beyond this, to the *actual*), then the object signified will actually exist in this possible sphere unless it be an *impossible* object such as "square circle," etc.

the various mythologies and folklore legends, or is on the contrary a sphere of beings that are really existent quite independently of human beliefs ; and although these controversies sometimes find their way into logic, as, for example, when they lead authors to disagree about the *real* definition of "cause," "substance," "energy," "free will," "spirit," "True Church of Christ," etc. (54) ; nevertheless these questions are in themselves metaphysical rather than logical. All that logic demands is the recognition of the indisputable fact that in every judgment "there is a reference to *some* system of [actual] reality which is to be distinguished from the uncontrolled course of our own ideas ".[1] And, this being granted, logic goes on to inquire whether the "existence" or "presence" or "occurrence" or "actual happening" of the things or events denoted by the terms of the judgment, *in that sphere* of reality, whatever it may be, referred to by the judgment, forms a necessary part of the import or implications of the judgment. The question is not whether these things exist or occur in other spheres ; they may or may not. The subject and predicate, taken apart and considered by themselves, may refer us to quite different spheres ; our present question is whether the judgment implies as part of its import or meaning that they exist in the sphere to which the judgment refers.

Again, this question whether or not a given propositional form affirms or denies *as part of its import or meaning* the existence of certain classes of things (S, P, $\overline{S}$, $\overline{P}$, SP, $\overline{SP}$, etc.) in the universe of discourse, is distinct from the question whether the existence of any or all of these classes in that universe is *assumed collaterally* with the proposition, independently of the latter, and as something *over and above its meaning*. It was an assumption of this latter kind that appears to have underlain the traditional treatment of immediate inferences in Aristotelean logic.[2]

125. PLACE OF THE INQUIRY IN LOGIC.—Objection is sometimes taken to the modern treatment of this whole question of *Existential Import*, on the ground that it is metaphysical rather than logical, and that the only existence of objects that comes within the proper purview of logic is their existence as objects of thought, in the sphere of the objectively possible. But even though it is with the latter kind of existence that logic primarily deals, it is no less true that the *possible* cannot be treated without reference to the *actual ;* that we can determine what *must be*, or *cannot be*, only through concepts derived from our experience of what actually *is ;*[3] that if logic is concerned

[1] KEYNES, *op. cit.*, p. 76. [2] *Cf. infra*, 125.

[3] For example, the *contrapositive* of the proposition "All future free acts are foreseen by God" is (abstracting from question of existential import) "All things that are not foreseen by God are things other than future free acts". But is the subject of the latter proposition an impossibility ? That will depend on our concept

not merely with the internal consistency of our concepts and judgments, but with their truth (10, 17, 54), not merely with what *may be* but with what *is*, it should certainly take account of the whole general question whether or how far the objects, of which we think and judge, are supposed by us to be not merely *possible* but *actual*, and whether such a supposition even enters into our judgments as a part of their very meaning. Even from the narrower point of view of *formal* logic, the propriety of dealing with the question is thus defended by Dr. Keynes:[1] "It is, of course, no function of logic to determine whether or not certain classes actually exist in any given universe of discourse, any more than it is the function of logic to determine whether given propositions are true or false. But it does not follow that logic has therefore no concern with any questions relating to objective existence. For, just as certain propositions being given true, logic determines what other propositions will as a consequence also be true; so given an assertion or a set of assertions to the effect that certain combinations do or do not exist in a given universe of discourse, it can determine what other assertions about existence in the same universe of discourse follow therefrom."

Nor can it be contended that the traditional Aristotelean logic always treated the import and implications of our judgments without making any suppositions as to the existence, in any *actual* sphere, of the objects compared in those judgments: that it merely supposed these objects to exist in the sphere of *possible* or *thinkable* things, without inquiring into their *actual* existence. For, without indeed raising this latter question explicitly, it made certain tacit assumptions by the very fact that it accepted the opposites and eductions given in Chaps. V. and VI. as valid unconditionally and without qualification.

For instance, the process of converting an A or an I proposition does not seem to be valid unless the existence of *S* in the universe of discourse is either taken as implied by the A or I proposition, or assumed as an independent *datum*. The propositions "*All S's are P's*" and "*Some S's are P's*" imply at all events that *if* there are any *S's* there must be some *P's* in the sphere referred to. But they do not seem necessarily to imply that *if there are P's there are S's*. If, however, we convert them to "*Some P's are S's*" we *have* in this converse the implication that *if there are P's there are S's* in the sphere referred to: *i.e.* in the conversion of an A or an I proposition we have in

of the Deity. But does this concept represent the reality aright? Proof based on the *actual facts* of our experience will alone determine this.

[1] *op. cit.*, p. 215.

the converse an implication of existence not contained in the convertend, *unless this latter be supposed to imply the existence of S;* or, *unless the existence of S be otherwise guaranteed.*

Again, the tacit assumption on which alone *all* the traditional eductions in Chap. VI. are valid (as categoricals) is the assumption that S, P, $\bar{S}$ and $\bar{P}$ represent classes *actually existing* in the universe of discourse of the proposition (*cf.* 120): whether this was considered to be *implied* in each of the four propositions, A, E, I, O, or merely *assumed* concomitantly by the person judging. In regard to the traditional doctrine on Opposition (Chap. V.), Dr. Keynes says this will be found to hold good universally only on the understanding *that universal propositions be supposed to imply, and particulars not to imply the existence of their subjects in the universe of discourse.* And he points out that this supposition "conflicts altogether with popular usage; it renders the processes of simple conversion and simple contraposition illegitimate; and whilst making universals double judgments, it destroys the categorical character of particulars altogether".[1]

But the traditional doctrine on opposition will also hold good if the existence of *S* be assumed independently of the import of the propositions themselves altogether. And this appears to have been the assumption really, if only implicitly, made in scholastic logic. We shall find on examination that the traditional doctrine on *contradictory* and *subcontrary opposition* holds good on the tacit assumption that *S* at least represents an actually existing class, and that we are therefore precluded from supposing *S* to be non-existent;[2] while if this assumption—that *S* represents an existing class—were *made part of the meaning* of the proposition, so that "*All S is P*" would mean "*S exists and is always P*," then, were *S* non-existent, the three propositions "*All S is P*," "*Some S is P*," and "*Some S is not P*," would be all false together. For instance, if the propositions "*All* (*or* some *of*) *the answers to the questions showed originality*" and "*Some of the answers to the questions did not show originality*" *implied as part of their meaning* the existence of "answers," they would all be *false* in case there were no answers. In such a case the real contradictory of "*All the answers to the questions showed originality*" would be "*Either some did not . . . or there were no answers at all*". The traditional doctrine on opposition did not consider the existence of *S* (or *P* or $\bar{S}$ or $\bar{P}$) to be *implied* as part of their meaning by the A, E, I, and O propositions, but to be presupposed by them, so that the possible consequences of the non-existence of any such class could not arise at all for discussion.

126. STATEMENT OF THE QUESTION OF EXISTENTIAL IMPORT IN CATEGORICAL PROPOSITIONS.—The question is formulated in practically the following terms by Dr. Keynes.[3] It is part at least of the meaning of every *universal* proposition to deny the existence of a certain class of objects of thought in some universe or sphere of being.[4] *S e P* denies that there are such

[1] *op. cit.*, p. 241, note 3. [2] *op. cit.*, pp. 228, 229. [3] *op. cit.*, pp. 212, 213.

[4] And, by implication, to affirm the existence of the contradictory class. For, since the sphere referred to by any judgment is objectively real, it cannot be sup-

things as *S P's*, *S a P* denies that there are such things as *S* $\bar{P}$*'s*, in a certain universe. This universe, in which the existence of S P (or S $\bar{P}$) is denied by the universal proposition, is the universe of discourse of that proposition. The universe of discourse for the particulars, *S i P* and *S o P*, is that universe, whatever it be, in which the existence *S P*, or of *S* $\bar{P}$, would be understood to be denied by their respective contradictories, *S e P* and *S a P*.

Now, *given a categorical proposition with S and P as subject and predicate, does it formally imply the existence of S or of P in that sphere in which it* (*or its contradictory*) *denies the existence of S P* (*or S* $\bar{P}$).

Or, in other terms, *S P* (*or S* $\bar{P}$), as an object, of thought is a certain complex of attributes: so is *S* itself, and so is *P* itself: *does the categorical proposition imply the existence of these latter complexes in the same sense as it denies the existence of the former.*[1]

Since the question is one of interpreting the meaning we are to attach to propositional forms, it is obvious that to some extent different alternative solutions may be agreed upon; that, on account of the conventional element there is in the ordinary use of language, and the absence of rigidly fixed meaning, no one solution can be pronounced correct to the exclusion of all others (*cf.* 86, 87). But it is no less clear that in such interpretation we should be guided to our results *mainly* by observing *what people generally mean to convey* by the use of the ordinary propositional forms. This is a *primary* consideration. A *secondary* one is the influence each of the various possible suppositions may have upon ordinary processes of inference, whether mediate or immediate. *Ceteris paribus*, the interpretation that would lend itself best to the logical treatment of inferences from judgments should have our preference.

127. Influence of Various Suppositions on Validity of Logical Inferences.—Of the very many suppositions

posed to be entirely empty of content. Hence, any judgment which denies the existence of S, or of P, implicitly affirms the existence of $\bar{S}$, or of $\bar{P}$,—by the Principle of Excluded Middle. But the affirmation of the existence of a class does not imply the denial of the existence of the contradictory class.

[1] When it is remembered we are dealing with *assertoric* propositions, it may reasonably be asked how we could reach a knowledge of the universals which deny the existence of S $\bar{P}$ or S P, except we experienced both S and P (or $\bar{S}$ and $\bar{P}$) as actually existing in the universe of discourse. *Cf. infra*, p. 256, n. 2.

that might be made, we will examine just a few, seeing in the first place what influence they would have on the ordinary doctrine regarding *Opposition* and *Eductions*.

(1) We have seen already that *if every proposition be taken to imply the existence of its subject, predicate, and their contradictories* (S, P, $\bar{S}$, and $\bar{P}$), all the ordinary *eductions* will be valid; but the forms given as *contradictories* and *subcontraries* in the square of opposition will be no longer valid[1] (125).

(2) *If every proposition be taken to imply the existence of its subject* merely: the forms given as *contradictories* and *subcontraries* in the square of opposition will, as in (1), be no longer valid (125). With regard to *eductions*, though the conversion of A and I is valid, since the existence of *S* involves, *in an affirmative* proposition, the existence of *P*, the conversion of E is invalid for the opposite reason. "*No women are now hanged for theft in England*" must therefore be converted to a *hypothetical*: "*No people now hanged for theft in England*, if there be any such, *are women*". As a consequence of the invalidity of converting E, the contrapositives of A, and the inverses of A and E, are also invalid, if stated categorically, since these involve the conversion of an E proposition. But the contrapositives of E and O remain valid, for the existence of *S* guarantees the existence of $\bar{P}$, which is the subject of these contrapositives.

(3) *If no proposition implies the existence either of its subject or of its predicate*, then (*a*) as regards *opposition*: *S a P* denies the existence of *S* $\bar{P}$*s*, whereas *S o P* merely asserts that *if there are S's* there are *S* $\bar{P}$*s*[2]—a statement which does not deny the truth

[1] These forms will, of course, remain valid if the "existence" of the classes in question be not taken as *contained* in the *import* of the proposition, but as assumed independently. This latter is what is probably understood by writers who, like Professor Welton, assume the existence of *S* as given with the categorical proposition, and yet hold the ordinary contradictories and subcontraries of the square of opposition as valid.

[2] *Cf.* Dr. Keynes (*op. cit.*, pp. 221, 225, 229), whose treatment of the whole question is largely followed in the present chapter. In connexion with the supposition in (3) above, it should be clearly borne in mind that if we regard a proposition as not *implying* the existence of S or P, but *take for granted independently* that S, P, $\bar{S}$ and $\bar{P}$ exist actually in the sphere to which the proposition refers us, then all the traditional laws of opposition will hold good of the forms set down in the square of opposition, and all the ordinary eductions will be valid. This seems to have been the assumption acted on in the traditional Aristotelean logic: and wherever a doubt arose as to the propriety of making such assumption about any class, such doubt was expected to find expression in the form of the proposition.

Indeed, the supposition made in (3) above—*if it be understood to exclude the*

of the former, but is compatible with it: in the case, namely, in which S is non-existent. The forms given as contradictories in the square of opposition are therefore not contradictories. Nor are *S a P* and *S e P* contraries, for the former merely asserts that there are no *S* $\overline{P}$*'s* and the latter that there are no *S P's:* two statements which *may be true together, viz.*, when *S* is non-existent. (*b*) As regards *eductions:* the conversion of A and I will be, as we have seen (125), invalid; for in either case the converse will imply that if there are any *P's* there are some *S's*—an implication not contained in either original. Hence, too, the contraposition of E and O and the inversion of E and A are invalid, since all these involve the conversion of an A or an I proposition.

(4) *If particulars be taken to imply, and universals not to imply the existence of their subjects*, then (*a*) as regards *opposition:* the forms given as *contradictories* are really contradictories, for it is exactly what the universal denies (the existence of *S P's* or *S* $\overline{P}$*'s*) that the particular of opposite quality affirms. But the forms given as *subalternants and subalterns*, as *contraries*, and as *subcontraries*, are not really such. From *All S is P* we cannot infer *Some S is P; All S is P* and *No S is P* may be true together, their combined force being to deny the existence of *S's*; and both *Some S is P* and *Some S is not P* will be false if *S* does not exist. (*b*) Of the *eductions:* the conversion of A is invalid;[1] and, consequently, the contraposition of E, and the inversion both of A

collateral and independent assumption of *S*, *P*, $\overline{S}$, and $\overline{P}$, as existing classes: and it is on this understanding that its results are worked out by Dr. Keynes—seems to be an impracticable supposition, *i.e.* one that is never realized in the case of assertoric judgments, which are the only ones contemplated. The assertoric judgment, even when universal, is a judgment *of fact*, based on experience. How, then, can we know the truth of *S a P* or *S e P*—that there are no S $\overline{P}$'s, or that there are no S P's—unless we have experienced S or $\overline{S}$, P or $\overline{P}$, as actually existing classes? And so far as we have experienced the actual existence of any such class, we may reasonably *assume* its existence in examining the implications of our judgments in reference to it. The interpretation of the assertoric particular, *S i P* (or *S o P*), as excluding all assumption of the existence of S, P, $\overline{S}$ or $\overline{P}$, is still more strained. How can experience enable us to *assert as a fact* that some S's are P's (or not P's) unless we have experienced S, P, or $\overline{P}$, as actually existing. Such existence, if not indeed *implied*, is certainly assumed in *S i P* or *S o P*, so that these forms, in so far as they assert existence at all, assert the existence of S P or S $\overline{P}$ just as categorically as *S e P* or *S a P* deny such existence.

[1] Unless *S* be otherwise given as existing—in some such proposition, *e.g.* as *Some R is S*. So, also, can E be converted in supposition (2) above, if *P* be otherwise guaranteed as existing.

and of E. In other words, we may infer a universal from a universal, a particular from a particular, but not a particular from a universal. The inferences which remain valid on this supposition are those in which there is no loss of force, and which are therefore "simple" or "reciprocal".

Having noticed there various results, let us now see what the ordinary usage of language has to say to the question.

128. EXISTENTIAL IMPORT AND ORDINARY USAGE OF LANGUAGE.—Ordinary usage seems, on the whole, to be distinctly in favour of the view implicitly adopted in the traditional logic: the view, namely, that while the existence of S, P, $\bar{S}$, or $\bar{P}$, is not to be regarded as part of the *import* or *implications* of any proposition (unless one that predicates existence), still *S* at least is assumed independently to exist; and usually P, $\bar{S}$, and $\bar{P}$, also; and whenever a doubt arises about the existence of any of them, this doubt should be expressed in order to remove ambiguity. People usually *assume* the existence of the objects of their thought in some actual sphere, but do not usually intend to make this assumption part of the import or implication of their judgments. Perhaps modern logicians deviate too much from this usage by supposing that existence is ever *implied* in propositions (other than existential propositions).

Mill taught[1] that all real or synthetic propositions should be held to imply the existence of their subjects: "because in the case of a non-existent subject there is nothing for the proposition to assert"; and it must be admitted that as a rule men intend their statements to be understood of what does exist, rather than of what might exist. We shall find, however, that categorical forms of statement are oftentimes not meant to *imply* the existence of the subjects they refer to, and that, consequently, such forms cannot be interpreted as always and necessarily carrying such an implication with them.

Universals.—If we examine universal *affirmatives*, we find that *concrete* or *collective* universals, which are the result of observation and enumeration of instances, naturally involve, by implication or assumption, the existence of their subjects; but that the *abstract* or *generic* universal—which is reached by reasoning, or by an analysis of the notions compared: the universal which expresses a law—does not necessarily carry any such implication, and

[1] *Logic*, i., 6, § 2.

sometimes even makes the latter impossible. For example, the first law of motion, "*Every body not compelled by impressed forces to change its state, continues in its state of rest, or of uniform motion in a straight line*,"—makes a statement about a condition of things that is believed never to have been realized. Similarly, a rule or direction laid down in such terms as "*All trespassers are prosecuted*" does not imply the existence of its subject. Again, assertions about future events, *e.g.* "*Those who pass this examination will be eligible*," carry no necessary implication that the subject exists.

From those examples we see that the assertoric categorical form, *All S is P*, does not *necessarily imply* the existence of its subject, though perhaps it carries this implication (or assumption) oftener than not. Dr. Venn emphasizes the consideration that a *single-worded* term, standing as subject of a proposition, furnishes a very *strong presumption* that the subject exists: because "man has something more pressing to do with his vocal organs and inventive powers than to impose names upon objects which he merely contemplates as possible";[1] whereas universals with *many-worded* terms as subjects will carry no such strong presumption, but will be found rather to *suggest a doubt* about the existence of their subjects. These, he points out, are oftener, more easily, and certainly more appropriately, expressed in the *hypothetical* than in the *categorical* form—on account of this very doubt about the real existence or occurrence of their subjects (*cf.* 134 *infra*). This is true even of many categoricals with *simple-worded* terms. But though the hypothetical may be the more appropriate form for all such, it is no less true that the assertoric categorical, *S a P*, is used to express them, and that, therefore, this form cannot be interpreted as necessarily implying the existence of its subject: the Law of Parsimony forbids us to read any more into a propositional form than the least amount of meaning it may express.

What has been said about the universal affirmative applies equally to the universal *negative:* except that the latter is even more frequently used without any implication of existence. For example: "*A planet moving in a hyperbolic orbit can never return to any position it once occupied*," "*No unicorns have ever been seen*" (the universe referred to being the visible, material universe),

[1] *Empirical Logic*, pp. 258 *sqq.*

"*No person condemned for witchcraft in the reign of Queen Anne was executed*".

Particulars.—Particular propositions, whether affirmative or negative, almost invariably carry with them, in ordinary usage, if not an implication, at least an assumption, of the existence of their subjects. The reason is that these judgments are almost invariably based on observation of actually existing instances. And a very natural way for asserting the existence of some class is by means of the particular proposition. We assert the existence of a class of cattle without horns by saying "*Some cattle have no horns*".

129. CHOICE OF ALTERNATIVE INTERPRETATIONS.—It would appear, then, that we should not be departing very much from popular usage by interpreting particulars as implying, or assuming, the existence of their subjects. If, in any case, we wished to avoid this implication, we might say "*If there be any S's*, some of them are (or are not) *P*".

In interpreting universals as *never* implying, or assuming, the existence of their subjects, we should, no doubt, be departing somewhat more from ordinary usage; but the advantages to be gained from making this supposition in processes of inference would seem to compensate for the disadvantages of such a deviation.[1]

We have already seen that this supposition regarding universals and particulars is the only one among those considered which leaves the most important form of opposition, namely, *contradictory* opposition, valid;[2] and that it also leaves the validity of the two most important eductions, namely, *simple conversion* and *simple contraposition*, untouched. Of course, the fact that A and E may be true together, and the fact that I and O may be false together, on this assumption, will be a drawback from the point of view of logical convenience. But the fact that I cannot be inferred from A, nor O from E, merely renders unlawful what is a useless inference in any case, while it gives the particulars a *locus standi* of their own, independently of the universals.

Finally, the present assumption is the only one which makes each of the propositions A, E, I, and O, a *simple* categorical proposition, containing one statement only: *viz.* that there *are* (or

[1] *Cf.* KEYNES, *op. cit.*, pp. 239 *sqq.*

[2] Except the converse supposition referred to in 125, and the independent assumption—outside the import—that S, etc., exist.

are not) *S P's*, or S $\bar{P}$'s, as the case may be. For, were universals to imply the existence of their subjects, each universal would be a *double* assertion: *e.g. S a P* would mean "*There are S's, and all of them are P*"; while if particulars did not imply the existence of their subjects they would really be *hypothetical* propositions: *e.g. S i P* would mean "*If there are any S's some of them are P*".

130. EXISTENTIAL IMPORT OF MODAL PROPOSITIONS.—It is precisely those universals which, though expressed assertorically (*All S's are P, No S's are P*), would be more appropriately expressed as modals, that do not imply the existence of their subjects. Propositions which express some necessary connexion —of identity or incompatibility—between *S* and *P*, and which are reached by an analysis of the notions compared, not by any experience of actual instances: these are obviously *apodeictic* in character, and they do not imply the existence of their subjects. And the same, of course, is true of the propositions which *deny* any such necessary connexion. But these latter are the *problematic* modals (90). Hence these forms also—*S may be P*, *S need not be P*—do not necessarily imply the existence of their subjects. If we accept this view, and combine it with the view that assertoric particulars do, and assertoric universals do not, imply the existence of their subjects, we shall still be able to infer "*All S's are P*" from "*S must be P*," but we can no longer infer "*Some S's are P*" from "*S may be P*".

131. FORMULATION OF EXISTENTIAL PROPOSITIONS.—An *Existential Proposition* is one which *directly and explicitly affirms or denies the existence or occurrence of something in the universe to which the proposition refers.* For example, "*God exists*," "*It is raining*," "*Once upon a time there was a man who . . .*," "*There are no such things as ghosts*," "*There is no rose without a thorn*," "*It came to pass that . . .*," "*The Resurrection of Christ is an historical fact*," "*The dodo is now extinct*" (*cf.* 86).

We have already seen (109) that the existence of any class, X, may be expressed by the formula $X > 0$, and its non-existence by the formula $X = 0$; and, furthermore, that on the assumption of particulars implying and universals not implying the existence of their subjects, the four traditional forms may be expressed:—

$$S\ a\ P \text{ as } S\bar{P} = 0$$
$$S\ e\ P \text{ as } SP = 0$$

$$S\ i\ P \text{ as } S\,P > 0$$
$$S\ o\ P \text{ as } S\,\bar{P} > 0$$

Analogous results may be worked out on the other suppositions already considered.

KEYNES, *Formal Logic*, pt. ii., chap. viii. JOSEPH, *Logic*, pp. 146-52. VENN, *Empirical Logic*, pp. 258-62. WELTON, *Logic*, i., pp. 211-14. JOYCE, *Logic*, pp. 111-16.

CHAPTER VIII.

CONDITIONAL AND HYPOTHETICAL JUDGMENTS AND PROPOSITIONS.

132. ANALYSIS OF THE "IF" JUDGMENT : ITS RELATION TO THE SIMPLE OR CATEGORICAL JUDGMENT.—So far we have been dealing mainly with the *simple* or *categorical* judgment. We must now revert to the division of judgments on the basis of "Relation" as given in a previous chapter (83, 84). The most important classes of *compound* judgment are the *hypothetical*, and the *disjunctive* or *alternative* (including *copulative* and *remotive*). With the *hypothetical* we shall deal in the present chapter; with the remaining kinds in the next chapter.

The propositions which we now purpose to examine are all characterized by the presence of the conjunction "*if*," introducing a conditional clause called the *antecedent* or *protasis* (or *conditio*), on which depends a second part or clause called the *consequent* or *apodosis* (or *conditionatum*). For example, *If all prophets spoke the truth some would be believed.* Their force or function is to assert some sort of relation or connexion of dependence between antecedent and consequent. Their truth or falsity will therefore depend, not at all on the truth or falsity of the constituent parts taken separately, but on *whether the relation* between these parts *is or is not what it is asserted to be.*

As a process of thought, the "*if*" judgment is not fundamentally different from the simple or categorical judgment: the former as well as the latter is a *judgment*, and must fulfil the definition of a *judgment:* it must make a predication or statement which will be verifiable in some sphere or other of objective reality: but, while the predication made in the categorical judgment is simple and unqualified, that which is made in the "*if*" judgment is qualified, conditioned, limited in a certain way. The latter "asserts a predicate of the subject of the consequent, under a condition expressed in the antecedent; and if that condition can be expressed as an adjective of the subject of the consequent, then of that subject, so qualified, we may assert the predicate in the consequent categorically. But we do not thus reduce hypothetical to categorical judgments: the hypothetical meaning remains under the

categorical dress . . . *If corn is scarce, it is dear*, becomes *Scarce corn is dear* . . . [but this latter] is not really a judgment about scarce corn but about corn: we realize that corn is something which may be scarce, and is dear when scarce; and so the dependence in corn of a consequent on a condition is the burden of our judgment about it."[1] This may be expressed in some such categorical form as "Corn is such that its scarcity will involve dearness": where, although the form is categorical, yet the real judgment is contained in the complex predicate, and remains a conditional judgment—an assertion of the dependence of a consequent upon a condition, without any assertion as to the reality or fulfilment of the condition. Indeed, the claim to truth in an "*if*" judgment necessarily implies that it has underlying it some simple, categorical affirmation or denial: that in its ultimate analysis we must be brought face to face with an absolute "*is*" or "*is not*". We have seen (84, 123) that the categorical judgment makes a predication about some subject belonging to some sphere of reality, and that this latter might be called the *ultimate* subject—as distinct from the *logical* subject—of the judgment. The same is true of the "*if*" judgment, so that even in its purest form (133) it involves a categorical statement about the sphere of reality to which it refers. Where *A* and *C* represent two simple judgments, the "*if*" judgment may be expressed "*If A then C*"; but it might also be expressed by a categorical judgment of this sort: *Things are such* (or *Reality is such*, or *The sphere of things referred to is such*) *that if A be true in reference to this sphere of things or reality, C is also true in reference to it.*[2]

We point out this merely to show that, as acts of the mind, there is no *fundamental* difference between these various kinds of judgment. But the assertion of a predicate about a subject is *logically* distinct from the assertion of the dependence of one judgment upon another.[3] The *logical* subject of

[1] JOSEPH, *op. cit.*, p. 164.

[2] The fact that a hypothetical judgment may be true though neither condition nor consequent be ever fulfilled or realized—"*If Hannibal had marched on Rome after Cannae, he would have taken it*"—raises a problem about the nature of the reality asserted by the hypothetical. What is asserted in the example is that "Rome *was* in such a state that it could not have resisted Hannibal after Cannae. This is true; but it still leaves us with the question, how can there be the ground, in the real universe, of something which nevertheless does not happen? We speak freely of unrealized possibilities, as if they existed as well as realized actualities. We are not always conscious of the metaphysical difficulties involved: how are we to *think* of what we so freely *speak* of? When we reflect, in Logic, upon the hypothetical form of judgment, we become conscious of the problem" (JOSEPH, *op. cit.*, p. 166). This problem at least suggests the thought that "what does not happen," what is not "actual," is nevertheless *real* if it is *possible*. Our study of the negative judgment (98) revealed to us that reality as known by us contains the ground not only for knowing what *can* be but *is not* (the *possible*), but also for knowing that which cannot be (the *impossible*), *i.e.* the unreal. *Cf.* a kindred problem in connexion with disjunctive judgments.—JOSEPH, *op. cit.*, p. 168; *infra*, 146 n.

[3] It has been said that the difference between the hypothetical and the categorical judgment, being material, should not be noticed in logic [*cf.* MANSEL, *Prolegomena Logica*, pp. 282, 251, *apud* JOSEPH, *op. cit.*, p. 165]. "For both assert; they differ in what they assert; the difference is therefore in the matter and not in the form. . . . But it will be readily admitted that the distinction between categorical and hypothetical assertion is formal in the sense that it meets us whatever be the

the categorical proposition just given is different from the logical subject of the "*if*" judgment. Indeed, in the latter we have two judgments compared, instead of two concepts; and this we saw to be the criterion of distinction between the simple and the compound judgment (84): corresponding to *subject* and *predicate* in the *simple* judgment—or, rather, analogous to them—we have here *antecedent* and *consequent*.

The consequent is *somehow* dependent on the antecedent: but if we ask *how* exactly? or *what is the relation* between the two simple judgments *A* and *C*, or between *W is X* and *Y is Z* in the judgment "*If W is X, Y is Z*," where we give full symbolic expression to all the *terms?* or whether all such judgments, even though they contain four distinct terms, may be reduced to the form in which antecedent and consequent have the *same logical subject*, the form "*If S is M it is P*"? when we ask these questions and endeavour to answer them by the study of examples, we shall find that there is room to draw a distinction between two great classes of these "*if*" judgments; although the ultimate grounds of the distinction may not be at first quite clear.

133. Two Classes of "If" Judgments, the "Conditional" and the "Hypothetical".—Examples of one class would be the following: *If a child is spoilt, its parents suffer; if the government is good, the people are happy; if the barometer falls, we shall have rain; if a lighted match is applied to gunpowder, there will be an explosion; if employers and workmen disagree, the trade of the country will be injured; if a triangle be inscribed in a semi-circle, it will be right-angled; if a triangle be right-angled, the square on its hypotenuse will be equal to the sum of the squares on the other two sides.*

The main characteristics of this class of judgments are (1) that they connect two *events*, or two *groups of properties*, so that "*whenever*," "*wherever*," "*as often as*," "*in all cases in which*"[1] we have the first, we have the second; (2) that the consequents, if taken apart and expressed fully, have not a complete import of their own, but refer us back inevitably, for the full understanding of them, to the antecedents; (3) that they can be *easily* reduced to the form "*If S is M it is P*," in which the four original terms (if

subject we may think about. And to exclude it from Logic on the ground that, as compared with the common form of assertion in both, it is material, only shows the impossibility of making Logic a *purely* formal science. It is claiming to consider the genus and refusing to consider the species: a procedure which would be tolerated in no other science, and cannot be tolerated in Logic."—Joseph, *ibid.*

[1] All these expressions may be substituted for "if" in the conditional judgment.

there were four) are reduced to three, the same term being now subject of both antecedent and consequent; (4) that they can be easily *quantified*, or written in the concrete, denotative, form "*If any S is M it is P*," or "*Whenever an S is M*," etc., or "*Sometimes if S is M*," etc.; (5) that they can be easily expressed in the *categorical* form "*S which is M is P*," or "*S, because it is M, is P*," or "*S M is P*". Propositions of this class we shall call *conditionals*: to distinguish them from the following class, which we shall call *hypotheticals*,[1] or *pure hypotheticals*.

Examples of this second class would be: *If there is a just God, the wicked will be punished; if patience is a virtue there are painful virtues; if virtue is voluntary, so is vice; if the earth is immovable, the sun moves round the earth; if all savages are cruel, the Patagonians are cruel.*

Now these examples cannot be distinguished, *so far as form goes*, from those of the former class; but they will be found on examination to differ from the former in certain ways. (1) They appear to connect together not so much two *events in time or space*, or two *groups of attributes* in one subject, but rather *two abstract truths* that are seen to hold good together once and for all, independently of any time or space relations; (2) then, the antecedent and the consequent, taken separately, express each a full judgment, complete in itself, independently of the other; (3) they cannot be so easily reduced to the form "*If S is M it is P*"; nor (4) do they admit of quantitative expression; nor (5) can they be reduced to categoricals without so changing the judgment and its subject as to modify the import of the original judgment.

It would, however, be a mistake to imagine that these points of difference reveal two fundamentally different types of "*if*" judgments. More especially it must be noted that distinctions (2) and (3) are not absolute. They are rather distinctions of degree. Even though antecedent and consequent be in themselves judgments of "independent import," still it is the very function of the "*if*" judgment to establish some sort of a relation of dependence, some link or bond of connexion, between them. It is the mental perception of this bond, whatever it be, that forms the *ground* or *reason* for making the "*if*" statement; and, consequently, it must always be possible to bring out explicitly this ground of predication by expressing the judgment in the form "*If S is M it is P*". This process will, it is true, often involve the *explicit* statement of what was *implicit*, the making part of the *import* what was rather an *impli-*

[1] The two words, "conditional" and "hypothetical," have been commonly used as synonyms by logicians. It will be convenient to appropriate one of the two titles for each of the classes indicated above.

cation (82), and will therefore sometimes entail considerable modification of the original propositional terms. The examples given might be expressed thus: *If a just God exists, He will punish the wicked; if the virtues include patience, some of them are painful; if vice is a habit, like virtue, it is voluntary; if the earth is immovable, it has the sun moving around it; if the Patagonians are savages, they are cruel.*

It is this form, *If S is M it is P*, that reveals most clearly the relation between the "*if*" judgment and the categorical judgment. The former brings out explicitly some condition on which the predication made in the latter is grounded. In virtue of the *Principle of Sufficient Reason* (16), we must have a "sufficient reason" for formulating the judgment *S is P*. This reason is something which we see in *S*, or connected with *S*, when we compare the latter with *P* (97, 98). This something, which furnishes us with the ground for predicating *P of S*, let us call *M*. Now, we may make this ground explicit by stating our judgment: *S, because it is M, is P;* or, *S which is M is P.* And if we abstract from every concrete *S* of our experience, and fix our attention on the abstract relation between *M* and *P*—which is the ground of our original judgment—the form of statement which will best bring out this relation is the "*if*" proposition, "*If S is M it is P*". Thus, while the *categorical* form emphasizes rather the *concrete reality* of the terms compared, the "*if*" form emphasizes the *abstract relation* between them.[1]

There is, consequently, no really sharp line of demarcation between the categorical judgment and the "*if*" judgment, nor between the two classes we have distinguished in the latter. But these two classes do differ sufficiently to justify us in dealing with each separately. They might be distinguished symbolically by writing the *conditional* in some such form as: "*If any S is M it is P*";[2] and the *hypothetical:* "*If A is true C is true*," or simply "*If A then C*".

134. "DOUBT" AND "INFERENCE" IN THE "IF" JUDGMENT.—The force of the conjunction "*If*" seems to be to express a combination of *doubt* and *inference.* It expresses that condition of things "in which we know that two elements,—events, objects, or what not,—are connected together, but are uncertain about the first member of such connexion. It is as if we knew that there were two links of a chain which held together, but were not quite secure in our grasp of the nearest of them."[3] Indeed this combination of doubt and knowledge is necessary for all inference: were all things doubtful inference would be impossible; were all things certain it would be superfluous. Of these two

[1] *Cf.* WELTON, *Logic*, pp. 183-86.

[2] Compare the above with the proposition "*If this S is M it is P*," which, though derivable from the former, a *conditional*, is itself put down by Dr. Keynes (*op. cit.*, p. 252, footnote) as a *hypothetical*, thus forming "a kind of junction between" the two forms. While the former may be easily expressed by the *categorical* "*All S's that are M are P*," can the latter be identified with the categorical "*This S which is M is P*"? *Cf. infra*, 135, 144.

[3] VENN, *Empirical Logic*, p. 249.

elements, however, *doubt* and *inference*, either may, in a given judgment, almost entirely overshadow the other. When the element of doubt is present it falls, of course, not on the connexion between the consequent and the antecedent, but *on the actual happening or occurrence of the latter*.[1] In the case of conditionals it is certainly the inferential element—the connexion of consequent with antecedent—that is primarily asserted. The element of doubt about the occurrence of the antecedent is only secondary and incidental. At the same time, it is rarely, if ever, altogether absent when "*If*" is used. At all events, *when we want to imply that we have no doubt* about the existence of the antecedent we usually substitute the word "*Since*". The "hypothesis of inference," as it is called, is not appropriately introduced by "*If*". The doubt insinuated in the remark of Colonel Morden in *Clarissa Harlowe:* "If you have the regard for my cousin which you say you have, you must admit," etc.—provoked the anger of Lovelace, notwithstanding the colonel's assurance "that his 'if' prefaced a conclusion and did not necessarily suggest a doubt. . . . Had Colonel Morden really felt no doubt about the existence of the regard he would surely have prefaced his sentence with a 'since' instead of an 'if'".[2] If the latter word does not always imply, it always suggests, some doubt about the antecedent, or, at the very least, leaves its occurrence an open question. This is true, too, of the various substitutes for "*if*" in conditional propositions,—"*Whenever*," "*Wherever*," "*As often as*," etc. Perhaps the most that can be said of these is that they do not necessarily imply the occurrence of their antecedents.

Dr. Venn contends, further, that they always imply a doubt at least as to whether the antecedent in the given case belongs to the class of things which involve the consequent. Taking the example: "*If* (*when*, *where*), the husband is a drunkard, the home is a wretched one,"[3] he points out that "there is a connexion asserted between antecedent and consequent,—the drunkenness and the squalor,—and a doubt implied about its occurrence in certain cases . . . any husband taken at random might or might not fit the designation. . . ." To this he likens all examples taken from geometry, examples in which the element of doubt about the occurrence of the antecedent seems at first sight to be entirely absent. The doubt is there, he maintains, and it falls upon this point: whether a given instance will really have the necessary characteristics for membership of the class of things that involve the consequent; whether, for instance, "the learner with his ruler, compass, and paper before him"[4] *will* inscribe a triangle in a semi-circle, or draw a straight line

[1] *Cf.* JOSEPH, *op. cit.*, p. 315.
[2] VENN, *op. cit.*, pp. 262, 263.
[3] *ibid.*, pp. 255, 263.
[4] *ibid.*, p. 263.

cutting two parallel lines, etc. "He may draw what straight lines and angles he pleases, so it is uncertain what we may find drawn in any particular case. We are therefore perfectly in order in employing the hypothetical form and saying that *if* he draws such and such figures, such and such properties will necessarily be found to be involved in them."[1]

135. CONDITIONALS AND CATEGORICALS.—The question whether the same judgment may be expressed indifferently in the conditional or in the categorical form, without any loss of, or addition to, its meaning, has met with different solutions according to the various views adopted about the *existential import*, and the *modality*, of these two forms.

Were we to interpret the conditional form as always implying a doubt about the occurrence of its antecedent, and the categorical form as always implying certainty about the existence of its subject, then, plainly, we could never lawfully pass from the conditional to the categorical form of statement—from "*If any S is M it is P*" to "*All S M is P*". If, however, we interpret the universal categorical as not implying the existence of its subject, then the universal forms "*If Any S is M it is P*," and "*All S M is P*" would be mutually interchangeable.[2]

Particular conditionals for the most part merely assert the results of concrete experience. Hence these may be taken as implying the existence of their antecedents, and as, therefore, mutually interchangeable with particular categoricals. The proposition: "*Sometimes if heavy rains are followed by high winds forests are seriously damaged*," may be expressed—"*High winds following heavy rains sometimes cause serious damage to forests*".

Dr. Venn maintains[3] that although the categorical form sometimes prevails where the subject is doubtful, and the hypothetical where the antecedent is certain, nevertheless the "original and fundamental distinction between these forms ("*X is Y*" and "*if X then Y*") is "that the presence or existence of the *subject* is taken for granted, whilst that of the *antecedent* is recognized as being doubtful".

But even though all conditional judgments involve *some* element of doubt, at least about the identification of the case with the class, in the sense explained by Dr. Venn, still it cannot be denied that some universal conditionals—like some universal categoricals—imply the occurrence rather than the non-occurrence of their antecedents: those, namely, that are based upon concrete

[1] *ibid.*, p. 263.

[2] The transference of a judgment from any one to any other of the classes based on RELATION (88)—*Categorical*, *Hypothetical*, *Disjunctive*, etc.—is called *Change of Relation*, or *Transversion*.

[3] *ibid.*, pp. 257 *sqq.*

experience. Dr. Keynes gives the example:[1] "If we descend into the earth, the temperature increases at a nearly uniform rate of 1° Fahr. for every fifty feet of descent down to almost a mile". This, indeed, is rather the rule than the exception in the case of particular conditionals.

So far, then, as existential import is concerned, there seems to be practically no room for distinguishing between conditionals and categoricals. We have already distinguished, however, between the *modal* and the *assertoric* categorical (89, 90). We have seen also that as a rule the *modal* judgment does not imply the existence of its subject (130). But a categorical judgment which is really apodeictic is often expressed in the form of the assertoric universal, *All S is P*. The question may therefore be asked whether all *conditional judgments* are modal, or all assertoric, or some the one and some the other? The *particular* conditional is, as a rule, merely assertoric; but the *universal* conditional is far oftener, though not always, the expression of an *apodeictic* judgment. Not always: for we occasionally form such obviously assertoric judgments as "*If any book be taken down from that shelf, it will be found to be a novel*". But all conditional statements of mathematical truths and necessary laws are apodeictic. The conditional proposition may therefore be interpreted either assertorically or modally. We shall recur to this point in dealing with pure hypotheticals (138).

136. Opposition of Conditional Propositions.—Regarding *antecedent* and *consequent* as analogous to *subject* and *predicate* in the categorical, we may apply to the conditional proposition the ordinary distinctions of *quantity* and *quality*, and so construct the square of opposition. The conditional will be *universal* or *particular* according as the consequent is stated to accompany the antecedent in *all* or in *some cases* (indefinitely). It will be *affirmative* or *negative according as the* CONSEQUENT[2] (*not* the *antecedent*) is an *affirmative* or a *negative* proposition. Thus we have:—

A—*If any S is M that S is always P.*
E—*If any S is M that S is never P.*
I—*If an S is M that S is sometimes P.*
O—*If an S is M that S is sometimes not P.*

Here, "sometimes" has the same indefinite meaning as the logical "some". Judgments expressed in the *indesignate* conditional forms, *If S is M it is P*, or, *If A is B, C is not D*, are interpreted as *universals:* and hence the very great danger of confounding their *contradictories* with their *contraries*. The contradictories of the two forms just given are *not* "*If S is M it is not P*" and "*If A is B, C is D*". These are the respective *contraries* of the former propositions. The respective contradictories

[1] *op. cit., p.* 253.
[2] *Cf.* 139 for import of the negative "if" judgment.

are "*Sometimes if an S is M it is not P*" and "*Sometimes if A is B, C is D*". We may take, as a material example, the following :—

A—*If any country is well governed its people are happy.*

E—*If any country is well governed its people are not happy.*

I—*Sometimes if a country is well governed its people are happy.*

O—*Sometimes if a country is well governed its people are not happy.*

Here A means that happiness always *de facto* [1] accompanies good government ; E that whenever we have good government we have never *de facto* happiness accompanying it ; I that good government is sometimes accompanied, O that it sometimes is not accompanied, by happiness.

If we interpret the conditional *modally*, we have only to substitute for the signs of quantity the modal signs "*must*," "*cannot*," "*may*," and "*need not*". Remembering, then, that the force of the problematic is to deny the necessary connexion asserted by the apodeictic, we might write, in the example just given :—

A—*If a country be well governed its people must be happy.*

E—*If a country be well governed its people cannot be happy.*

I—*Though a country be well governed nevertheless its people may be happy.*

O—*Though a country be well governed nevertheless its people need not be happy.*

In the latter pair of propositions the conjunctions "*though*" and "*nevertheless*" ("yet," "still") make their appearance for the first time. Their force is to connect or conjoin two statements whose mutual compatibility excludes some (suggested) necessary combination of judgments. Propositions of this kind are commonly called *adversative* or *discretive propositions.*

The *modal* statement of the conditional brings out its inferential element better than the assertoric form. The latter, too, is open to ambiguity. Take the proposition, "*If any parents gamble some children will be ill-treated*". Is this an A or an I proposition? It will be A if it be interpreted to mean that the ill-treatment of some children *always* follows the gambling of parents ; it will be I if it be taken to mean that the ill-treatment of some children results, *at least sometimes*, from the gambling of parents. In the example given the latter is probably meant, for it alone is true. The proposition, "*If a man plays recklessly he sometimes loses*," is interpreted by Professor Welton [2] as an A proposition, on the ground that it means "If any man plays recklessly *it always follows that he has some losses*". This latter, however, would scarcely be admitted as universally true, and hence

[1] *i.e.* interpreting the forms *assertorically.* [2] *op. cit.*, i., p. 273.

we may doubt whether that is the meaning of the proposition. If it were, the latter would probably be expressed, "*If a man plays recklessly he always loses*". If the original proposition be interpreted as A, how shall we classify the form just stated?

137. EDUCTIONS FROM CONDITIONAL PROPOSITIONS.—Taking antecedent and consequent in the same way as we have taken subject and predicate in the categorical, we can derive the same eductions from a conditional as from the corresponding categorical.

Take the A proposition, *If any S is M, that S is P.* The *obverse* will be got by changing the quality both of the proposition itself and of its consequent. The form arrived at will be awkward: "*If any S is M, it does not happen that that S is not P*"; but the obverse is needed mainly for the contrapositive and the inverse. In conversion, the original consequent will be the new antecedent and the original antecedent will be the new consequent.[1] A converts to I. Hence the *converse* of the example given above will be "*If an S is P, sometimes that S is M*". The *contrapositive*—which is the most important eduction from A—will be "*If any S is not P, that S is not M*". The *inverse* will be "*If an S is not M, then sometimes it is not P*".

If the A proposition, "*If S is M it is P*," be interpreted *modally* (to mean "*If S is M it must be P*"), its *converse* will be "*If S is P it may be M*"; its *contrapositive* "*If S is not P it cannot be M*"; its *inverse* "*If S is not M it need not be P*".

The conditional propositions, E, I, and O, yield inferences analogous to the corresponding categoricals, just as in the case of A. These inferences[2] illustrate the two rules commonly given in scholastic text-books on logic:—

(1) Affirmata conditione affirmari potest conditionatum, *sed non vice versa;*

(2) Negato conditionato negari potest conditio, *sed non vice versa.*

The student is recommended to take the following or some other concrete examples, and to work out all the eductions from each, firstly in the quantified, denotative, or *assertoric* form, and secondly in the abstract, connotative, or *modal* form:—

[1] The most important converse is the simple converse of E. "*If any S is M that S is not P*": "*If any S is P that S is not M*".

[2] The validity of these inferences is influenced by the various suppositions regarding existential import in precisely the same way as in the case of categoricals.

A—*If a swan is not white it is black.*
E—*If a man is wicked he is not wise.*
I—*If a story is believed it is sometimes true.*
O—*Sometimes if a man kills another he is not a criminal.*

How far categoricals can be inferred from conditionals, and *vice versa*, depends on how we interpret each form in regard to modality and existential import (135). Apart from implications of existence, the *assertoric universal* can be inferred from the *apodeictic*, but not *vice versa*, whether we are dealing with two categoricals, two conditionals, or one of each; while, on the other hand, the *problematic* proposition may be inferred from the *assertoric particular*, but not *vice versa*.

138. HYPOTHETICAL PROPOSITIONS: THEIR MODAL IMPORT. —Since pure hypothetical judgments do not differ fundamentally from conditionals (133), we need not re-examine the relation of the former to categoricals. We have seen that their reduction to the categorical form of proposition is more difficult than in the case of conditionals: that, in fact, instead of *reducing* them *to* categorical form, we rather *substitute* a *new* judgment, categorical in form, differing in import from the hypothetical itself, and having for *its* import what was only an implication of the latter—a statement about the latent grounds or conditions of the predication contained in the consequent of the latter. The hypothetical, *If A then C*, would yield the categorical: *The reality about which the assertion A is made is such that the assertion C is also* [*de facto*] *true* [or, *must necessarily be true*] *about that reality.*

The alternatives here offered as the import of this latter judgment suggest at once the possibility of a twofold interpretation of the hypothetical, *If A then C*. Does this proposition merely mean that *A* is a judgment with whose truth the falsity of *C* is *de facto incompatible*, or does it mean that *A* is a judgment from the truth of which the truth of *C is a necessary consequence?* Does it mean "If *A* is true then *C* is *de facto* true," or does it mean "If *A* is true then *C must be* true"? Does it merely deny the *actual conjunction* of *C false* with *A true*, or does it deny the *possibility* of such conjunction? In other words, is it *assertoric*, or is it *modal?* The difference between the two interpretations will be evident at once, if we are asked to contradict the proposition "*If A then C*". If it only denies the *actual* conjunction of *not-C* with *A*, we contradict it by the proposition "*A is true but C is false*," or "*A but not C*,"—which affirms that actual conjunction.

If, however, it denies the *possibility* of combining *not-C* with *A*, then its contradictory will merely assert that possibility (without asserting the *actuality*), and will therefore be "*If A is true C may be false*" or "*If A then perhaps not C*".

Although we meet with occasional examples of hypotheticals which may be interpreted *assertorically*, these are by no means typical of the pure hypothetical, which has usually a *modal* force. The following few instances of the hypothetical form used assertorically are given by Dr. Keynes:[1] "*If the flowers I planted in this bed were not pansies, they were violets*"—where I know that one of two propositions is true but do not know or remember which; "*If that boy comes back I'll eat my head*" (*vide Oliver Twist*)—where we emphatically deny a proposition by making its contradictory the antecedent of a manifestly false consequent; "*If he cannot act, he can at any rate sing*"—where we emphasize the truth of a proposition by making it the consequent of an admittedly true antecedent.[2] But these examples are not typical of the ordinary hypothetical. A sufficient proof that the latter is interpreted modally is found in the fact that in order to contradict "*If A then C*" we consider it quite sufficient to be able to say "*If A then not necessarily C*," without *categorically* affirming *A* and denying *C* in the copulative form "*A but not C*". We shall therefore interpret "*If A then C*" *apodeictically*, *i.e.* to mean that *C* is a *necessary consequence* of *A*.

This, of course, does not necessarily imply that *C* is an *immediate inference* from *A*. Where *C* is an immediate inference from *A*, where *A* gives explicitly the adequate ground for *C*, we might call the proposition a *formal* or *self-contained* hypothetical;[3] *e.g.* "*If all men are mortal, and the Pope is a man, then the Pope is mortal*". But, far oftener, *A* is only a part, though a necessary part, of the adequate ground of *C*, the suppressed past being taken for granted. The name *referential* has been suggested for this class of hypotheticals.[4]

The question has been raised whether, in the ideally perfect hypothetical —or conditional—proposition, the antecedent should give what is at once the *adequate* and the *only possible* ground for the consequent, and neither more nor less than this. If, in the proposition "*If S is M it is P*," the "*M*" gave us explicitly the *sufficient* and *only possible* ground for "*P*," and nothing else, the proposition would be *reciprocal*: we should infer from consequent to antecedent as necessarily as from antecedent to consequent, and we should be enabled to convert our A proposition *simply*, to "*If S is P it is M*".

[1] *op. cit.*, pp. 262, 263.

[2] Compare such forms as "*If Cromwell was an Englishman he was an usurper*".

[3] KEYNES, *op. cit.*, p. 264, footnote. [4] *ibid.*

So long as P can follow from *other grounds* besides M, we are not at liberty to convert "*If S is M it is P*," *simply*, just as we are not at liberty to convert "*All S's are P*," *simply* (118). And so far as the form of expression goes—so far, therefore, as *formal* logic is concerned—the statement that "*If S is M it is P*" does not in the least suggest that the fact of S being M is the *only possible ground* of S being P. Thus, while SM conditions or involves SP, SP does not condition or involve SM.

It is pointed out that the only reason for the onesidedness of this relation must lie in the fact that SM contains something over and above the ground of SP, something irrelevant to the latter; that if it contained only the ground of SP we could pass back from SP to SM as securely as we passed from SM to SP, inasmuch as *the necessity of conditioning does not differ from the necessity of being conditioned.* It is pointed out, furthermore, that there *are* such reciprocal hypotheticals known to us, *e.g.* "*If a triangle is equilateral it is equiangular*"—and indeed all those propositions in geometry which admit of a simple, "geometrical" converse (118). And it is suggested that our general aim should be, in every department of knowledge, to discover truths, laws, principles, which we could formulate in reciprocal hypothetical propositions whose antecedents would give a most exact and perfect knowledge of their consequents by revealing to us the *adequate* and *only possible* ground of the latter. At these we are invited to arrive by *eliminating* from the antecedent everything that is irrelevant to the consequent; the assumption being that if we could do this we should have a reciprocal hypothetical, inasmuch as any single, individual consequent will be seen, if we know it fully in all its relations, to be grounded in one single, individual, corresponding antecedent. In the inductive sciences this same attitude is expressed in the statement that the relation of cause and effect is reciprocal; that if we knew any phenomenon *fully* we should see that it could not be indifferently the outcome of any one of a number of alternative causes,[1] but must have sprung from one only: and the existence of coroner's inquests is given as an apt illustration of the soundness of this contention.

To the theoretical soundness of this view, in so far as it regards the *objective facts* of causality in the world, no exception can be taken. But in so far as it regards the *reasons* or *grounds* on which we base our knowledge and our inferences about these objective facts, it seems to confound the latter with our necessarily limited and imperfect knowledge of the former. It is not a true view of human knowledge—whether looked at as a process, or as a product—which regards our knowledge of any given truth as imperfect so long as we can prove that truth in more ways than one, so long as we can connect it as a consequent with more than one antecedent. No doubt, the different channels of proof for one and the same truth—its different logical antecedents—must be themselves interrelated as parts of one and the same rational system—so that if our knowledge of all things were direct, intuitive, and adequate, we should see how every single element of the whole of reality involved everything else—but then our knowledge would not be human: it would be divine.

At such a stage, or in such a state, of knowledge as that, besides the

[1] *Cf.* Part iv. 221.

fact that inference would be superfluous (134), every judgment would be likewise superfluous, because everything would be already seen to involve everything else.

Moreover, the attempt to approach such an ideal by eliminating what is irrelevant from the antecedents of our hypothetical judgments, would make these judgments tautological. When we connect SM with SP in the non-reciprocal hypothetical, "*If S is M it is P*," our aim is to connect SP with SM including whatever is irrelevant in the latter. It is just the "irrelevant element" in SM that makes it to be SM as distinct from SP, so that were we to eliminate this "irrelevant element" from SM, it would become SP, and thus "the judgment is made a tautology, that is, destroyed".[1]

In fact, it is the presence of such an element in the premisses or antecedents of our reasoning processes that makes discursive reasoning possible at the same time that it renders this necessary. Take the typical syllogism, "*If M is P and S is M, then S is P*". It is the very presence of the "irrelevant" M in the antecedent that enables us to reach the consequent SP, from which the M is eliminated. "But how in this case," writes Dr. Keynes, "we are to eliminate the irrelevant from the antecedent it is difficult to see. Our object is to eliminate M from the consequent, and if in advance we were to eliminate it from the antecedent the whole force of the proposition would be lost.[2]

Finally, there is obviously another way of reaching the reciprocal hypothetical besides by eliminating from SM what is irrelevant to SP, and that is to *expand* SP by putting into it all that does follow from the antecedent SM.

We may, perhaps, aim at expressing the abstract truths of the exact or mathematical sciences by means of reciprocal hypotheticals; but if we set up such an ideal in the concrete, inductive, historical and social sciences, or seriously ambition the realization of it, we are not likely to achieve any notable measure of success.

Besides all this, the hypothetical judgment is not superior to the categorical as a form of human knowledge; indeed it emphasizes more pointedly than the categorical that imperfection which consists in doubt or ignorance as to the *real existence* of the objects thought about.

139. Opposition of Hypothetical Propositions.—If it be the function of the pure hypothetical to *affirm* one judgment to be a *necessary consequence* of another, there would seem to be no room for a *negative* hypothetical.[3] The form "*If A then not C*" would be really affirmative, for its force would be to *affirm* that the negation of C (or the *falsity* of C, or the *truth* of $\bar{C}$) *is* a necessary consequence of the truth of A. There would thus be no room for distinctions of *quality*; nor, therefore, for a square of opposition.

There is, in the next place, certainly no room for distinctions

[1] Bosanquet, *Logic*, i., p. 261;—*apud* Keynes, *op. cit.*, p. 272.
[2] *ibid.* [3] *Cf.* Joseph, *op. cit.*, p. 164.

of *quantity* in the pure hypothetical; for its antecedent is not an event, nor a group of attributes occurring in time or space (133), but an abstract judgment which is either true or false once and for ever (84). *Quantity* is here replaced by *modality*. But, turning to modality, we encounter another obstacle to the formation of a square of opposition: for our conception of the hypothetical judgment—as one that asserts a *necessary* consequence—is applicable only to the apodeictic forms, *If A then C* and *If A then not C*, and not to the problematic forms, *If A then perhaps C*, and *If A then perhaps not C*.[1]

It would be desirable, if possible, to overcome these two difficulties, in *quality* and *modality*, and to construct for hypotheticals a square of opposition analogous to the modal square for conditionals and categoricals. The distinctions of quality can be preserved, as in the case of conditionals, by regarding the form which leads up to a *negative consequent—If A then not C*—as negative. The distinction between *apodeictic* and *problematic* can be introduced only by so widening our definition of the hypothetical as to admit of the latter forms (*If A then perhaps C; If A then perhaps not C*). There is no reason why we should not do this. The four forms, A, E, I, and O, will be hypotheticals, if, with Dr. Keynes,[2] we "define a hypothetical as *a proposition which, starting from the hypothesis of the truth (or falsity) of a given proposition, affirms (or denies) that the truth (or falsity) of another proposition is a necessary consequence thereof*". We then have the following four forms:—

A—*If A* (or *not A*) *then* (*necessarily*) *C*.
E—*If A* (or *not A*) *then* (*necessarily*) *not C*.
I—*If A* (or *not A*) *then possibly* (*perhaps*) *C*.
O—*If A* (or *not A*) *then possibly* (*perhaps*) *not C*.

What corresponds to *actual existence* in the universe of discourse, in regard to the existential import of assertoric categoricals, is *possible truth* in regard to these modal hypotheticals. Thus, if we start with the independent assumption of the *possible truth* of *A*, the doctrine of opposition holds good here, just as it did in the analogous assumption in the case of categoricals (124). Without this assumption, however, A and O, E and I, will not be really contradictories unless A and E be interpreted *differently* from I and O in regard to implying the *possible truth* of their antecedents. There is no need to

[1] In these problematic forms, as in conditionals, it will be observed that "*Though*" may with advantage be substituted for "*If*".

[2] *ibid.*, p. 266.

examine the results of the various possible suppositions ; for the same principles apply here as in the case of categoricals.

We referred above (136) to the danger of mistaking the contrary for the contradictory of a conditional proposition. The same danger exists in the case of the hypothetical. The proposition *If A then not C* cannot possibly be the *contradictory* of the proposition *If A then C.* If both be interpreted *modally* and as implying the possible truth of their antecedents, they cannot be true together ; but they may both be *false* together : for neither the truth nor the falsity of *C* may be a necessary consequence of the truth of *A* :[1] in which case they would be contraries. If, being modal, they be not understood to imply the *possible truth* of their antecedents, not only might both be false together, but both might be true together, thus uniting to establish the *impossibility of A*, just as in the analogous case *All S's are P's* and *No S's are P's* might both be true while establishing the non-existence of *S* (127).

If both be interpreted *assertorically*, *If A then C* merely denies $A\ \bar{C}$, and *If A then not C* merely denies *A C :* hence both cannot be *false* together, for that would mean the simultaneous truth of $A\ \bar{C}$ and *A C*. But both may be *true* together ; for since they merely deny *A C* and *A C* they are simultaneously compatible with $\bar{A}\ C$, or with $\bar{A}\ \bar{C}$. For example, neither of the propositions "*If this pen is not cross-nibbed it is corroded with ink*" (*If* $\bar{A}$ *then C*) and "*If this pen is not cross-nibbed it is not corroded with ink* (If $\bar{A}$ then $\bar{C}$) can be said to be false in case the pen *is* cross-nibbed.[2]

Hence, on the assertoric interpretation, *If A then C* and *If A then not C* are *subcontraries.*

140. Eductions from Hypothetical Propositions.—As in the case of conditionals, the most important eductions here are those analogous to the contrapositive of A and the converse of E in categoricals.

A—From "*If A then C*" we infer by *contraposition* "*If not C then not A*". For example, from the proposition, "*If there is a just God the wicked will be punished*," we infer "*If the wicked are not punished there is no just God*".

E—From "*If A then not C*" we infer by *conversion* "*If C then not A*". For example, from the proposition "*If the scriptures speak truly the human soul will never die*" we infer "*If the human soul dies the scriptures do not speak truly*".

The *converse* and *inverse* of A, and the *contrapositive* and *inverse* of E are, of course, obtainable ; but they are of little importance, seeing that they give us practically no knowledge : for they are all *problematic*, merely stating that something *may*, or *need not*, follow from something else.

[1] *If A then not-C* means that *not-C* follows necessarily from *A*—which is much more than to say merely that *C* does *not* follow necessarily from *A*.

[2] Keynes, *op. cit.*, p. 267.

The most common and dangerous fallacy in eductions from hypotheticals is that which is analogous to the simple conversion of a categorical A proposition, *viz.* the inferring of "*If C then A*" from "*If A then C*". Similar to this is the fallacy of inferring a simple contrapositive from E, *i.e.* of inferring "*If not C then A*" from "*If A then not C*". Finally, the inferring of a simple inverse from either A or E is a fallacy not always avoided. "*If A then C*" does *not* yield "*If not A then not C*," but only "*If not A then perhaps not C*"; nor does "*If A then not C*" yield "*If not A then C*," but only "*If not A then perhaps C*".

KEYNES, *Formal Logic*, part ii., chap. ix. JOSEPH, *Logic*, pp. 163-66. WELTON, *Logic*, i., pp. 181-86, 244, 271-73. VENN, *Empirical Logic*, 248-64.

CHAPTER IX.

DISJUNCTIVE AND ALTERNATIVE JUDGMENTS AND PROPOSITIONS.

141. VARIOUS FORMS OF DISJUNCTIVES AND ALTERNATIVES. —The titles "disjunctive" and "alternative" are indiscriminately applied to those forms of judgment and proposition *which offer us alternative subjects or alternative predicates, or a choice of simple judgments*. They may be represented symbolically by "*Either A or B is C*," "*A is either B or C*," "*Either A is B or C is D*," or, more briefly, by "*Either X or Y*," where X and Y stand for simple propositions. It would be better to call this form of proposition *alternative*, and to reserve the name *disjunctive* for the form "*Not both X and Y*"—which *disjoins* or separates two propositions by asserting that both are not simultaneously true. We shall follow this usage. But it must be borne in mind that logicians generally draw no distinction between the two titles (84); and, moreover, the type of judgment just referred to as disjunctive is not really different from the alternative: for every such disjunctive involves an alternative, and *vice versa*. "*Not both X and Y*" is equivalent to "*Either X or* $\bar{Y}$": and "*Either X or Y*" is equivalent to "*Not both X and Y*". "*No man can serve God and Mammon*," may also be expressed, "*Every man must either not serve God or not serve Mammon*"; "*Every swan is either black or white*" may be expressed "*No swan is both not-black and not-white*, or—which is the same—"*There is no swan that is neither black nor white*".

If we distinguish between the disjunctive and alternative forms we shall have (83): (1) the *conjunctive*[1] or *copulative* proposition. "*Both X and Y*," denied by the *disjunctive* proposition "*Not both X and Y*"; and (2) the *alternative* proposition "*Either X or Y*" denied by the *remotive* proposition "*Neither X nor Y*".

[1] Some logicians use the term "conjunctive" to designate the "If" proposition, inasmuch as this form also *conjoins* two simple propositions by asserting a dependence of one upon the other. *Cf.* JOSEPH, *op. cit.*, p. 163.

142. PROPOSITIONS WITH ALTERNATIVE SUBJECTS.—We have purposely defined the alternative or disjunctive judgment very widely, so as to take in all the forms discussed by logicians. The first of these—the proposition with an alternative subject—is not usually regarded as an alternative. However, it requires recognition somewhere, for it occasionally occurs; and it may be appropriately referred to here, seeing that the general function of the alternative judgment is, as we shall see, to express knowledge which is too vague and undefined in its reference to admit of simple categorical statement, and that the form "*Either A or B is C*" expresses one way in which our knowledge may be indefinite. The force of introducing an alternative into the subject of a proposition is to make the proposition *particular* or *indefinite* (93) if it were universal, and more indefinite still if it were already particular. Compare, for example, "*John will be present*," with "*Either John or James will be present*"; "*All the clergy will support the candidate*," with "*Either all the clergy, or all the barristers, or all the doctors, will support the candidate*" [*i.e.* (categorically) "*One at least of the learned professions* will support him"]; "*Some of the Liberals will vote for us*," with "*Either some of the Liberals or some of the Home-Rulers will vote for us*" (which might be expressed categorically, "*Some of the opposition* will vote for us.")[1] Obviously, judgments of this class differ in language rather than in thought from categoricals.

143. PROPOSITIONS WITH ALTERNATIVE PREDICATES.—These form a more distinctive class than the former; although here, too, the divergence from the ordinary categorical is not great. In the affirmative categorical the predicate is indefinite or undistributed: the introduction of an alternative into it makes it still more indefinite, and widens the reference of the subject accordingly. Compare the proposition "*He is a lawyer*" with the proposition "*He is either a lawyer or a doctor or a clergyman*". The latter has the *categorical* equivalent "*He is a member of a learned profession*," because the *genus*, to which the sub-classes enumerated in the predicate belong, happens to have a name of its own: so, too, the reference in "*He is either a solicitor or a barrister*" is exactly the same as in the categorical "*He is a lawyer*," though wider than the reference in either of the categoricals "*He is a barrister*" or "*He is a solicitor*".

Thus we see how little these judgments differ from categori-

[1] *Cf.* VENN, *Empirical Logic*, p. 247.

cals; and when the proposition is negative the introduction of an alternative into the predicate does not give any choice of predications at all: it cannot have the effect here of widening or making more indefinite the class to which the subject is referred, for this is already distributed, and thus wholly excluded from the subject, in a negative proposition. When we introduce an alternative into the predicate of the judgment "*No X is Y*," and say "*No X is either Y or Z*" we do not increase but diminish the vagueness of the reference of the subject. This form, "*No X is either Y or Z*," is called the *Remotive Proposition*—because it *removes*, or *denies altogether*, the two simple judgments (*X is Y* and *X is Z*) that are *conjoined* in the *copulative or conjunctive proposition.*

Thus we see that not every proposition which has an alternative predicate gives an alternative predication, or a choice of judgments. We may point out here that, similarly, not every proposition which has a conjunctive subject gives a conjunctive or copulative proposition (87), in the sense of a compound proposition which asserts the simultaneous truth of two judgments. For example, "*Two and two are four*," "*Black and white are incompatible terms*,"—these are not copulative, but simple, categorical propositions.

From the examples given above we see that it is possible to substitute for the judgment with an alternative predicate a simple, categorical judgment, whenever the wider class, under which the alternatives are sub-classes, happens to have a special name. More usually, perhaps, this wider class has no special name. We may instance: *He is either a knave or a fool; We must be either vaccinated or run the risk of small-pox; The election will turn either on the eight-hours question or on the Home Rule question.* Although we might conceivably invent a name for every group of sub-classes, thus brought together in alternative predication, it would be useless and absurd to do so in the cases of sub-classes which are too disparate to be conveniently grouped together. The alternative judgment enables us to recognize them as co-existing classes, to the combined extension of which we may refer a given subject; and this is sufficient for our needs.

When the subject of any judgment with an alternative predicate is preceded by the distributive *All* (*Every*), or by the indefinite *Some*,—*e.g.* "*All* (or *Some*) *X's are either Y or Z*"—the form is ambiguous, for it does not tell us whether the "*All*" or "*Some*"

in question will be found exclusively in the class *Y*, or exclusively in the class *Z*, or partially in one and partially in the other. This ambiguity prevents us from resolving the *universal* judgment "*Every X is either Y or Z*" into the *two alternative judgments* "*Either every X is Y or every X is Z*". But the corresponding ambiguity in the case of "*Some*" does not prevent us from resolving "*Some X's are either Y or Z*" into "*Either Some X's are Y or Some X's are Z*". When the subject is a *collective*, or a *singular*, term, no such ambiguity can arise.

Dr. Keynes points out that when the predicate of an affirmative proposition is *conjunctive*, a similar obstacle to regarding the proposition as *compound* (88) arises in the case of *particulars*, but not in the case of *singulars* or of *universals*. "Thus, *This S is P and Q = This S is P and this S is Q; All S is P and Q = All S is P and All S is Q.* From the proposition *Some S is P and Q* we may indeed infer *Some S is P and Some S is Q;* but we cannot pass back from this conclusion to the premiss, and hence the two are not equivalent to one another".[1]

144. THE ALTERNATIVE BETWEEN TWO JUDGMENTS OF INDEPENDENT IMPORT.—It is possible, as in the case of "*If*" propositions, to draw a distinction between two classes of "*Either . . . or*" propositions. The choice may be between two or more predicates which signify properties or events occurring in time and space; or between two or more judgments of independent import, whose truth or falsity is independent of time and space. An example of the former would be: *Every blood vessel is either a vein or an artery.* An example of the latter: *Either there is a future life or wickedness remains unpunished.*

Propositions of the former of these two classes are called by Dr. Keynes *complex* propositions, or *propositions* with *complex terms*, as opposed to *compound propositions* (84). A *complex term* he defines as a combination of two or more simple terms; and such combination can be either *alternative* or *conjunctive*.[2] Care must be taken to distinguish the "*complex term*" in this technical sense from the *many-worded term* (or *terminus complexus* of scholastic logic) which also results from the combination of two or more simpler terms, as, for example, "Highest mountain in Asia" (22). The difference consists in this, that in the latter the simpler elements or "notes" are constituents of *one mental object*, and are held as one whole in the mind, whereas in the former case the combining elements are held apart, as distinct objects, in the mind. *S is both P and Q; S is P Q; S is either P or Q:* would be examples of propositions with complex predicates. *Whatever is S and*

[1] *op. cit.*, p. 276. [2] KEYNES, *op. cit.*, pp. 276, 468 *sqq.*; *supra*, 94.

M is P; S M is P; whatever is either S or M is P: would be examples of propositions with complex subjects.

Propositions with alternative predicates Dr. Keynes would regard not as "*compound*" but as "*simple*" propositions "with complex terms," and as equivalent to categoricals: whereas the second class of alternatives, to be referred to presently, are "*compound*," and not reducible to categoricals. Similarly, he regards the "conditional" proposition (*If S is M it is P*, or *Whenever S is M it is P*, or *S M is P*) as a "*simple*" proposition with a complex subject, and equivalent to the categorical; while he regards the "pure hypothetical" (*If A is true C is true*) as a compound proposition, not so reducible.

By means of these distinctions he leads up to a very symmetrical classification of judgments into *simple assertoric*, *simple modal*, *compound assertoric* and *compound modal*.[1] The alternative proposition will, as we shall see, be best interpreted assertorically; hypotheticals we have seen to be always modal; conditionals to be usually so; while the categorical form is the natural expression of the simple assertoric judgment. Thus, we have the

SIMPLE JUDGMENT expressed	*Assertorically* by the *Categorical* proposition;
	Modally by the *Conditional* Proposition;
COMPOUND JUDGMENT expressed	*Assertorically* by the *Alternative*[2] proposition;
	Modally by the *Hypothetical* proposition.

As in the case of "*If*" propositions, so also here, the distinction between the class called "simple" with alternative predicate, and that called "compound," is not fundamental. It may lend itself well to logical treatment, but it does not remove the fundamental sameness there is in the act of judgment as such—whether this be expressed by a logically "simple," or by a logically "compound," proposition. The fact that "*This S is either P or Q*" may be regarded as logically "compound," while "*Every S is either P or Q*" must be regarded as logically "simple"; or that "*If this S is M it is P*" is "compound," while "*If any S is M it is P*" is "simple" (133 n.): shows clearly enough that the distinction is not a fundamental one, but rather conventional, and established for convenience of logical treatment.

In the sections that follow we shall have regard both to the propositions which merely give alternative predicates and to those which offer a choice between independent judgments.

145. IMPORT AND FUNCTION OF ALTERNATIVE JUDGMENTS.—We must now determine whether in the alternative proposition the alternatives are meant to be mutually exclusive: whether "*Either X or Y*" is to be interpreted as "*Either X or Y or possibly both*," or as "*Either X or Y but not both*". In many, if not in most, cases, the matter or meaning will determine this question for

[1] *op. cit.*, p. 282; *cf. supra*, 83, 84.

[2] To which we may add the *Remotive*, *Disjunctive*, and *Conjunctive* or *Copulative*, forms.

us. We meet with both sorts of alternatives. For instance, the propositions "*He will either pass or fail*," "*He was either first or second in the race*," "*This book is to be bound in half-calf or in morocco*," "*Every swan is either black or white*," give us mutually exclusive predicates. But this springs from the *matter*, not from the *form* of these statements. So far as the *form* of the proposition is concerned, there is nothing to indicate that the alternatives are meant to be exclusive. Nor are they exclusive in every case. For example, the propositions "*He has either used bad text-books or has been badly taught*," "*Either the witness is perjured or the prisoner is guilty*," "*Every candidate must be a graduate either of the Queen's or of the National or of Trinity*," "*He is either very timid or very modest*,"—all offer us alternatives which are not, and are not meant to be, mutually exclusive.

Hence, since the form of expression "*Either X or Y*" is capable of both interpretations, we must in logic—in virtue of the Law of Parsimony—accept the one which bears the lesser amount of meaning,—and this, as we shall see presently, is the *non-exclusive* interpretation.

In the latter, too, we are keeping more closely to ordinary usage, for here the alternatives are not as a rule meant to be mutually exclusive. It is only when they are incompatible that they are mutually exclusive, and in such cases we may express the judgment symbolically in the form "*Either X or Y but not both*". Mr. Joseph, in his *Introduction to Logic*[1] expresses the view that even where we cannot know from the subject-matter "that the alternatives exclude each other . . . it is perhaps safer to assume that they are intended as mutually exclusive, unless the contrary is stated; a legal document is careful so to write it, where 'A or B or both' is meant, or to write 'A and/or B' with that signification". But such an assumption would not be safe if applied universally to the ordinary usage of language. Moreover, the universal adoption of this exclusive interpretation would increase unduly the compound character of the form in question, and would lead to complicated and curious results. For example, the denial of "*Either X or Y but not both*" would be "*Either both of them or neither of them*". Then, again, on the exclusive interpretation of "*Either X or Y*," the two alternatives are $X\,\overline{Y}$ and $\overline{X}\,Y$; but these also are the two alternatives in "*Either* $\overline{X}$ *or* $\overline{Y}$" on the same interpretation: from which it follows that "*Either X*

[1] p. 167.

or Y" would have exactly the same meaning as "*Either* $\overline{X}$ *or* $\overline{Y}$". "*Anyone who acts thus is either dishonest or unwise*" would mean exactly the same as "*Anyone who acts thus is either honest or wise*".

The exclusive interpretation of the alternative proposition is advocated by some logicians on the ground that the function of the judgment with alternative predicates is *to express the results of a logical division or classification:* that, therefore, the alternative predicates, which give the sub-classes within the logical whole or system, should, according to the rules of logical division, be *mutually exclusive*, and at the same time *collectively exhaustive* of the denotation of the subject-term (62). Hence, they say, the ideally perfect alternative proposition should fulfil these conditions. No doubt, the alternative proposition *can* be made to fulfil this function, and, in so far as it does, it must verify the conditions referred to. But the fact remains that the alternative proposition is *not* limited to, or even used primarily for, this purpose, but is used to express alternatives independently of logical division. And so long as it is used in this latter way it must be interpreted in the non-exclusive sense.

Of course, *if* we assume that the alternative predicates of such judgments *always* give us what we can make out to be *co-ordinate sub-classes under the genus* indicated by the subject term, then indeed the alternatives must always not merely *differ* somehow from one another but be *exclusive* of one another and *incompatible with* one another. But they are not always mutually exclusive, co-ordinate classes; though they must always *differ somehow* from one another: else the alternatives "would merge into one".[1] But *difference* is not *exclusion* or *incompatibility*. When we say of a "candidate" that he must be "either a graduate of the Queen's or of the National, etc.," we assert that he must have the common or *generic* attribute of "graduateship" in which all these alternatives agree; we see, too, that the alternatives *differ* as to "place of graduation"; but we see at the same time that the candidate may combine in himself a number of these "*differences*" —that they are *not mutually exclusive* differences. Hence it is scarcely accurate to say that the points in which such attributes differ from one another are "points of exclusion".[2] They are not *the same* of course; but the same subject may possess them all, and belong simultaneously to the classes denoted by each of them respectively.

146. Relation of Alternative to Hypothetical and Categorical Judgments. Opposition and Eduction in Alternative Judgments.—Since every alternative proposition

[1] Welton, *Logic*, i., p. 190. [2] *ibid.*

gives some sort of choice between two or more alternatives, it follows that by denying any one member (or more) we can affirm the remaining member (or members—*alternately*—if there remain more than one). Thus "*Every blood vessel is either a vein or an artery*" is equivalent to the conditional proposition "*If any blood vessel is not a vein it is an artery*" (or its contrapositive, "*If any blood vessel is not an artery it is a vein*"). "*Either there is a future life or wickedness remains unpunished*" is equivalent to the hypothetical "*If there is no future life wickedness remains unpunished*" (or its contrapositive). In general: "*Either X or Y*" is equivalent to "*If not X then Y*" (with its contrapositive, "*If not Y then X*").

Were we, however, to interpret the form "*Either X or Y*" in the *exclusive* sense, we should need, in order to give its full import, not only the hypothetical "*If not X then Y*" (with its contrapositive), but also this other hypothetical "*If X then not Y*" (with its converse, "*If Y then not X*"). This shows clearly how much more meaning is contained in the *exclusive*, than in the *non-exclusive*, interpretation.

Owing to the fact that the disjunctive judgment contains two, or possibly four, hypotheticals, it "has been suggested that the disjunctive judgment is in reality a combination of hypotheticals. . . . Doubtless these [hypotheticals] are involved [in the disjunctive]; but we do not therefore get rid of the peculiar nature of the disjunctive judgment. For they are not four independent hypothetical judgments; and their force is not appreciated, unless it is seen that together they make up a disjunction, that they offer us a choice between alternative hypotheses. Thus disjunctive judgment at once includes and goes beyond hypothetical, in the same sort of way as hypothetical includes and goes beyond categorical. An hypothetical judgment makes an assertion, like a categorical; but what it asserts is a relation of a consequent to a condition. A disjunctive judgment involves hypotheticals; but it presents them as alternatives and asserts the truth of one or other of them." [1]

The alternative judgment gives, moreover, not merely a choice between hypothetical judgments, but a choice between categorical judgments (144), or between predicates of a subject (143), or subjects of a predicate (142). In some of these cases it may be easily resolved into a categorical judgment by substituting for the

[1] JOSEPH, *op. cit.*, p. 168.

alternatives a wider genus including them; and in *all* cases it has some categorical judgment underlying it, exactly as in the case of the "*If*" judgment (132). In all cases, too, it is an expression of *limited* knowledge, of knowledge combined with ignorance or doubt: a limited knowledge of some genus or universe of reference, and ignorance about the species or sub-classes in that sphere.[1] But the categorical which we may substitute for it is subject to precisely the same limitations.

Some writers have claimed that the disjunctive "goes beyond" the hypothetical, and this in turn beyond the categorical judgment, not merely in the sense explained by Mr. Joseph, but in the sense that the disjunctive judgment is the ideal or most perfect attainable form of human thought—and the categorical presumably the least perfect. This, however, is a view which seems to have been advocated without sufficient grounds.

Professor Welton, following Bosanquet, holds that the disjunctive contains some positive or categorical element not contained in the hypothetical. "Were we confined to the latter," he writes,[2] "thought would be condemned to an endless regress. For though *If S is M it is P*, gives us in *M* the ground of *P*, yet we must go on to similarly ask for the ground of *M*. *This regress can only be avoided by assuming that the judgment refers to a more or less self-contained system.* It is such a system that the disjunctive judgment *in its ideal form* makes explicit in its enumeration of the sub-species under the subject genus. It is in *the exhaustive character* of this enumeration that the sufficiency of the hypothetical as a statement of a condition is found. Hence we find in the disjunctive *the mode of expressing that systematic connexion* which is *the only form in which we can think reality*."

[1] When our knowledge about the real state of things is indefinite, *i.e.* mixed with doubt or ignorance, this indefiniteness cannot of course belong to the facts themselves, but only to the state of our knowledge—in whatever form of judgment we express the latter: categorical, hypothetical, or disjunctive. The facts themselves are *definite*, *determinate*. And just as the study of each of the former kinds of judgment leads to metaphysical problems about our mode of conceiving the real (*cf.* 98, 132), so does the study of the disjunctive judgment (*cf.* JOSEPH, *op. cit.*, p. 168). The problem here would be: Is the disjunctive *in the facts*, or is it merely *between our partial and limited mental views* of the facts? And the correct answer would seem to be that it is always only in our knowledge, or in *possible*, *ideal* reality, in reality only as *determinable*, and never in *actual* facts. According to Mr. Joseph, such a proposition as "Number is either odd or even" "seems to express a disjunction in the facts; and the species of the same genus are a kind of real disjunction" (*ibid.*). But species and genus as such belong to the ideal or conceptual order only, and there alone is disjunction admissible. Whatever *exists* must be a determinate individual, itself and no other, as Mr. Joseph himself immediately asserts: "If a colour is to exist, it must be blue or red or some other colour, and if it is one it can be none of the others. We come back here upon the same truth which met us in considering negative judgments, that a thing is definitely this or that by not being something else; we have to recognize also that there is often a limited number of possibilities, in the way, for example, of colour, or of animal species, but why or how there should be a limit to what is possible in the universe is a hard question" (*ibid.*).

[2] *op. cit.*, p. 191, italics ours.

On this we must observe that ordinary usage forbids us to set up as the ideal of the alternative judgment the one that would make our knowledge of a system explicit by enumerating all the classes contained in it; and it is only on the assumption that the alternative judgment attains to this ideal of exhaustive enumeration that Professor Welton claims for it superiority over the hypothetical.

If we could know things only in so far as we classified them and gave them a "systematic connexion" with all other things, if classification were the only ideal of human knowledge, "the only form in which we [could] think reality," then, perhaps, the alternative form of proposition could be set up as the expression of our highest knowledge. But our highest knowledge is not based on classification, nor does it tend to such an ideal. The law of gravitation, for example, has no affinity with any process of classification or of disjunctive or alternative thought.[1] Its natural form of expression is the conditional, or the categorical, proposition—the latter being, perhaps, superior to the former in so far as it may be held to eliminate any element of doubt (about the reality, existence, occurrence of the things or events thought about), suggested or implied by the "*If*" judgment; or at all events in so far as it is the form of thought which, in ultimate analysis, is found to underlie both the "*If*" judgment (132) and the alternative judgment.

The transition from the "*If*" form, "*If S is M it is P*," to the categorical "*S M is P*," or "*S because it is M is P*," does not necessarily involve the mediation of an alternative judgment. If we ask why is *M* a ground for *P*, and reply "*M because it is N is P*," we are showing that the evidence for our former statement is *mediate*, and that we are tracing it back to its sources. Now it is quite true that this process of backward search in justification of our original statement, "*If S is M it is P*," must come to an end somewhere. *Mediate* evidence must ultimately be based on evidence that is *immediate*. We must come to some statement, *e.g.* "*N because it is O is P*," *the ground for which is self-evident:* to a point at which, when we are asked why *O is P*, we can answer: *because it is so, self-evidently.* The recognition of some truth, *e.g.* "*O is P*," as *self-evident*, is all that is involved in the grounds of any "*If*" judgment. But that the only way of reaching such self-evident truths, and thus avoiding an endless "regress," is "by assuming that the judgment refers to some self-contained system," and by making that system explicit in an "ideal" alternative judgment—that is what has not been shown, and cannot be shown, to be universally true.

Provided we interpret "*If*" judgments and alternative judgments similarly as regards modality, we may pass from either form to the other. "*If S is M it is P*" will yield "*S is either* $\bar{M}$ *or P*". Its contrapositive, "*If S is* $\bar{P}$ *it is* $\bar{M}$," will yield exactly the same alternative, "*S is either P or* $\bar{M}$": which shows that *equivalent* propositions in one form may yield *identical* propositions in another. Similarly, the forms "*If A then C*" and "*If* $\bar{C}$ *then* $\bar{A}$" yield "*Either* $\bar{A}$ *or C*".

[1] *Cf.* KEYNES, *op. cit.*, p. 283, n. 2.

Opposition of Alternative Judgments.—Before considering the opposition of these judgments we must decide whether they are to be interpreted *modally* or *assertorically*. Both interpretations are possible, but the assertoric seems to keep more closely to common usage. This will be seen by asking how a person would contradict the ordinary alternative forms "*Every S is either P or Q*," and "*Either X or Y*". Interpreted modally, their contradictories would be "*An S need not be either P or Q*," and "*Possibly (Perhaps) neither X nor Y*," respectively. Interpreted assertorically, they would be "*Some S's are neither P nor Q*" and "*Neither X nor Y*" respectively. The latter forms would in all probability be regarded as necessary for denial of the original propositions, thus showing that the assertoric interpretation is the more usual one.

The typical modal forms would be:—

A—*Either X or Y*
E—*Neither X nor Y* } Necessary.

I—*Perhaps Either X or Y*
O—*Perhaps Neither X nor Y* } Contingent.

The typical assertoric forms would be

A—*All S's are either P or Q*
E—*No S's are either P or Q* } Universal.

I—*Some S's are either P or Q*
O—*Some S's are neither P nor Q* } Particular.

It will be observed that in both cases the negative forms are not *alternative* but *remotive* propositions, and are equivalent to the *copulative* or *conjunctive* categoricals: *Both* $\bar{X}$ *and* $\bar{Y}$; *Perhaps both* $\bar{X}$ *and* $\bar{Y}$; *All S's are both* $\bar{P}$ *and* $\bar{Q}$; *Some S's are both* $\bar{P}$ *and* $\bar{Q}$.[1]

Eductions from Alternative Judgments.—In order to secure eductions analogous to those from categorical propositions, we

[1] For a fuller treatment of inferences from *complex* and *compound* (alternative and conjunctive) propositions, see Keynes, *op. cit.*, *Appendix C*, chap. ii., pp. 478 *sqq.* When dealing with propositions that have simple predicates, we cannot distinguish between *merely incompatible* and *contrary* propositions. We can, however, when the predicate is a complex term. Thus the propositions, "He is both sober and industrious" and "He is either not sober or not industrious" (or "He is not both sober and industrious"), are *contradictories*. The propositions, "He is both sober and industrious" and "He is neither sober nor industrious," are *contraries*. The propositions "He is both sober and industrious" and "He is not sober" (or "He is not industrious") are *incompatible* or *repugnant*, simply.

must take the assertoric form which gives alternative predicates. Care must be taken, in obversion, to secure the correct *negative* or *contradictory* of the original alternative predicate. The required form is secured by substituting for the simple terms their contradictories, for alternative combinations conjunctive ones, and for conjunctive combinations alternative ones.[1] Thus, from the A proposition we may derive the following eductions :—

Original Proposition	*Every S is either P or Q.*
Obverse . . .	*No S is both $\overline{P}$ and $\overline{Q}$.*
Converse . . .	*Some things that are either P or Q are S.*
Obverted Converse .	*Some things that are either P or Q are not S.*
Partial Contrapositive	*Nothing that is both $\overline{P}$ and $\overline{Q}$ is S.*
Full Contrapositive .	*Everything that is both $\overline{P}$ and $\overline{Q}$ is $\overline{S}$.*
Partial Inverse . .	*Some $\overline{S}$'s are neither P nor Q.*
Full Inverse . .	*Some $\overline{S}$'s are both $\overline{P}$ and $\overline{Q}$.*

From the I proposition we may infer as follows :—

Original Proposition	*Some S's are either P or Q.*
Obverse . . .	*Some S's are not both $\overline{P}$ and $\overline{Q}$.*
Converse . .	*Some things that are either P or Q are S.*
Obverted Converse .	*Some things that are either P or Q are not $\overline{S}$.*

Were we to take the alternative form which gives us a choice between two independent judgments—"*Either X is true or Y is true*"—we might set down as a form analogous to the obverse the *disjunctive* form "*$\overline{X}$ and $\overline{Y}$ are not both true*". But, since these forms have no elements analogous to subject and predicate in the categorical, we cannot derive any of the ordinary eductions from them.

KEYNES, *Formal Logic*, pt. ii., chap. x. ; Appendix C., chap. i. WELTON, *Logic*, i., pp. 187-92, 246, 274. JOSEPH, *Logic*, pp. 167-8. VENN, *Empirical Logic*, pp. 242-48.

[1] KEYNES, *op. cit.*, p. 488.

PART III.

REASONING AND SYLLOGISMS.

CHAPTER I.

NATURE, STRUCTURE, AND AXIOMS OF THE PURE SYLLOGISM.

147. MEDIATE AND IMMEDIATE INFERENCES.—We have already distinguished (116) between the process of *Immediate* Inference—by which we draw out the various implications of a *single* judgment, without having recourse to the "medium" of a second judgment—and the process of *Mediate* Inference, or Reasoning proper. Mediate inference is so called because the conclusion is reached by comparing each of the terms of the latter, *S* and *P*, alternately, with a third, intermediate or *middle* term, *M*. The two propositions in which *M* is compared with *P* and *S*, respectively, are called *Premisses* [*præmissæ*, "sent before"]—or, both together, the *Antecedent*, or *Reason*, or *Ground;* the third or inferred proposition is called the *Conclusion* or *Consequent*—or, if stated first, the *Question* or *Thesis*. The mental process itself is called *Reasoning* [*Ratiocinatio* or *Argumentatio*]; the mental product or result is also called a *Reasoning* [*Ratiocinium*]; the verbal expression of the mental process or result is called an *Argument* [*Argumentum*].

We see, then, that while immediate inference starts from a *single* proposition, mediate inference starts from *two*. We might, therefore, perhaps, define *Mediate Inference* as a *reasoning process or argument in which two judgments are seen to be so related that a third necessarily follows from them*. This definition, however, while emphasizing the *necessary sequence* which constitutes the formal aspect of all inference, gives us no insight into the ground or reason for this necessity.

It is practically Aristotle's definition of the *Syllogism* (Συλλογισμός): Λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα

εἶναι :[1] a "discourse in which certain things being posited, something else than what is posited necessarily follows on their being true". But this definition is far too wide for the forms of reasoning described in the *Prior Analytics* as forms of the *syllogism;* it embraces, in fact, all forms of mediate inference (191).

The confusion of the syllogism, by many writers, with mediate reasoning of whatsoever form (*cf.* chap. vii., *infra*), is no doubt partially due to this initial ambiguity in the use of the term.

Suppose we were to define mediate reasoning as *the process in which we compare two objective concepts*, [called *extremes*] *S and P, with some third concept*, [called the "*mean*" or "*middle term*"] *M, in order thus to discover whether or how those two extremes are related to each other:* we should be giving *some* insight into the *reason* for our conclusion; *S* and *P* are related in a certain way to each other because each is related in a certain way to *M*. This definition, too, has been given as a definition of the syllogism. But, like the former, it is too wide; nor does it give us a *full* insight into the grounds of the necessity by which conclusion follows from premisses; for each of the concepts compared in the syllogism has two sides to its meaning, *extension* and *intension;* each *stands for* things, and *implies* attributes; and the relation or comparison expressed in each of the premisses is open to a variety of interpretations;[2] from which it follows that an adequate analysis—of the ground on which the conclusion follows necessarily from the premisses—must take into account both the extension and the intension of the concepts compared: and this brings us to the real Aristotelean conception of the syllogism.

148. NATURE AND STRUCTURE OF THE SYLLOGISM: ITS MATTER AND ITS FORM.—The Syllogism may be properly defined as *the expression of a reasoning process in which from two judgments that contain a common concept, and one at least of which is universal, a third judgment, distinct from either of the former, necessarily follows.*

[1] *Anal. Pri. a.* i. 24b, 18: *cf. Top. a.* i. 100a, 25—*apud*, JOSEPH, *op. cit.*, p. 225. The term *συλλογισμός* was probably borrowed by Aristotle from mathematics, where it meant a *computation*, or *adding* or *putting together*.

[2] *Cf.* Part ii., chap. iv. Some writers restrict the term *syllogism* to inferences from two simple or categorical propositions expressive of judgments in which the concepts are compared with one another *as subject and predicate or attribute*. Thus, Mr. Joseph defines the syllogism as "an argument in which, from the given relation of two terms, *in the way of subject and predicate*, to the same third term, there follows necessarily a relation, *in the way of subject and predicate*, between these two terms themselves" (*op. cit.*, p. 225). So that inferences involving hypothetical, disjunctive, or otherwise compound judgments, would not be deemed "syllogistic". No doubt, the strict Aristotelean syllogism is concerned only with subject-predicate relations. But seeing that many of the other modes of relating concepts in thought are reducible to the subject-predicate relation, and in view of the almost universal nomenclature that recognizes pure and mixed hypothetical and disjunctive "syllogisms," we do not think it advisable to adopt such a restricted meaning of this term (*cf.* 149).

This definition embraces forms of syllogism to which Aristotle's fundamental axiom of syllogistic reasoning—the *Dictum de omni et nullo*—does not apply: though these latter forms of reasoning may all be expressed—absolutely speaking, though not quite naturally—in the typical Aristotelean syllogism to which the axiom referred to does apply. We shall examine the definition presently in connexion with the axiom in question.

Meantime we must distinguish between the *matter* or *material elements*, and the *form* or *formal element*, of the syllogism.

The material elements of the syllogism are its terms (*remote* matter), and its propositions (*proximate* matter). Take as a typical syllogism this example: "A free agent is responsible for his conduct; voters are free agents; *therefore* voters are responsible for their conduct". This reasoning is thus expressed symbolically:—

M *is* P . . . major premiss.
S *is* M . . . minor premiss.
$\therefore$ S *is* P . . . conclusion.

There are *three* propositions. These contain *three* terms, each occurring *twice:* the *middle term* twice in the premisses and *not at all in the conclusion*, the extreme terms once each in the conclusion and once in either premiss. The *predicate of the conclusion*, P ["responsible for conduct"], is called the *major term* or *major extreme* of the syllogism. The *subject of the conclusion*, S ["voters"], is called the *minor term* or *minor extreme.*

The *subject* and *predicate* of the conclusion are called respectively the *minor* and *major* terms, because as a rule, when the judgment is expressed in its *natural order* (78), the subject is more concrete, and narrower in its extension, than the predicate.[1] The *middle* term is so called because in the typical Aristotelean syllogism, of which the example given is an illustration, the *extension* of the concept expressed by M, the *middle* term, is *intermediate* between that of the lesser, smaller, or *minor* class-notion, S, and that of the larger, greater, or *major* class-notion, P. However, though the nomenclature is retained and employed universally, the relation indicated between the respective concepts in point of extension does not hold in all forms of the syllogism: sometimes the extension of S is not narrower than that of P; sometimes M is not intermediate, but is either the smallest, or the largest,

[1] *Cf.* JOSEPH, *op. cit.*, pp. 235-37.

of the three in point of extension. The middle term is, however, always "middle" in the sense that it is a mediating term, or common standard of reference, with which two other terms are compared, and is thus the *means* by which we pass from premisses to conclusion. When it has fulfilled the function of intermediary, or middle term of comparison, in the premisses, we let it fall or drop out, as it were, from the mind, retaining only *S* and *P* in our conclusion. This shows that the conclusion is not a mere *summing up* of the premisses; and, furthermore, it explains why mediate reasoning is called *Discursive Reasoning:* for we run on, as it were [*discursus*], to the conclusion, dropping, or leaving behind, the premisses with their common element.

Passing from *remote* to *proximate* matter, the student must note that the *Major Premiss* [sometimes called the *Principle*] is *that which contains the major term, P;* and the *Minor Premiss* [sometimes called the *Assumption* or the *Reason*], *that which contains the minor term, S.* The order in which the premisses are expressed *has no influence whatever* on the force or validity of the reasoning. The minor premiss *may* be expressed first. But *it is usual to express the* major *in the first place.*

Of course, if two premisses are given, without a conclusion, the question as to which premiss is major and which is minor will be determined by the conclusion drawn from them. For instance, the two premisses, "*Some honest men are poor; no highwaymen are honest,*" yield no conclusion about *highwaymen* in terms of *poor*, but only a conclusion about *poor people* in terms of *highwaymen, viz.* that *Some poor people are not highwaymen.* Hence it is the minor premiss (*i.e.* the one which contains the *subject of the conclusion*) that is expressed in the first place. Were we to symbolize *poor people* by *P*, and *highwaymen* by *S*, our conclusion would be *Some P's are not S*, thus showing that the premiss stated in the *first* place contains *the subject of the conclusion*, and is therefore the *minor premiss.*

When two premisses are given as true, the reasoning in the syllogism will consist in drawing a *consequence* or *conclusion* from them. But sometimes it is a conclusion that is proposed, for which premisses are wanted, *i.e.* an assertion is made for which proof or disproof is sought. The assertion here presents itself not as a *consequence* or *conclusion* from *given* premisses, but as a *question* or *problem*, for determining the truth or falsity of which premisses are *wanted.* The construction of a syllogism for such a purpose is a process of *finding a proof* (or a *refutation*). Even here, however, the premisses discovered for the purpose are themselves *assumed to be true*, and not proved to be true in the syllogism itself (*cf.* 167).

The *form* of the syllogism, as distinguished from its matter, consists simply in *the necessity with which the conclusion follows from the premisses.* This necessity constitutes the *formal force* or

formal validity of the syllogism as a process of reasoning: and it is a characteristic not of the syllogism alone, but of all valid forms of cogent logical inference, whether mediate or immediate. It is expressed by the conjunction "*Therefore*" ["*Ergo*"], which introduces the conclusion, and the function of which is to express that *if the premisses are true*, and *on this assumption*, the conclusion MUST *be true*. The nature of the syllogism as a process of *formally correct* reasoning, *i.e.* a process every step of which is *consistent* with every other, and with the assumed starting point, consists wholly and entirely in the accuracy of the hypothetical assertion made by the "*Therefore*"—the assertion, namely, that *if* the premisses are true the conclusion must be true: *e.g.* that if what develops the mind is educational, and logic develops the mind, *then* logic is educational (138).

This hypothetical guarantee—that *if* the premisses or antecedent be true the conclusion or consequent must be true—is the *only* guarantee of truth involved in the *formal* correctness of the syllogism. Representing premisses and conclusion by *A*, *B*, and *C*, respectively, we may express the *formal force* of the syllogism by the hypothetical "*If A is true and B is true, then C is true*". And, just as we cannot derive from this the inverse "*If A or B or both be false C must be false*," but only the worthless inverse that "*If A or B or both be false C may be true or may be false*" (140); so, also, *the nature of the syllogism does not authorize us to infer that if one or both premisses be false the conclusion will be false:*[1] it may be false or it may be true. Hence the axiom: *Ex falso sequitur quodlibet*. For example, from the two false premisses: "*All lions are herbivorous; all cows are lions*"; we *validly* draw the *true* conclusion, "*Therefore, all cows are herbivorous*". Similarly, from the premisses, "*All good angels are happy; some men are good angels*"—a false premiss,—we *validly* draw the *true* conclusion, "*Therefore, some men are happy*".

These syllogisms are *formally valid*, because the formal assertion in each, —that the premisses, if true, would be a sufficient ground for the truth of the conclusion—is itself a true statement. But the syllogism, *considered in its formal aspect*, does not say whether the premisses are *de facto* true, nor that they form the only possible ground for the conclusion, nor that this could not be true on other grounds, nor whether it is *de facto* true at all, but only that *if* the given premisses be true they form a sufficient ground for inferring the truth of the conclusion.

It is possible, then, to reach a true conclusion by reasoning validly from false premisses. In assenting to the latter the mind is, of course, assenting to a false judgment or judgments; but nevertheless, *since the reasoning is*

[1] We could of course infer this, if the premisses of the syllogism were assumed to give us the *only possible ground* of the conclusion; so that the syllogism could be resolved into a *reciprocal* hypothetical (135). And so of the other inferences in the text. *But no such assumption can be made for the syllogism as a form of inference.*

discursive, since *the false premisses may be dropped* with the middle term, and since the mind, after accepting the conclusion on false grounds while *inferring* it, may now abandon the grounds altogether and *assent* to the conclusion in itself and for its own sake, the mind may now be considered to be—accidentally of course—in possession of the truth.[1]

Reverting again to the expression of the formal force of the syllogism by the hypothetical "*If A is true and B is true C is true*," we see further that, just as this proposition yields the contrapositive "*If C is false then either A or B or both must be false*," and does not yield the simple converse "*If C is true then A is true and B is true*," but only worthless one "*If C is true then perhaps A and B are true* or perhaps not": so, also, *the nature of the syllogism authorizes us to infer that if the conclusion of a formally valid syllogism be false then one at least of the premisses must be false; but it does not authorize us to infer that if the conclusion is true the premisses must be true.*

Notwithstanding all this, it is a very common fallacy for the controversialist to imagine that he has disproved or refuted his opponent's thesis or contention when he has merely succeeded in showing that the reasons alleged by his opponent in support of the latter are untrue. He forgets that even if they are untrue this does not prove the contention itself to be untrue. Many a good cause is supported by bad arguments.

Scarcely less dangerous is the temptation to be too easily assured of the soundness of one's own arguments and reasons from the mere conviction that the position one maintains is a sound one. Though our cause be good, we may be guilty of supporting, or rather injuring, it by bad arguments.

Of course, in order that the syllogism be a means of discovering and proving truth, not merely must it be formally valid, but its *matter* also, the judgments embodied in it, must be *true*. The *material* requirements of the syllogism will be treated in connexion with *Demonstration* (Part v.). Its *formal* aspect alone will be treated in the present context.

149. KINDS OF SYLLOGISM.—Much of what has been said so far in the present chapter applies primarily to that form of syllogism which is constituted exclusively by simple categorical propositions. But we have seen that there are other important types of proposition—most notably the hypothetical (or conditional) and the disjunctive (or alternative). It will be necessary to treat explicitly the separate forms of syllogism into which these three kinds of proposition enter. If the two premisses of a syllogism be of the same class, or *Relation* (83, 84), we have a *Pure Syllogism*. Of this, therefore, there are three kinds: the *Pure Categorical*, the *Pure Hypothetical*, and the *Pure Disjunctive*. If the premisses be propositions of different classes, we have a *Mixed Syllogism*. Of this also there are three kinds. The

[1] Of course, the mind, in so far as it *retains* the false premisses, is in error, and may be led by them otherwise into further error. *Cf.* JOSEPH, *op. cit.*, p. 342.

combination of a categorical with a hypothetical premiss gives the *Mixed Hypothetical Syllogism;* the combination of a categorical with a disjunctive premiss gives the *Mixed Disjunctive Syllogism;* and, finally, the combination of a hypothetical with a disjunctive premiss gives the peculiar form of syllogism called the *Dilemma.* This classification gives us the following species:—

Syllogisms
- 1 Pure
  - (*a*) Categorical.
  - (*b*) Hypothetical.
  - (*c*) Disjunctive.
- 2 Mixed
  - (*a*) Hypothetical.
  - (*b*) Disjunctive.
  - (*c*) Dilemma.

The axioms and canons of the pure categorical syllogism—which will be considered in the present and immediately following chapters—apply almost without any modification to pure hypothetical and pure disjunctive syllogisms. But mixed syllogisms will receive separate treatment subsequently.

150. The Syllogism and the Laws of Thought.—The formally valid syllogism must presuppose and involve the fundamental laws of thought (12-16): these are involved in every formally correct thinking process.[1] The separate constituent judgments involve the principles of identity, contradiction, and excluded middle. The formal dependence of the conclusion on the premisses may be expressed in a hypothetical judgment (148), and every such judgment is an application of the principle of sufficient reason. So, too, is the reasoning in hypothetical syllogisms (149) an explicit application of the latter principle.

The syllogism, therefore, is an expression or application of the laws of thought. But it must not be inferred that these laws give sufficiently clear expression to the *immediate* rational grounds on which syllogistic inference is based, or that the rules or canons which are to guide us in this process may be directly and immediately derived from those fundamental laws. These rules or canons cannot be derived from the laws of thought without the aid of certain other principles which may be appropriately called the *Mediate Axioms* of the syllogism. The characteristic feature of syllogistic reasoning is a *middle term*, with which two extremes are compared, in regard to extension and intension, through

[1] *Cf.* Keynes, *op. cit.*, p. 467.

relations of subject to attribute. To this the laws of thought make no explicit reference. Hence the need of *Mediate Axioms*, which expressly recognize and state the function of this characteristic element. These *Mediate Axioms* we shall now proceed to examine.

151. THE SYLLOGISTIC AXIOMS OF IDENTITY AND DIVERSITY. —We referred above (147) to a definition of the syllogism as *a mental process in which two concepts are compared with a third in order thus to discover whether they agree or disagree with each other.* This definition or description notes a comparison which is necessarily made in all possible forms of the syllogism. On it are based two axioms, applicable, therefore, to all forms of syllogistic reasoning. They may be stated as follows :—

1. If two ideas agree with a same third, they agree with each other.—*Quae sunt eadem cum uno tertio sunt eadem inter se.*

2. If one of two ideas agrees, while the other disagrees, with a same third, they disagree with each other.—*Quorum unum est idem, et alterum non est idem, cum uno tertio, non sunt eadem inter se.*

Instead of "ideas" [or "concepts"] some logicians prefer to use the word "terms," others, again, to speak of "things". The usage is optional, provided the proper view about the scope of logic be retained behind whatever set of words we may employ (26).

The two axioms just stated present an obvious and striking analogy with the *mathematical* axioms that "*Two things which are equal to the same are equal to each other*," while "*Two things, one of which is equal, the other unequal, to a same third thing, are unequal to each other*". But the axioms of the syllogism are not mathematical: they have not to do with *quantities, magnitudes, numbers.* They are *logical* axioms: they have to do with *ideas* (109). It would be interesting to inquire, however, to which side—the extensive or the intensive—of the meaning of the concepts compared, the logical axioms mainly refer. Primarily, it would appear, to their extension[1]. But the concepts have intension as well as extension. Hence, though the axioms of identity and diversity are simple and instructive so far as they go, they do not rest upon any detailed analysis of syllogistic reasoning.

[1] *Cf.* WELTON, *op. cit.*, i., pp. 282, 283.

152. THE ARISTOTELEAN SYLLOGISM AND THE "DICTUM DE OMNI ET NULLO". Aristotle and the mediaeval Scholastic logicians regarded as the *perfect* sort of syllogism only that form in which the middle term is *subject* in the *major premiss*—which should lay down a universal principle—and *predicate* in the *minor premiss*—which should apply the general principle to some particular case or class of cases. Syllogisms in this form—"*All* (or *No*) *M is P*; *S is M*; *therefore S is* (or *is not*) *P*"—are said to belong to the *first figure* of syllogism (159).

This is the form in which we most naturally argue from a general principle to some narrower application of the latter, and apply it to specific instances. Moreover, all other forms of syllogistic inference must, as we shall see, agree with the present form in this, that they are all inferences which involve among their premisses a *general principle: one* of their premisses *must* be universal (148). Furthermore, all the other forms may be "reduced" by the application of processes of immediate inference to their premisses (*infra*, Chap. IV.), though sometimes not very naturally, to the present form. Hence it is not surprising that this form should have been set up as the *standard form* of the syllogism, that an axiom should have been formulated which would express or typify *it*, so that the *other* forms could be tested by this axiom only by first "reducing" them to the present form, and applying its axioms to them when so "reduced".

Such an axiom is the Aristotelean "*Dictum de omni*": an abbreviation of the expression "*Dictum de omni—Dictum de nullo*": which is itself, in turn, an abbreviated expression of the two axioms: (*a*) *Dictum de omni dicitur de singulis*, or, *Quidquid dicitur de omni dici potest et de singulis*; and (*b*) *Dictum de nullo negatur de singulis*, or, *Quidquid negatur de omni negari potest et de singulis*.[1]

[1] The *Dictum* is apparently founded on the following passage in Aristotle's *Prior Analytics* (*a.* i. 24b, 26-30): τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν. λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι, ὅταν μηδὲν ᾖ λαβεῖν τῶν τοῦ ὑποκειμένου καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσάυτως. "That one term should be contained in another as in a whole is the same as for one to be predicated of all another. And it is said to be predicated of *all* anything, when no part [*i.e.* logical part] of the subject can be found, of which the other term [the predicate] will not be true; and to be predicated of *none* similarly." *Cf.* JOSEPH, *op. cit.*, p. 274 n. The *Dictum* has been variously stated by the Scholastic followers of Aristotle. The formula *Quod valet de omnibus valet etiam de singulis* is misleading, inasmuch as it suggests the erroneous view that the middle term is regarded as *a collection of particular instances*, and the major premiss as a mere *enumerative* universal (*cf.* 153, 195, 198).

This double axiom may be appropriately rendered as follows: *Whatever is predicated, whether affirmatively or negatively, of any logical whole* (or *universal*), *may be predicated in like manner of whatever is a logical part of that whole.*

153. ANALYSIS OF THE "DICTUM": ALTERNATIVE AXIOMS. —It has been objected that the Aristotelean *Dictum* bases the syllogism exclusively on the *extension* of the concepts compared, whereas *intension* is the more important and more fundamental feature of their meaning. This will depend on how we understand the *Dictum;* and if it be rightly understood the objection cannot be sustained. If we analyse the process of thought as indicated in the *Dictum*, we shall find that it is the *intension* of the *major extreme* (*P*) that is thought of throughout: both the major premiss "*All* (or *No*) *M is P*," and the conclusion "*S is* (or *is not*) *P*," are read *predicatively* (100), *P* being thought of as an attribute or group of attributes. The *minor premiss* is, no doubt, read *in extension, i.e. according to the class-inclusion view* (104), for in it the minor extreme *S* is "asserted to belong to" the class *M*. It is, therefore, the *extension* of the *minor extreme* that is uppermost in the mind both in the minor premiss and in the conclusion. There remains the middle term, *M*, to be examined. It is *predicate* in the *minor premiss* of the typical Aristotelean syllogism, and here, unquestionably, it is thought of as a "class," to the extension of which *S* is found "to belong". But which aspect of *M* is before the mind when *M* occurs as *subject* of the *major premiss?* This is a more important question. There can be no doubt that Aristotle laid stress, perhaps undue stress, upon the extensive aspect of the middle term in the major premiss; but neither can there be any doubt that, according to his general logical teaching, the major premiss must be an *abstract* or *generic* universal, and not merely a *concrete* or *collective* universal, expressing the result of an actual enumeration of instances (92, *a*). It must be a *principle* or *law*, expressing some kind of *necessary* relation (195, 198), between the attribute (or group of attributes), *P*, and the attributes which make up the *intension* of *M*, which, therefore, constitute the *nature* of *M* in the abstract, and, accordingly, of all the concrete individuals in which the attributes *M* are realized.[1]

We have already seen that it is because such propositions are "*necessary*" that they are "universal" (92); that the *ground* or

[1] *Cf.* JOSEPH, *op. cit.*, pp. 224, n. and 284.

reason for *universalizing* such propositions lies in our apprehension of a *necessary* relation between the *intension* of their predicates and the *intenison* of their subjects; that, consequently, even when such propositions are interpreted *predicatively*, and their subjects thought of as classes, they *presuppose*, as more fundamental, the *connotative* or *implicational* view, in which we compare the attribute *P* with *the attributes implied* by the subject, *M* (102).

All this is presupposed, and indeed necessarily involved, in the Aristotelean view of the syllogism as a *real process of inference* from a *general law* to the particular application of the same. The *intension* of the middle term must be compared with that of the major extreme, at least *antecedently* to the formulation of the major premiss as a universal, and *as a ground of its universalization*, if not actually and formally in the statement of the major premiss in the syllogism itself. We therefore pass, in syllogistic reasoning, from considering the *intension*, to considering the *extension*, of the middle term. It cannot be maintained, then, that the Aristotelean view of the syllogism is based exclusively on the *extension* of the concepts compared.

Some logicians suggest a modification of the Aristotelean *Dictum*, which would bring out more clearly the dependence of the syllogistic process on the *intension*, as well as on the *extension*, of the middle term. The major premiss, they suggest, should be read not *predicatively*, but *connotatively*. This would be an improvement; for it would at the same time show us wherein exactly lies the forward step, or *advance*, of thought, which constitutes the syllogism a genuine inference: namely, in the transition from the *intension* of *M* in the major premiss to its *extension* in the minor premiss, where it is *formally universalized*. "In our opinion," writes Cardinal Mercier, "the logical connexion between premisses and conclusion demands that the terms should be simultaneously considered from both points of view —the extensive and the comprehensive. The predicate of the conclusion, forming part of the *comprehension* of an abstract term which contains *in its extension* the subject of the conclusion, may be definitely predicated of this subject."

"In the major premiss, the major extreme is considered in its relation of *comprehension* to the middle term; in the minor premiss the same middle term is considered from the point of view of its *extension*, and put into a relation of extension with the minor extreme."

"In passing from the major premiss, where the middle term is considered in its abstract comprehension, to the minor premiss, where its extension is considered in relation to its 'inferiors' [sub-classes or individuals], a distinct work of thought has been accomplished: *the universalization of the abstract type.*"[1]

. . . "*Reasoning* [syllogistically] *is placing some definite subject under the extension of an abstract type in order to infer that something which is predicable of the abstract type as such is similarly predicable of the definite subject.*"[2]

[1] *Logique* (4th edit., Louvain, 1905), pp. 184-5. *Cf.* JOSEPH, *op. cit.*, p. 284.
[2] *ibid.*, p. 182.

. . . "*Quidquid affirmatur de subjecto abstractim considerato, affirmandum est de omnibus et singulis ejus inferioribus*, uno verbo, *universaliter*. —*Quidquid negatur de subjecto abstractim considerato, negandum est de omnibus et singulis inferioribus ejus*, uno verbo, *universaliter*".[1]

The significance of this modified presentation of the Aristotelean *Dictum* will be better understood when we come to examine the objection so often urged by philosophers, from Sextus Empiricus to John Stuart Mill, against the syllogism as a process of reasoning: that, in so far as it purports to be a process of genuine inference, it involves the fallacy of *Petitio Principii* (198).

Some of those logicians who object to the Aristotelean *Dictum* as basing the syllogism exclusively on what is a secondary aspect of the meaning of its terms, *viz.* their extension, endeavour to base the syllogism themselves exclusively on the *comprehension* or connotation of its terms.

Thus, for example, Cardinal Mercier quotes from a French Scholastic writer the following axiom analogous to the *Dictum*, but based on the *comprehension* of the terms: *Id quod includit continens includit etiam contentum; id quod excludit continens, excludit etiam contentum.*[2]

Similarly J. S. Mill adopted an axiom analogous to the *Dictum*, but based on *connotation: Nota notae est nota rei ipsius; repugnans notae repugnat rei ipsi:* Whatever characterizes (or is affirmed of) an attribute, characterizes (or is affirmed of) the subject of that attribute; Whatever is incompatible with (or must be denied of) an attribute, is incompatible with (or must be denied of) the subject of that attribute. Thus, the reasoning in the syllogism "Man is mortal; Socrates is a man; therefore Socrates is mortal" would be "Mortality is a characteristic of humanity, which is a characteristic of Socrates, therefore it characterizes Socrates"; and in the syllogism "No men are omnipotent; all kings are men; therefore no kings are omnipotent," it would be "Omnipotence is incompatible with humanity, which is an attribute or mark of kings; therefore it is incompatible with kings." The formula, according to Mr. Joseph (*op. cit.*, p. 285, n. 2, and 274, n.) "is really an abridged equivalent of [a] passage in Ar. *Cat.*, 1b, 10-12: ὅταν ἕτερον καθ' ἑτέρου κατηγορῆται ὡς καθ' ὑποκειμένου, ὅσα κατὰ τοῦ κατηγορουμένου λέγεται πάντα καὶ κατὰ τοῦ ὑπακειμένου ῥηθήσεται, 'When one thing is predicated of another as of a subject *de quo*, all that is asserted of the predicate will be asserted of the subject as well. . . .'" This passage some have wrongly supposed to be the basis of the *Dictum de omni.* Mr. Joseph shows clearly[3] that Aristotle was not contemplating syllogistic inference here at all. The *Nota notae*, however, which is based upon it, gives a fairly good alternative to the *Dictum*, or, rather, emphasizes the important aspect of *intension* which the *Dictum* leaves in the background.

Mill interprets the *Nota notae* in a different way from that given above.

[1] *ibid.*, p. 185. Aristotle points to the necessity, and fundamental identity, of both points of view, when he writes Τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον, ταὐτόν ἐστι. (In toto autem inesse alterum altero, et alterum de altero omni prædicari, idem est.)—*Anal. Prior.*, i., 1.

[2] RABIER, *Logique*, chap. v., p. 52;—*apud* MERCIER, *op. cit.*, p. 184. [The second portion of the axiom seems to be misstated. It should read: *Id quod continet excludens, excludit etiam exclusum: M excludit P: S continet M; Ergo S excludit P*].

[3] p. 274, n.

He makes it mean not "that what qualifies an attribute qualifies the subject of it," but "that what indicates the presence of an attribute indicates what the latter indicates"[1]: *whatever* [*S*] *is a mark of any mark* [*M*], *is a mark of that* [*P*] *which this last* [*M*] *is a mark of; whatever* [*P*] *is repugnant with a mark* [*M*] *is repugnant with that* [*S*] *which this last* [*M*] *is a mark of.* If we bear in mind that an attribute may be a mark of the *absence* of other attributes, we may combine both parts of this formula in the single statement "*Whatever* [*S*] *has any mark* [*M*] *has that* (P or *not P*) *which it is a mark of*".[2]

WELTON, *Logic*, I., bk. iv., chaps. i., ii. JOSEPH, *Logic*, chaps. xi., xii., xiv. MILL, *Logic*, II., ii. MERCIER, *Logique* (4th edit., Louvain, 1905), pp. 177 *sqq*. JOYCE, *Logic*, chaps. xi., xii.

[1] JOSEPH, *op. cit.*, p. 285, n. 2. [2] MILL, *Logic*, bk. ii., chap. ii., 4.

CHAPTER II.

GENERAL RULES OR CANONS OF THE SYLLOGISM.

154. DERIVATION OF THE GENERAL RULES OR CANONS FROM THE DICTUM.—We have pointed out that the *Dictum de omni et nullo* applies directly only to syllogisms in which the middle term is subject in the major premiss and predicate in the minor premiss. The middle term may, however, occupy other positions, thus giving rise to other forms or "figures" (159) of the syllogism.

If the *Dictum* be analysed it will be found to involve a number of requirements which are usually stated as *general rules or canons* of the syllogism, and which are directly and immediately applicable to *all* figures of the categorical syllogism. These latter being all reducible to the Aristotelean figure, it will need only a little generalization of the special requirements of the *Dictum* to make the rules so derived applicable to the syllogism whatever the position of the terms may be.

(1) Thus, the *Dictum* mentions *three* and *only three* terms: "whatever is predicated"; "the class-concept of which it is predicated"; "what is asserted to belong to that class". Hence the rule: *A syllogism must have three and only three terms.*

(2) Similarly, the *Dictum* mentions *three* and *only three* comparisons or propositions. Hence the rule: *A syllogism must have three and only three propositions.*

(3) The *Dictum* says that something must be predicated *distributively* about a *logical whole* or *class*-concept. This latter is the middle term, occurring as subject of the major premiss. The *Dictum* therefore demands that the *middle term be distributed in the major premiss.* Generalizing this, so that it will apply to all forms of syllogism, we have the rule: *The middle term must be distributed in one at least of the premisses.*

(4) The *Dictum* says that the original predication can be made [in the conclusion] of "anything" [*S*] which can be asserted

to belong to the class [*M*]: not therefore of anything *more definite*, of *anything more*, than this *S* [of the minor premiss]. Hence if the *S* be undistributed in the minor it must remain undistributed in the conclusion. Similarly, the *Dictum* requires that the predication be made *in the same way* in the conclusion as in the major premiss. If, therefore, the predication is made here by means of an undistributed term [*P*], this term must remain undistributed in the conclusion. Hence the rule: *No term may be distributed in the conclusion which was undistributed in its premiss.*

(5) The *Dictum* provides that the *minor* premiss be affirmative, for it asserts that something belongs to a certain class. Generalizing this, so as to make it applicable to other forms of syllogism, we have the rule: *One at least of the premisses of a syllogism must be affirmative.*

(6) The *Dictum* recognizes the possibility of the original predication being negative, but demands that in such a case the predication in the conclusion be also negative, and that if the predication in the major (as well as in the minor) be affirmative the predication in the conclusion must also be affirmative: what is predicated in the major must always be predicated *similarly*, *in like manner*, in the conclusion. Generalizing this we have the rule: *A negative premiss necessitates a negative conclusion, and vice versa.*

Summing up these results we have the following six GENERAL RULES OR CANONS OF SYLLOGISM.

A. *Rules of Structure :—*

(1) A SYLLOGISM MUST CONTAIN THREE, AND ONLY THREE, TERMS;

(2) A SYLLOGISM MUST CONTAIN THREE, AND ONLY THREE, PROPOSITIONS.

B. *Rules of Quantity :—*

(3) THE MIDDLE TERM MUST BE DISTRIBUTED AT LEAST ONCE IN THE PREMISSES;

(4) NO TERM MAY BE DISTRIBUTED IN THE CONCLUSION WHICH WAS NOT DISTRIBUTED IN ITS PREMISS.

C. *Rules of Quality :—*

(5) ONE AT LEAST OF THE PREMISSES MUST BE AFFIRMATIVE;

(6) A NEGATIVE PREMISS NECESSITATES A NEGATIVE CONCLUSION, AND *VICE VERSA*.

The following English rhymes may help the student to remember these rules (*cf. infra*, 158):—

Of terms have but three; proposition as term;
Distribute the middle—in this be most firm;
Distribute no term in conclusion, beside,
Unless in a premiss 'tis equally wide;
One premiss affirmative, this you must learn,
For negative premisses nothing affirm;
A negative head has a negative tail,
And the converse of this is of equal avail.[1]

155. Examination of the General Rules of Syllogism. —A (1), (2). The two rules of structure are not, properly speaking, *rules* of the syllogism, but rather a statement of its *essential requirements*, of its very *nature*. If there be valid forms of mediate inference which, as they stand, are not in keeping with these requirements,—*e.g.* "*B is greater than C; A is greater than B; therefore A is greater than C*,"—and which cannot be reduced to any form in which they will conform to these requirements (147), then they *are not syllogisms*. When, however, a mediate reasoning *does* conform to these requirements, by having three terms and three propositions connecting those terms as subject and predicate by the logical copula *is* (*are*, *not*), then it is not valid *unless it also conforms to the four remaining rules*.

The first rule of structure, however, is of great *practical* utility as a *canon* or *guide* to correct reasoning, inasmuch as it explicitly forbids *ambiguity* in the use of the terms of a syllogism. It puts us on our guard against *apparent* syllogisms which *really* contain *four* (or more) terms instead of three, because of some term or terms being used *in different senses* in the different propositions. This *apparent* syllogism—with four (or more) terms masked as three, is a very familiar form of fallacy, technically called *Quaternio Terminorum* or the *Fallacy of Four Terms*.[2]

"A good example of an ambiguous middle is given by De Morgan (*Formal Logic*, pp. 241-2):—

'*All criminal actions ought to be punished by law;*

'*Prosecutions for theft are criminal actions;*

'*∴ Prosecutions for theft ought to be punished by law;*

'Here the middle term is doubly ambiguous, both *criminal* and *action* having different senses in the two premisses'."[3]

The reason why the middle term must be *the same*, have the *same sense*, in both premisses, is sufficiently obvious: otherwise

[1] *Questions on Logic*, Holman and Irvine, p. 65.

[2] Also nicknamed "the *logical quadruped*".

[3] Welton, *op. cit.*, p. 290.

there would be no common element in the comparisons, no real connecting link between the extremes. To make the inference *possible*, *M* must be identical with *P* and identical with *S*; to make the inference *needful*, this identity must be combined with *diversity*—diversity of aspect in *M* as a connecting link between two *diverse* concepts, *S* and *P*.

The reason why the latter terms, *S* and *P*, must be unambiguous, is no less obvious: it is only about *the same S* as occurred in the premisses that the predication can be made—and only *the same* predication—in the conclusion.

B (3). The violation of this first rule of quantity is called the *Fallacy of Undistributed Middle*. The middle term must be *distributed*, *i.e.* taken universally, in its whole denotation, at least once in the premisses: for this simple reason, that were it *undistributed*, *i.e.* taken indefinitely, in both premisses, we could not be sure that the two extremes, *S* and *P*, were being compared with *the same portion* of the extension of *M*. *S* might be compared with one portion, and *P* with another portion, of the indefinite "*some M*". And, of course, we could draw no conclusion about the relation between *S* and *P* unless we were sure of having compared each of these with *exactly the same M's*. In order to secure this it is sufficient to have "*all M*" compared with *either* extreme; for the "*all M*" will overlap the "*some*, possibly *all M*" compared with the other extreme, and will thus secure an identical point of reference for both extremes.

Thus, from the premisses "*Some M's are P*" and "*Some M's are S*": "*Some artisans are married*" and "*Some artisans are drunkards*": we cannot infer whether or not any "*S's are P*," any "*Drunkards are married*": for the *M's* that are *P* may or may not be identical with those that are *S*. But if either premiss makes an assertion about "*all M*," then we know that *S* and *P* co-exist as attributes in some of the *M's*, and we can therefore infer that "Some *S's* are *P*" or that "Some *P's* are *S*".

If the middle term is singular there can be no ambiguity: it will be necessarily the same in both premisses. If we take "*Most*" to mean *at least one more than half*, then, from the premisses "*Most M's are P; Most M's are S*" we may draw the conclusion that "*Some S is P*; for, the two *most's* overlap, thus yielding a common identical element with which to compare *S* and *P*.

It is not necessary that the middle term be distributed in *both* premisses; but if it be, the identity of reference of the extremes is made doubly secure.

It is only when, as in the above examples, *M* is *subject* in both premisses [*i.e.* in the *third* figure (159)] that its extension is explicitly thought of in both, and so made an identical point of reference for *S* and *P*. There the reason for having *M* distributed at least once is clear.

If, however, *M* is *predicate* of two affirmative premisses (*i.e.* in the *second* figure), the extension of *M* is, as a rule, not explicitly thought of at all in either premiss. And if such premisses as "*P is M*," "*S is M*," "*Birds fly*," "*Bats fly*," yield no conclusion as to whether or not "*S is P*," "*Bats are birds*," the reason is "that it does not follow, because the same predicate attaches to two subjects, that these can be predicated one of the other".[1] It is not the *extension* of *M* that is here thought of as the common point of reference for *S* and *P*. But if the same predicate, *M*, is affirmed of one extreme and denied of the other, these can be denied of each other; and in thus requiring one negative premiss we are distributing its predicate *M*, so that "for working purposes"[2] we may say of this form of syllogism too that its validity requires "distributed middle".

Again, if *P* be predicated of *M* and *M* of *S* (as in the *first* figure)—"*M is P, S is M, ∴ S is P*"—*M* must be distributed in the major premiss: "unless *P* is connected necessarily and universally with *M*, it is clear that what is *M* need not be *P*".[3]

B (4). The violation of the second rule of quantity is called the *Fallacy of Illicit Process*—of the major, or of the minor, or of both extremes, as the case may be. The reason of the rule is plain enough. *S* and *P* are related in the conclusion *because and in so far as* they were related to *M* in the premisses. Hence, we have no right to take *S* or *P* any more *definitely* in the conclusion than they were used in the premisses: to refer to *all S*, or to *all P*, in the conclusion, if the minor or major referred only to *some S*, or to *some P*, indefinitely. We cannot infer that because *All criminals are wicked* and *some Irishmen are criminals* therefore "*All* Irishmen are wicked": to do so would involve the fallacy of *illicit minor*; but only that "*Some* Irishmen are wicked". Similarly, we cannot infer that because *All Spaniards speak Spanish* and *No natives of Maynooth are Spaniards*, therefore *No natives of Maynooth speak Spanish*. The conclusion, here, may be true, or it may not; but, whether it is or not, it does not follow from the premisses. For the major does not state that there are no other people besides Spaniards who speak Spanish; there *may* be others: and among these others *may* be some natives of Maynooth. In other words, the syllogism is invalid on account of the fallacy of *illicit major*. The major extreme,

[1] JOSEPH, *op. cit.*, p. 250. [2] *ibid.*

[3] *ibid.* The distribution of *M* in the fourth figure (159) needs no separate discussion.

"speakers of Spanish," is *distributed* in the conclusion, being the *predicate of a negative proposition*, whereas it was *undistributed* in the major premiss, being there *predicate of an affirmative proposition*. If we could convert the major premiss *simply* (to "*All who speak Spanish are Spaniards*") the conclusion would be valid; but we cannot do so, for an A proposition cannot be converted *simply*.

It will be noted that the fallacy of illicit minor can occur only when the conclusion is *universal* (and S undistributed in the minor premiss); and that the fallacy of *illicit major* can take place only when the conclusion is *negative:* for P, being *always the predicate* of the latter, is not distributed unless when the latter is negative. The fallacy then arises if P was not distributed in its premiss (the major premiss) as well.

C (5). The first rule of quality appears to be very simple. From two negative premisses of a syllogism nothing can follow. If S is not M, and if P is not M, we evidently cannot know whether S and P are identical[1] or not. They may be, or they may not be, identical. Neither of the extremes is connected with the middle term: there is, therefore, really no common bond or link between the extremes. They may indeed agree *de facto*, wholly, partially, or not at all, with each other: but nothing of this can we know from the fact that neither of them is identical with a certain third thing.

A reference to Euler's circles (104) will illustrate this. If we simply draw a circle representing the class M alongside (and outside) each of the five combinations of S and P, every single trio so formed will illustrate the two premisses "*No S is M; no P is M*": thus showing that every possible relation between S and P is compatible with those premisses, and that therefore no conclusion whatsoever about S and P can be drawn from them. If, instead of two E premisses, we have E and O, or two O premisses, we may see, by drawing the M circle so as to intersect *either* or *both* of the circles S and P, that, while we make the number of alternative pairs of premisses still greater, all the five possible relations of S and P are still compatible with every pair of premisses so formed. Besides which we must remember that an O proposition never excludes the possibility of E; and that therefore the conclusion from E and O, or from two O premisses, cannot possibly be any more definite than from two E premisses.[2]

[1] The terms, "identity," "agreement," "disagreement," etc., do not express *exactly* the affirmative or negative relation of predicate to subject in a categorical proposition. (*Cf.* JOSEPH, *op. cit.*, pp. 248, n., 257, n.) The relation is *sui generis*, and the terms used are the best to be found.

[2] WELTON, *op. cit.*, pp. 294-5.

Simple, however, as the present rule appears, the *apparent* exceptions to it are, nevertheless, so striking that they have led logicians from time to time to raise doubts about its validity. But the rule *is* certainly valid, and the exceptions *are only apparent.* For example, in the syllogism, *Whatever is not material is not mortal; The human soul is not material; ∴ The human soul is not mortal:* the two premisses are, as they stand, negative in form; but the middle term is, in reality, "*not-material*"; it is only by regarding this as the class-concept to the extension of which the human soul *is asserted* in the minor to belong, that the conclusion in question can be drawn about the latter. The minor premiss is thus in reality *affirmative:* its force or function, as it stands, is affirmative, *viz.* to assert that a thing belongs to a certain class. The substitution of the term "immaterial" will at once convince us of this. What we have to remember, therefore, is this, that it is not so much the affirmative or negative *form* of the premisses we must look to: this may always be altered by the simple process of *obversion.* We must look rather to their *function* in the context, remembering that a proposition which is negative in form is sometimes really affirmative in force and function.

The present rule does not state simply and absolutely that *from two propositions which are negative in form* nothing can follow, but that *no inference may be drawn about either of two extremes, S and P, from comparing these in two really negative judgments with a single third term.*

The premisses of the syllogism given above, as it stands, may be expressed symbolically thus: *No not-M is P; S is not M.* Here, with the two negative propositions, we have *four* terms *not-M, P, S, M; i.e.* we have not a syllogism at all. In order to make the given propositions *premisses of a syllogism*, with a common middle term, we must obvert the minor, *thus making it an affirmative proposition.* From which we see that before we can, *by valid syllogistic reasoning*, draw a conclusion, the premisses must conform to the present rule.

Similarly, for example, from the two propositions *No M is P; No M is S* we can draw no conclusion—about either *P or S.* But by obverting the minor, or both, premisses, we get a valid *syllogism*, whose terms are *M, P,* and *not-S,* or *M, not-P* and *not-S,* and from whose respective premisses we can infer that *Some not-S is not P* or that *Some not-S is not-P.*

The rule therefore holds, in all cases, *that*—if we have two propositions which are really premisses of a syllogism as they stand, *i.e.* which connect two terms with a common third—in order to draw a conclusion about either of these extremes in terms of the other, *one of the premisses at least must be affirmative.* If, as they stand, they are two negatives, they cannot yield a conclusion about either actual extreme, from comparison of these with the single actual middle (if there be such).

The premisses of *any* valid syllogism can be made to stand as two negatives by the simple process of obversion; but in that negative form they do not yield the conclusion. Thus, the premisses of the syllogism: "*All M is P; All S is M; therefore, All S is P*": may be written: *No M is not-P; No S is not-M;* but it is only by obverting them back *to their original form* that we get the original conclusion from them.

This whole difficulty is an old one. It was considered by the mediaeval Scholastic logicians; and it was probably their consideration of it that led to their treatment of the process now called *Obversion*, under the name of *Aequipollence of Judgments* (117).

But it is a curious fact, to which Dr. Keynes calls attention,[1] that those logicians who have found this difficulty so troublesome "do not appear to have observed that, as soon as we admit more than three terms, other apparent breaches of the syllogistic rules may occur in what are perfectly valid reasonings. Thus, the premisses *All P is M* and *All S is M*, in which *M* is not distributed, yield the conclusion *Some not-S is not-P*;[2] and hence we might argue that undistributed middle does not invalidate an argument. Again, from the premisses *All M is P*, *All not-M is S*, we may infer *Some S is not P*,[3] although there is apparently illicit process of the major. . . . But of course none of the above examples really invalidate the syllogistic rules; for these rules have been formulated solely with reference to reasonings of a certain form, namely, those which contain three and only three terms. In every case the reasoning inevitably conforms to the rule which it appears to violate, as soon as, by the aid of immediate inferences, the superfluous number of terms has been eliminated."

A practical corollary from all this is that, when we are asked whether anything can be inferred from two given propositions containing *four* terms, we should see whether these latter might not be reduced, by the aid of processes of immediate inference, to *three*, and so, perhaps, yield a valid conclusion.

[1] *op. cit.*, pp. 297-8.

[2] By the contraposition of both premisses, this reasoning is reduced to the valid syllogistic form, *All not-M is not-P*, *All not-M is not-S*, therefore *Some not-S is not-P.*

[3] By inversion of the first premisses, this reasoning is reduced to the valid syllogistic form, *Some not-M is not P*, *All not-M is S*, therefore, *Some S is not P.*

C. (6). A negative premiss necessitates a negative conclusion, because if one premiss is negative the other, by the preceding rule (5), must be affirmative, and they will thus relate the extremes *in opposite ways* to the middle term: hence the extremes cannot agree with each other; and the conclusion, to express this disagreement, must be negative.

Conversely, if S and P disagree with each other—*i.e.* if the conclusion be negative—it must be that one of them agreed, and the other disagreed, with M—*i.e.* that one premiss was negative—for if both extremes agreed with M they would agree with each other and yield an affirmative conclusion.

156. COROLLARIES FROM THE GENERAL RULES.—The six rules just enumerated are in themselves sufficient for the detection of any fallacy in the formal aspect of syllogistic reasoning. From them, however, are derived three other simple canons, the explicit and distinct remembrance of which will aid the student considerably in detecting such fallacies. As we shall see presently, the first two of these are sometimes stated as independent rules. They are as follows:—

1. FROM TWO PARTICULAR PREMISSES NOTHING CAN BE INFERRED;

2. IF ONE PREMISS IS PARTICULAR THE CONCLUSION MUST BE PARTICULAR;

3. FROM A PARTICULAR MAJOR AND A NEGATIVE MINOR NOTHING CAN BE INFERRED.

Cor. 1. The following proof of the canon that from two particulars nothing can be inferred, is given by De Morgan (*Formal Logic*, p. 14). Since both premisses are particular, the middle term, in order to be distributed once (Rule 3), must be predicate of a negative premiss. Consequently the other premiss must be affirmative (Rule 5), and, being also particular, will distribute neither of its terms. Hence both extremes are undistributed in the premisses. But since one premiss is negative (to distribute M), the conclusion must be negative (Rule 6), and will therefore distribute its predicate P, which, however, was undistributed in its premiss. Hence we have *Illicit Major*. Therefore, with two particular premisses a conclusion is impossible.

We may state what is practically the same proof by examining each possible case separately. We will premise this important truth: that in a valid syllogism the premisses must always contain *one distributed term more than the conclusion;* for,

over and above the distributed extreme or extremes, the premisses must distribute *the middle term at least once.* Now, if both premisses be O propositions, Rule 5 forbids a conclusion. If both be I propositions, they distribute no term at all, and hence we have the fallacy of *Undistributed Middle.* If one be I and the other O, then, as we saw in De Morgan's proof, the O premiss must have M for predicate in order that the syllogism may avoid *Undistributed Middle.* But it avoids this fallacy only by falling into the other fallacy of *Illicit Major*; for I and O distribute only the one term, M, between them, thus leaving P undistributed, whereas the conclusion, being negative, will necessarily distribute it.

Cor. 2. If both premisses are *affirmative*, one being particular, they will distribute only one term, namely, the subject of the other or universal premiss, and this distributed term must be the middle term. Hence, S and P will be undistributed in the premisses. Therefore the conclusion must be *particular* (and affirmative).

Secondly, if one premiss be affirmative and the other *negative* [*both* cannot be *negative* (Rule 5)], the premisses distribute two terms, namely, the subject of the universal premiss and the predicate of the negative premiss. One of these distributed terms must be the middle term (Rule 3); and the other must be the major extreme, since this is distributed in the negative conclusion (Rules 4 and 6). Hence the minor extreme, remaining undistributed in the premisses, must be undistributed in the conclusion; that is, the conclusion must be *particular* (and negative).

Therefore, in every case when one premiss is particular the conclusion must be particular.

Another brief demonstration of the present corollary is based on the previous corollary, combined with a consideration put forward in the preceding chapter (148). Representing the propositions of a valid syllogism by A, B, and C, respectively, we pointed out that the force of the syllogism may be expressed by the hypothetical "*If A and B, then C*". This may also be expressed: "*Given A, if B then C*," which yields the contrapositive "*Given A, if* $\bar{C}$ *then B*". In other words, if two propositions, A and B, prove a third, C, then either of the two (A) and the denial of the third ($\bar{C}$) will prove the denial of the remaining one ($\bar{B}$). "Now, if possible, let A (particular) and B (universal) prove C (universal), then A (particular) and the denial of C (particular) prove the denial of B. But two particulars can prove nothing."[1] Hence a universal and a particular can prove only a particular.

[1] De Morgan, *op. cit.*, p. 14, using P, Q and R as symbols.

Cor. 3. If the major is particular, and the minor negative, the major must be also affirmative (Rule 5), and, therefore, distributes neither of its terms. The major extreme is therefore undistributed. Hence it must be undistributed in the conclusion (Rule 4). But it cannot; for the conclusion, being negative (Rule 6), will distribute its predicate. Hence, from a particular major and a negative minor no conclusion can be drawn.

157. Simplification and Restatement of the General Rules.—The three corollaries just examined have been shown to be dependent on the general rules. These latter are themselves not independent of one another. The student will, of course, find it useful to remember and utilize both the rules and the corollaries, each for itself and independently of the others. But it will be instructive to see how far the general rules themselves are derivable from one another.

Abstracting from the first two rules, which merely give the nature of the syllogism, we have four rules which govern the validity of syllogistic inference proper: two rules of quantity and two of quality;—given above as (3), (4), (5), and (6), respectively. Now it can be shown, *firstly*, that the violation of (3) involves *indirectly*[1] the violation of (4) and *vice versa; secondly*, that (5) may be similarly deduced from (3); *thirdly*, that the first part of (6) is deducible from (5) and *vice versa:* so that we thus arrive at *two* ultimate and *fundamental* rules, one of *quantity*, and the other of *quality*, the former of which may be stated in either of two alternative ways. They are as follows:—

(1) Rule of Quantity: *The middle term must be distributed in one, at least, of the premisses:*

or,

No term may be distributed in the conclusion which was not distributed in its premiss.

(2) Rule of Quality: *To prove a negative conclusion requires a negative premiss.*

Dr. Keynes points out[2] that "the only syllogism rejected by this rule [2] and not also rejected directly or indirectly by the preceding rule [1] is the following:—*All P is M, All M is S*, therefore, *Some S is not P*. In the technical language explained in the following chapter, this is A A O in figure 4. So far, therefore, as the first three figures are concerned, we are left with a single rule, namely, a rule of distribution, which may be stated in either of the alternatives given above."

[1] *i.e.* not in the given syllogism itself, but in another essentially involved in the former, *v. infra*, 169; *cf.* Keynes, *op. cit.*, p. 294, n. 2.

[2] *ibid.*, p. 294, n. 1.

The above results have been reached by the employment of the principle referred to in the proof of *Corollary* 2, above (156). The principle will be again invoked in a subsequent chapter, in connexion with the process of *Indirect Reduction* (169). The following proofs, borrowed from Dr. Keynes,[1] will illustrate its application in the present context.

(*a*) "*Any syllogism involving directly an illicit process of major or minor involves indirectly a fallacy of undistributed middle, and vice versa.*[2]

"Let P and Q be the premisses and R the conclusion of a syllogism involving illicit major or minor, a term X which is undistributed in P being distributed in R. Then the contradictory of R combined with P must prove the contradictory of Q. But any term distributed in a proposition is undistributed in its contradictory. X is therefore undistributed in the contradictory of R, and by hypothesis it is undistributed in P. But X is the middle term of the new syllogism, which is therefore guilty of the fallacy of undistributed middle. It is thus shown that any syllogism involving directly a fallacy of illicit major or minor involves indirectly a fallacy of undistributed middle."

"Adopting a similar line of argument,[3] we might also proceed in the opposite direction, and exhibit the rule relating to the distribution of the middle term as a corollary from the rule relating to the distribution of the major and minor terms."

(*b*) *Rule* (5) *is a corollary from Rule* (3). "This is shown by De Morgan (*Formal Logic*, p. 13). He takes two universal negative premisses, E E. In whatever figure they may be, they can be reduced by conversion to *No P is M*, *No S is M*. Then by obversion they become (without losing any of their force) *All P is* $\bar{M}$, *All S is* $\bar{M}$, and we have undistributed middle. Hence rule [5] is exhibited as a corollary from rule [3]. For if any connexion between *S and P* can be inferred from the first pair of premisses, it must also be inferable from the second pair.

"The case in which one of the premisses in particular . . . may . . . be disposed of by saying that if we cannot infer anything from two negative premisses both of which are universal, *a fortiori* we cannot from two negative premisses one of which is particular."

In this latter case, if M is the *predicate* of the particular premiss, the syllogism may be shown to imply undistributed middle, as in the case of E E above. If, however, M is *subject* of the O premiss we cannot show that the syllogism indirectly involves *undistributed middle*, but we can show that it indirectly involves *four terms*, S, P, M and $\bar{M}$. This latter we can do either by obverting the E premiss after making M its predicate, as Professor Welton does,[4] or by also contraposing the O premiss as De Morgan does.

[1] *op. cit.*, pp. 291-4.

[2] The "invalid syllogism *All M is P*, *No S is M*, therefore *No S is P*, does not directly involve [undistributed middle]. But if this syllogism is valid, then must also the following syllogism be valid: *All M is P* (original major), *Some S is P* (contradictory of original conclusion), therefore *Some S is M* (contradictory of original minor); and here we have undistributed middle. Hence the rule relating to the distribution of the middle term establishes *indirectly* the invalidity of the syllogism in question. The principle involved is the same as that on which we shall find the process of indirect reduction to be based."—*Ibid.*, p. 294, n. 2.

[3] *Cf.* WELTON, *op. cit.*, p. 301.

[4] *ibid.*, pp. 299, 300.

If both are O premisses with *M* as predicate, the obversion of both will show undistributed middle just as in the case of E E. If two O premisses have *M* as subject the same result is reached by contraposition. Finally, if *M* be subject of one O premiss and predicate of the other, the obversion of the latter will show four terms.

Two negatives, therefore, always lead either to *undistributed middle* or to *four terms:* and the latter fallacy may be regarded as an extreme case of the former; for in *quaternio terminorum* neither extreme is compared with even one single instance of the class with which the other extreme is compared.

(*c*) *The first part of Rule* (6) *is deducible from Rule* (5), *and vice versa.* "If two propositions *P* and *Q* together prove a third *R*, it is plain that *P* and the denial of *R* prove the denial of *Q*, for *P* and *Q* cannot be true together without *R*. Now, if possible, let *P* (a negative) and *Q* (an affirmative) prove *R* (an affirmative). Then *P* (a negative) and the denial of *R* (a negative) prove the denial of *Q*. But by hypothesis two negatives can prove nothing.

"It may be shown similarly that if we start by assuming the second of the rules then the first is deducible from it." [1]

158. Alternative Statement of the General Rules of Syllogism.—It was remarked above (156) that the first two corollaries are sometimes given as general rules, increasing the number of these to eight.[2] These eight canons are condensed in the following mnemonic verses which are traditional in English works on logic.[3]

> "Distribuas medium, nec quartus terminus adsit;
> Utraque nec praemissa negans, nec particularis;
> Sectetur partem conclusio deteriorem;
> Et non distribuat, nisi cum praemissa, negetve."

"You must distribute the middle term, and not have a fourth; both premisses must not be negative, nor both particular; the conclusion must follow the weaker part of the premisses, and must not distribute a term, nor deny, unless one premiss does the same."

The pregnant phrase, that "the conclusion must follow the weaker part of the premisses," means (1) that it must be negative if one premiss is negative, (2) that it must be particular if one premiss is particular: the negative and the particular being

[1] Keynes, *ibid.*, p. 293.

[2] See, for example, *Palaestra Logica* (by Forbes and Hird, Oxford, 2nd ed. 1904), p. 46.

[3] *ibid.*, p. 47; *cf.* Joyce's *Principles of Logic*, p. 173, n. The lines are attributed to Petrus Hispanus, afterwards Pope John XXI. (Keynes, *op. cit.*, p. 291).

regarded as "inferior," in quality and quantity, to the affirmative and the universal, respectively.[1]

In scholastic treatises on logic, the following *eight* rules of syllogism are uniformly enumerated,[2] the first four applying to the terms, the second four to the propositions :—

1. *Terminus esto triplex, major, mediusque, minorque.*
2. *Latius hos quam praemissae conclusio non vult.*
3. *Nequaquam medium capiat conclusio opportet.*
4. *Aut semel aut iterum medius generaliter esto.*
5. *Utraque si praemissa neget nihil inde sequetur.*
6. *Ambae affirmantes nequeunt generare negantem.*
7. *Pejorem sequitur semper conclusio partem.*
8. *Nil sequitur geminis ex particularibus unquam.*

It will be noted that, on the one hand, these eight rules do not include among them what we have called the *second rule of structure :* that the syllogism contains three and only three propositions ; that the part of the sixth or last rule which states that a negative premiss necessitates a negative conclusion, is here involved in the rule 7, *Pejorem*, etc. ; while, on the other hand, the two canons given above (156) as corollaries are here stated as rules (7 and 8), together with the rather superfluous rule (3) that the middle term should not appear in the conclusion.

WELTON, *Logic*, i., pp. 282 *sqq.* JOSEPH, *Logic*, chap. xii. KEYNES, *Formal Logic*, part iii., chap. i. JOYCE, *Logic*, xi. and xii.

[1] In regard to the *material* aspect of syllogistic reasoning, this same phrase may be taken to mean that the conclusion cannot be more probable, or certain, than the less probable, or less certain, of the premisses.

[2] *Cf.* HICKEY, *Summula Philosophicae Scholasticae* (Dublin, Browne and Nolan, editio altera, 1908), vol. i., pp. 90, 91. REINSTADLER, *Elementa Philosophicae Scholasticae* (Herder, ed. altera, 1907), vol. i., p. 83. ZIGLIARA, *Summa Philosophica* (edit. tertia, 1880), vol. i., p. 113. "According to Prantl (*Geschichte der Logik im Abendlande* II, S. 275, Leipzig, 1861) these rules were first formulated by Psellus [the younger] in the eleventh century. He expressed them in five formulæ, which were afterwards increased by the explicit statement of their implications to the number of eight."—MERCIER, *Logique*, p. 207, n. The work ascribed to Psellus, in which they occur, is now believed to have been a Greek version, by Georgius Scholarius (Gennadius, 15th century), of the *Summulae Logicales* of Petrus Hispanus (1226-1277). *Cf.* DE WULF, *History of Medieval Philosophy*, p. 356 ; JOSEPH, *op. cit.*, p. 244, n.

CHAPTER III.

FIGURES AND MOODS OF THE SYLLOGISM.

159. DISTINCTION OF FIGURES.—By the *figure* of a syllogism we understand *the position of the middle term relatively to the extremes in the premisses.* Distinctions of *figure* result, therefore, from a consideration of the position of the *terms* of the syllogism —its *remote* matter—in its premisses.

Thus, the middle term may be subject in one premiss and predicate in the other; it may be predicate in both; or it may be subject in both. And, if we distinguish between *major* and *minor* premiss (148), the first of these three alternatives involves, theoretically at least, two distinct cases: that in which the middle term is subject in the major and predicate in the minor premiss, and that in which the middle term is predicate in the major and subject in the minor premiss. The former of these two cases gives us the *First* (or "Perfect" Aristotelean) *Figure;* the latter gives us the so-called *Fourth* (or "Galenian") *Figure*, or else what Aristotle recognized and as an *indirect* and less natural mode of drawing a conclusion in the first figure—a conclusion about what is naturally the major extreme (78, 148), in terms of the minor extreme (171). The case in which the middle term is twice predicate gives us the *Second Figure;* that in which the middle term is twice subject gives us the *Third Figure.* Thus we have the following empty schemes of the four figures:—

Fig. I.	Fig. II.	Fig. III.	Fig. IV.
M—*P*	*P*—*M*	*M*—*P*	*P*—*M*
S—*M*	*S*—*M*	*M*—*S*	*M*—*S*
∴ *S*—*P*	∴ *S*—*P*	∴ *S*—*P*	∴ *S*—*P*

For these we have the mnemonic line of Scholastic logic: "*Sub. prae.* PRIMA; ALTERA *bis prae.*; TERTIA *bis sub.*"; (to which we might add: QUARTA *prae. sub.*).

1. *M* is *subject* of the *major* and *predicate* of the *minor* premiss in the FIRST FIGURE;

2. *M* is *predicate* of *both* premisses in the SECOND FIGURE;

3. *M* is *subject* of both premisses in the THIRD FIGURE;

4. *M* is *predicate* of the *major* and *subject* of the *minor* premiss in the FOURTH FIGURE.

160. DISTINCTION OF MOODS.—By the *Mood* or *Mode* ("*Modus*") of a syllogism, we understand *the quality and quantity of its premisses and conclusion.* Distinctions of *mood* result, therefore, from a consideration of the quality and quantity of the *propositions* of the syllogism—its *proximate* matter. For example, AAA expresses a syllogism in which major, minor, and conclusion are *universal affirmatives;* E I O expresses a syllogism in which the major is a *universal negative*, the minor a *particular affirmative*, and the conclusion a *particular negative.*

Now, recognizing the distinction between *major* and *minor* premisses (148), and always writing the major first, we can easily understand that there are, *a priori*, sixteen possible arrangements of the propositions A, E, I, O, taken two at a time, as premisses of a syllogism. These are A A, A I, A E, A O, I A, (I I), (I E), (I O), E A, E I, (E E), (E O), O A, (O I), (O E), (OO). Furthermore, *each* of these *sixteen* combinations of premisses may be expressed in *four* different figures. For example, A A may represent (1) *M a P, S a M;* (2) *P a M, S a M;* (3) *M a P, M a S;* (4) *P a M, M a S.* Thus, we have in all *sixty-four* possible forms of syllogism ultimately distinct from one another when determined by figure and mood. The question now, therefore, arises: *How many of* these *possible* moods are *valid?* How many of them yield conclusions? This we shall proceed to determine by examining the sixteen forms given above, in the light of the *General Rules and Corollaries* (154-6), and by putting them into each of the four figures successively. The latter process will simultaneously bring to light certain rules which apply to the separate figures of the syllogism, and which are known as the *Special Rules of Figure:* to distinguish them from the general rules of syllogism (154), of which they are simply narrower applications.

161. DETERMINATION OF LEGITIMATE MOODS AND SPECIAL RULES OF EACH FIGURE.—If we apply the general rules and corollaries to the sixteen combinations given above, we shall find that E E, E O, O E and O O violate Rule 5, which forbids two negative premisses; that I I, I O, and O I are excluded by the first corollary, which forbids two particulars; and that I E is excluded by the third corollary, which forbids the combination of a particu-

lar major with a negative minor. Rejecting, then, those eight combinations [enclosed in brackets above, (160)], which, being invalid in all four figures, give us at once *thirty-two* as *invalid* out of the sixty-four possible forms, we are left with the following eight combinations: A A, A I, A E, A O, I A, E A, E I, O A. These we shall now examine in each of the figures.

I. *Special Rules and Lawful Moods of the First Figure.*— The scheme of the first figure is

$$\begin{array}{r} M—P \\ S—M \\ \hline \therefore\ S—P. \end{array}$$

(*a*) Its special rules—one of quality and one of quantity—are

(1) *The minor premiss must be affirmative;*

(2) *The major premiss must be universal;*

or, as expressed in Scholastic logic:—

Minor sit affirmans; major generalis.

(1) Were the minor negative the major would have to be affirmative (Rule 5) and the conclusion negative (Rule 6). The major extreme would therefore be undistributed in its premiss (being predicate of an affirmative) and distributed in the conclusion (being predicate of a negative). And this illicit process violates Rule 4.

(2) Were the major premiss a particular proposition the middle term, being its subject, would be undistributed; and it is also undistributed in the minor premiss where it is predicate of an affirmative. Hence we should have a violation of Rule 3.

Those two rules follow *directly* from the *Dictum de omni et nullo*, which applies directly to this figure.

(*b*) Of the eight combinations of premisses given above, the first special rule of this figure excludes A E and A O. The second excludes I A and O A. This leaves *four* combinations of premisses, *viz.*, A A, A I, E A, E I. What conclusions can these yield? Bearing in mind the general rules and corollaries, that the conclusion follows the weaker premiss, and that a negative conclusion requires a negative premiss, we find that A A will yield A or I; E A will yield E or O; A I will yield I only; E I will yield O only. Thus we have in the first figure *six* valid moods:—

A A A, (A A I), E A E, (E A O), A I I, E I O.

II. *Special Rules and Lawful Moods of the Second Figure.*— The scheme of the second figure is

$$\begin{array}{l} P—M \\ S—M \\ \hline \therefore\ S—P \end{array}$$

(*a*) Its special rules—one of quality and one of quantity—are

(1) *One of the premisses must be negative ;*

(2) *The major premiss must be universal ;*

or, as expressed in Scholastic logic :—

Una negans esto ; et major generalis.

(1) One of the premisses must be negative in order to distribute *M*—which is twice predicate—and thus avoid the fallacy of *undistributed middle* (Rule 3).

(2) Were the major particular, its subject, *P*, would be undistributed, while as predicate of the negative conclusion it would be distributed, thus involving *illicit major* (Rule 4).

(*b*) Of the eight combinations of premisses to be submitted to these two rules, A A, A I, I A are eliminated by the first rule ; I A (again) and O A by the second : thus leaving A E, A O, E A, E I. Tested by the general rules and corollaries, these forms yield the following conclusions respectively : A E yields E and O ; A O yields O alone ; E A yields E and O ; E I yields O alone. Thus we have in the second figure *six* valid moods :—

E A E, (E A O), A E E, (A E O), E I O, A O O.

III. *Special Rules and Lawful Moods of the Third Figure.* The scheme of the third figure is

$$\begin{array}{l} M—P \\ M—S \\ \hline \therefore\ S—P \end{array}$$

(*a*) Its special rules—one of quality and one of quantity—are

(1) *The minor premiss must be affirmative ;*

(2) *The conclusion must be particular ;*

or, as expressed in Scholastic logic :—

Minor sit affirmans ; conclusio particularis.

(1) Were the major negative, the conclusion would have to be negative (Rule 6), thus *distributing* its predicate, the major extreme, *P ;* and the major would have to be affirmative (Rule 5), thus leaving its predicate, the major extreme, *P*, undistributed : thus we should have *illicit major* (Rule 4).

(2) Were the conclusion universal it would distribute its sub-

ject, *S*, which is *undistributed* in its premiss, being there the predicate of an affirmative proposition: thus we should have *illicit minor* (Rule 4).

(*b*) Subjecting the eight combinations of premisses to these rules, we find that the first of them eliminates A E and A O, thus leaving *six* combinations, from each of which, according to the second rule, only a particular conclusion can follow, *viz.* I if both premisses be affirmative, O if one premiss be negative. These *six* valid moods of the third figure are:—

A A I, I A I, A I I, E A O, O A O, E I O.

IV. *Special Rules and Lawful Moods of the Fourth Figure.*—The scheme of the fourth figure[1] is

$$\begin{array}{l} P—M \\ M—S \\ \hline \therefore\ S—P. \end{array}$$

(*a*) Its special rules are somewhat complex. In each of the three preceding figures we were able to commence by laying down some *definite, categorical* rule about the *quality* of the premisses. This, then, helped to determine the other categorical rule for *quantity*.[2] Here, however, we can lay down no *special* categorical rule about the quality of the premisses (beyond the *general* rule that both cannot be negative). Both may be affirmative, or either negative and the other affirmative. These hypotheses give the following *hypothetical* rules of *quantity*.

1. *If the major premiss be affirmative, the minor must be universal;*

2. *If the minor premiss be affirmative, the conclusion must be particular;*

[1] The scheme for the indirect moods of the first figure would be

$$\begin{array}{l} M—P \\ S—M \\ \hline \therefore\ P—S \end{array}$$

—which really differs from the scheme of the fourth figure only in the order in which the premisses are written. Since the conclusion is not expressed in its natural order, Aristotle would still regard the premiss containing *P* as the major, and that containing *S* as the minor, *i.e.* he would describe the syllogism as belonging to the first figure with the conclusion drawn *indirectly*—about the major in terms of the minor. *Cf. infra*, 171.

[2] Whereas *quality* affects only the *propositions*, and is independent of *quantity*, this latter affects the *terms* of the propositions, and is, itself, so far as regards the predicate, dependent on, and determined by, the *quality* of the proposition. Hence the special rules of quality come first, helping to determine those of quantity.

3. *If either premiss is negative the major must be universal ;*

or, as expressed in Scholastic logic : —

1. *Si major affirmat, sit minor generalis ;*
2. *Si minor affirmat, sit conclusio particularis ;*
3. *Si una præmissarum neget, sit major generalis.*

(1) If the major be affirmative, *M*, which is its predicate, is undistributed. It must therefore be distributed in the minor premiss (Rule 3). It is subject in the latter : therefore this must be universal.

(2) This rule has the same reason as the second special rule of the third figure.

(3) This rule has the same reason as the second special rule of the second figure.

It follows as a corollary from the first and third of these rules that *neither of the premisses of a syllogism in the fourth figure can be a particular negative* (O) *proposition.*

(*b*) Applying these three rules to the eight combinations of premisses, we eliminate A I and A O by the *first*, and O A by the *third*, thus leaving five combinations. Of these five A E is the only combination which can here yield two conclusions, *viz.* E and O. For, the *second* of the rules given above forbids an A conclusion from A A, and an E conclusion from E A ; while E I and A I yield only particular conclusions (*Cor.* 2, 156).

Thus we have in the fourth figure *six* valid syllogisms,[1] namely : A A I, A E E, (A E O), I A I, E A O, E I O.

162. THE "ORIGINAL" OR "NAMED" MOODS AND THE "SUBALTERN" MOODS.—The result of our investigation, therefore, is this, that out of the *sixty-four* possible moods, *twenty-four* only are valid, *viz. six* in *each* of the *four* figures. These are respectively :—

[1] Mediate axioms, applicable to the second and third figures respectively, will be given below (169, 170). Various unsuccessful attempts have been made to frame an axiom expressive of the line of inference in the fourth figure—unsuccessful because the lines of inference embodied in its moods are not distinct from those of the other three figures (171). The following purely *extensive* or *class-inclusion* axiom, suggested by Mr. Johnson (*apud* KEYNES, *op. cit.*, p. 338), is not without interest : "*Three classes cannot be so related, that the first is wholly included in the second, the second wholly excluded from the third, and the third partly or wholly included in the first*". This is sufficiently evident, but how it applies to the fourth figure is not at first sight apparent. It "affirms the validity of two antilogisms; in other words it declares the mutual incompatibility of each of the following trios of propositions : *X a Y, Y e Z, Z i X ; X aY, Y e Z, Z a X ;* and it will be found that these incompatibles yield the six valid moods [including *Camenop*] of the fourth figure" (*ibid.*).

In Figure 1, A A A, E A E, A I I, E I O, (A A I), (E A O),
In Figure 2, E A E, A E E, E I O, A O O, (E A O), (A E O),
In Figure 3, A A I, I A I, A I I, E A O, O A O, E I O,
In Figure 4, A A I, A E E, I A I, E A O, E I O, (A E O).

Of these twenty-four, however, the five we have bracketed and brought to the ends of the lines are *superfluous*, inasmuch as a *particular* conclusion is derived in them from premisses which warrant a corresponding *universal* conclusion—from which latter the particular could be got by *subalternation*. Hence, these five moods are called *subaltern moods*.

A SUBALTERN MOOD is *one in which we derive a particular conclusion from premisses which warrant the corresponding universal conclusion about the same subject.*[1] We may distinguish these five, as "*superfluous*," from the other nineteen, as "*useful*," moods. And, since these nineteen have got each a special name, and are hence called the "*named*" moods, the five subaltern moods are sometimes called the "*unnamed*" moods—although, indeed, modern logicians have coined names for them too. The conclusion of a subaltern mood is also sometimes called a "weakened" conclusion.

There must, obviously, be a subaltern mood corresponding to each mood (of the twenty-four) which has a universal conclusion. There are five such: two in the first figure, two in the second, and one in the fourth. The third figure can have no subaltern mood because one of its special rules forbids a universal conclusion.

The subaltern moods are of no practical importance, because we do not usually draw a particular conclusion from premisses which warrant the corresponding universal conclusion; and if we want the particular, we can get it by subalternation from the latter. The inclusion of the subaltern moods gives, of course, a certain symmetry to the treatment of the syllogism.

Of the nineteen "named" moods, *four* belong to the first figure, *four* to the second, *six* to the third, and *five* to the fourth. Their "names" are given in the italicized words of the following traditional mnemonic lines—each "name" having three of the four vowels (*a*, *e*, *i*, *o*) to indicate the quality and quantity of the

[1] *i.e.* about *S* in terms of *P*. From A A in the fourth figure we may draw the conclusion *P a S* instead of *S i P*. But the latter is not the subaltern of the former; it is not *about the same subject* as the former (*P*). Hence A A I in the fourth figure is not a subaltern mood, though A A I in the first figure is such.

major and minor premisses, and conclusions, respectively. The "named" moods are arranged above in the same order as their "names" occur in these lines:[1]

Barbara, Celarent, Darii, Ferioque prioris;
Cesare, Camestres, Festino, Baroco secundae;
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bocardo, Ferison, habet: Quarta insuper, addit:
Bramantip, Camenes, Dimaris, Fesapo, Fresison.
Quinque Subalterni, totidem generalibus orti.
Nomen habent nullum, neque, si bene colligis, usum.

Logicians have found it convenient, nevertheless, to call the five subaltern moods *Barbari, Celaront, Cesaro, Camestrop,* and *Camenop,* after the universal moods to which they respectively correspond.

The significance of the *consonants* in the names of the moods will be explained below (166).

163. THE "FUNDAMENTAL" AND THE "STRENGTHENED" MOODS OF THE SYLLOGISM.—If we examine the twenty-four valid moods given above, we shall see that in certain cases, where a *particular* conclusion is drawn from *two universal* premisses, precisely the same conclusion is also arrived at in the same figure by substituting for *one or other* of the *universal* premisses the corresponding *particular* premiss.

In figure 1, (A A I) and (E A O) have the same conclusions as A I I and E I O respectively.

In figure 2, (E A O) and (A E O) have the same conclusions as E I O and A O O respectively.

In figure 3, A A I has the same conclusion as I A I and A I I; E A O the same conclusion as O A O and E I O.

In figure 4, A A I and E A O have the same conclusions as I A I and E I O respectively.

Now, these *eight* moods, which contain *universal premisses yielding a particular conclusion that could be obtained equally well if one of the premisses were reduced to a particular,* are called STRENGTHENED MOODS or STRENGTHENED SYLLOGISMS.

A Strengthened Syllogism may, therefore, be defined as *a syllogism which would still yield its (particular) conclusion even if*

[1] The lines, as given, are traced by Mr. Joseph (*op. cit.*, p. 261, n.) only as far back as Aldrich's *Artis Logicae Fundamenta.* The older form, with the indirect moods of the first figure instead of the moods of the fourth, is given below (171).

one of its universal premisses were reduced to the corresponding subaltern or particular proposition.

The eight *strengthened moods* are :—

(A A I) and (E A O) in the first figure
(E A O) and (A E O) in the second figure
A A I and E A O in the third figure
A A I and E A O in the fourth figure.

It will be noticed that of these eight, four are *subaltern* moods and four are *named* moods. Thus, of the five subaltern moods, (A E O), *Camenop*, in the fourth figure, is the only one which is not also a strengthened mood.

The *named strengthened* moods are *Darapti* and *Felapton* in the third figure, *Bramantip* and *Fesapo* in the fourth. Every strengthened mood has some term unnecessarily distributed in its premisses. Thus, *Darapti*, *Felapton*, and *Fesapo*, each distribute the *middle term twice ;* while *Bramantip* distributes the *major term* in the major premiss and not in the conclusion. In *Darapti* and *Felapton* we may reduce *either* premiss to its subaltern, thus giving us the "fundamental" moods *Disamis* or *Datisi* from the former, *Bocardo* or *Ferison* from the latter. In *Bramantip* we may reduce the major premiss, thus obtaining *Dimaris ;* and in *Fesapo* we may reduce the minor premiss, thus obtaining *Fresison.*

In opposition to the *eight strengthened* moods, the remaining *sixteen* are called "FUNDAMENTAL" MOODS.

A Fundamental Syllogism is one in which neither extreme is distributed in the premisses unless it is also distributed in the conclusion, and in which the middle term is distributed once only. In other words, it is a syllogism in which no term is unnecessarily distributed.

Summing up the results of the preceding and present sections, we see that of the twenty-four valid moods :

(1) Nineteen are "named" or "original," and five "subaltern" ;

(2) Sixteen are "fundamental" (fifteen named and one subaltern), and eight "strengthened" (four named and four subaltern).

164. DIRECT DETERMINATION OF THE VALID MOODS IN EACH FIGURE.—The way we have determined the nineteen valid and useful moods out of the sixty-four (161), is only one of many alternative ways. It is a more or less mechanical method, simple and easy to follow. We might reach the same

results somewhat more scientifically by inquiring directly, through the general rules of syllogism, or through the fundamental laws of thought (12, 14), in how many ways the A, E, I, and O propositions, respectively, may be proved. Professor Welton gives both methods.[1] They are so instructive that we take the liberty of reproducing here his *Determination of the Valid Moods by Reference to the General Rules of Syllogism.*

(1) *To prove* A. Both premisses must be affirmative (Rule 6), and will therefore distribute their subjects only. *S*, being distributed in the conclusion, must be distributed in its premiss, *i.e.* it must be *subject* of the *minor* premiss, leaving *M* undistributed as predicate. *M* must, therefore, be distributed in the major premiss, *i.e.* must be subject of the latter. Thus we get the one single form of syllogism

$$\begin{array}{l} M\ a\ P \\ \underline{S\ a\ M} \\ \therefore S\ a\ P \end{array}$$

That is to say, *the* A *proposition can be proved only in* ONE *way* —in one mood of one figure—*viz. Barbara* of the *first* figure.

A cannot be proved in the second figure, for the second figure proves *only negatives;* or in the third figure, for this proves *only particulars;* or in the fourth figure, because it would involve *Illicit Minor* (or what, in the corresponding *indirect* mood of the first figure, would be called *Illicit Major*).

(2) *To prove* E. One premiss must be negative (Rule 6) and one affirmative (Rule 5). *S* and *P*, being both distributed in the conclusion, must also be distributed in the minor and major premisses (Rule 4). *M* must also be distributed in one of the premisses (Rule 3). *But in order to distribute three terms between them the premisses must both be universal and one of them negative*—one A, the other E. Hence we may either have the *major* A and the *minor* E, or *vice versa;* and in either alternative the E premiss may be written *in two ways*, being simply convertible. This gives us *four* possible forms:—

(1)	(2)	(3)	(4)
$M\ e\ P$	$P\ e\ M$	$P\ a\ M$	$P\ a\ M$
$\underline{S\ a\ M}$	$\underline{S\ a\ M}$	$\underline{S\ e\ M}$	$\underline{M\ e\ S}$
$\therefore S\ e\ P$	$\therefore S\ e\ P$	$\therefore S\ e\ P$	$S\ e\ P$

That is to say, the E proposition *can be proved in* FOUR *ways, viz.* by E A E (*Celarent*) in the first figure;

[1] *op. cit.*, pp. 315-22.

E A E and A E E (*Cesare* and *Camestres*) in the second;
A E E (*Camenes*) in the fourth figure

E cannot be proved in the third figure, which proves only particulars.

(3) *To prove* I. Both premisses must be affirmative (Rule 6). As neither *S* nor *P* are distributed in the conclusion, it does not matter whether they are distributed in the premisses or not. But since we are searching for premisses which will prove *S i P only* (and not for premisses which would warrant *S a P*), we shall have to confine ourselves to those in which *S* is *undistributed*. Nor does it matter whether *M* is distributed *twice*. The *distribution* of *M* in one premiss, and the *non-distribution* of *S*, are all that need to be secured. Every combination of affirmative premisses in which *M* is, and *S* is not, the subject of an A proposition, will fulfil these conditions. *S* may, therefore, be either (1) subject, or (2) predicate of an I minor, with *M a P* as major; or it may be the predicate of an A minor (*M a S*), distributing *M*, and thus leaving *four* alternatives for *M* in the major, *i.e.* (3) subject of I, (4) subject of A, (5) predicate of I, (6) predicate of A. Thus we have *six* forms :—

(1)	(2)	(3)	(4)	(5)	(6)
M a P	*M a P*	*M i P*	*M a P*	*P i M*	*P a M*
S i M	*M i S*	*M a S*	*M a S*	*M a S*	*M a S*
∴ *S i P*	∴ *S i P*	∴ *S i P*	∴ *S i P*	∴ *S i P*	∴ *S i P*

That is to say, the I proposition *can be proved in* SIX *ways* (besides the *subaltern mood* of *Barbara*) *viz.* by

A I I (*Darii*) in the first figure;
AI I, I A I, AA I (*Datisi, Disamis, Darapti*) in the third figure;
I A I, A A I, (*Dimaris, Bramantip*) in the fourth.

I cannot be proved in the second figure, which proves only negatives.

(4) *To prove* O. One premiss must be negative (Rule 6). *P*, being distributed in the conclusion, must be distributed in the major premiss, and *M* must also be once distributed. Furthermore, as we are seeking premisses which prove O *only*, and not E, *S* will be undistributed in the minor.

(*a*) If the major premiss is E, both *P* and *M* are distributed in it whether it be *M e P* or *P e M*, and no term need be distributed in the minor premiss, though *M* may be. This allows an I minor written either *S i M* or *M i S*, or an A minor written *M a S*. Hence we have the six forms :—

(1)	(2)	(3)	(4)	(5)	(6)
M e P	*P e M*	*M e P*	*P e M*	*P e M*	*M e P*
S i M	*S i M*	*M i S*	*M i S*	*M a S*	*M a S*
S o P	*S o P*	*S o P*	*S o P*	*S o P*	*S o P*

(*b*) If the major premiss be O, *P* must be its predicate, and *M*, to be distributed in the minor, must be subject of an A proposition. Hence we have :—

(7)
M o P
M a S
S o P

(*c*) If the major premiss be A, *P* must be its subject, and *M*, to be distributed in the minor (which must be negative), must be predicate. The minor must therefore be O. Hence we have the form :—

(8)
P a M
S o M
S o P

(*d*) The major premiss cannot be I, for this would involve *illicit major*.

There are, therefore, EIGHT *possible ways* (besides four subaltern moods) *of proving an* O *proposition*. These are

E I O (*Ferio*) in the first figure ;

E I O, A O O (*Festino* and *Baroco*) in the second ;

E A O, O A O, E I O (*Felapton*, *Bocardo*, *Ferison*) in the third ;

E A O, E I O (*Fesapo*, *Fresison*) in the fourth.

Summing up our results we see that :—

(1) A can be proved in *one* way, E in *four*, I in *six*, and O in eight ways : making nineteen valid and useful moods in all.

(2) A can be proved in only *one* figure (the first), E in every figure except the third (which proves only particulars), I in every figure except the second (which proves only negatives), O in every figure without exception.

These results are interesting. The *universal affirmative*, which is the most valuable proposition scientifically, is shown to be *hardest* to prove—there being only one way possible. It is likewise easiest to *disprove ;* for it is disproved by *establishing* its *contradictory*, O, and this is the *easiest* of the four propositions to establish—there being *eight* ways possible. The E proposition,

which is next to A in scientific importance, is also next in difficulty to prove, having *four* ways. Then comes the I proposition, which gives us *indefinite* though *positive* knowledge, with *six* ways for proof. And least important of all is the O proposition, with its *indefinite* and *negative* information, and its *eight* ways of proof.

165. The Valid Moods and the Existential Import of Propositions.—In the chapter on the *Existential Import of Judgments and Propositions*, we examined the bearing of the various conceivable hypotheses, there suggested, on the doctrine of opposition and immediate inference (127). How do those suppositions affect the validity of syllogistic reasoning?

(1) If propositions be interpreted as implying the existence of their subjects, their predicates, and the contradictories of these, then all the moods given already as valid remain valid.

(2) If propositions be taken to imply the existence of their subjects only, then, since the negative proposition will not guarantee the existence of its predicate, any mood will be invalid in which the minor term, *S*, occurs as the predicate of a negative minor premiss. This cannot happen in the first (direct), second, or third figures. It does happen, however, in *Camenes*, and its subaltern mood of the fourth figure, (or *Celantes* with its subaltern, the corresponding indirect moods of the first; *cf.* 173): these moods are, therefore, invalid, if their conclusions be stated categorically, *i.e.* without the proviso, "If *S* exist".

It will be found, further, that on the present supposition the reduction of these moods (to *Celarent* of the first figure) is impossible (168). They involve the conversion of the new conclusion *P e S*, the existence of whose predicate, *S*, was not given in the original premisses, where *S* was predicate of an E proposition. In all other cases of reduction that involve the conversion of E (or the contraposition of A, as in *Faksoko*), the term whose existence is not guaranteed in these processes has its existence guaranteed in the other premiss. Hence the reduction of all the other moods is legitimate on the present supposition.

(3) On "the supposition that no proposition implies the existence either of its subject or of its predicate, all the ordinarily recognized moods of figures 1 and 2 are valid, but none of those of figures 3 and 4 excepting *Camenes* and the weakened form of *Camenes*".[1]

[1] Keynes, *op. cit.*, p. 393.

(4) If particulars be interpreted as implying, and universals as not implying, the existence of their subjects, then (*a*) all the moods with universal conclusions will be valid: (*b*) all the moods in which one premiss is particular will also be found to be valid; while (*c*) all the moods in which from two universals we infer a particular will be invalid. Therefore, all the *subaltern moods* and all the *strengthened syllogisms* are, on this hypothesis, invalid; while the *fifteen named fundamental moods* are valid.[1]

166. The Syllogism and the Quantification of the Predicate.—If Hamilton's doctrine of quantifying the predicate were strictly adhered to (108, 109), it would remove all distinction between subject and predicate, and, consequently, all distinction of figures in the syllogism. For this "unfigured syllogism" Hamilton gives the axiom of identity and diversity (151): "In so far as two notions either both agree, or one agreeing the other does not, with a common third notion, in so far these notions do or do not agree with each other".

But, apart altogether from the intrinsic unsoundness of the whole quantification doctrine, every attempt to carry its application into the theory of the syllogism leads to results which, as Dr. Keynes remarks,[2] are "the reverse of simplification".

A "table of valid moods resulting from the recognition of Y [*Only S is P*] and [its contradictory (108)] η [*Not S alone is P*], in addition to A, E, I, and O" is made out by Dr. Keynes himself. As each of these six propositions has some other form among the six to express its simple converse, it follows that "a valid mood in any figure is reducible to any other figure by the simple conversion of one or both of the premisses. Hence, if the valid moods in any one figure are determined, those of the remaining figures may be immediately deduced therefrom."

"It will be found that in each figure there are twelve valid moods, which are neither strengthened nor weakened."[3]

167. Finding Middle Terms and Constructing Formally Valid Syllogisms.—Scholastic logicians have been accustomed to incorporate, in their treatment of the syllogism, rules and directions for discovering middle terms which would serve to "prove" judgments put forward as theses.[4] This "discovery of a middle term"—*Inventio Medii*, as the process was called—is analogous to the *formation of an hypothesis* in inductive inquiry, and, like the latter, is a work of original endeavour

[1] Keynes, *op. cit.*, p. 393. [2] *op. cit.*, 378. [3] *ibid.*, p. 381.
[4] *Cf.* Zigliara, *Summa Philosophica*, i. (35), xi. Hickey, *Summula Philos. Schol.*, i., p. 102.

which cannot be aided by any logical rules. Besides, being a *material*, rather than a *formal* process, it will find its proper treatment in connexion with Demonstration (*cf.* 258), and not in the present context which deals only with the more *formal* aspect of the syllogism. The Scholastics felt the difficulty of framing rules to direct us in the discovery of those "middle terms" or "means of proof," and practically confined their attention to the first figure. To prove an affirmative conclusion in this figure, they advise us to look for a middle term which is *intermediate in extension between the major and the minor extremes*, or, *of which the major extreme can be predicated and which can itself be predicated of the minor extreme.* To prove a negative conclusion, we are to look for a middle term *about which the major extreme can be denied while it can be affirmed itself of the minor extreme.*

The work of "discovering" general truths, and the "means" of proving them, is not such a simple process as rules like those might lead one to believe. Whether in the abstract or in the experimental sciences, new truths of importance are discovered—and explained or proved—only by long and laborious research (*cf.* 197, 198).

The "proofs" that are often demanded from students, by means of syllogisms to be constructed in some particular mood and figure, are not to be confounded with the genuine proofs that are forthcoming in each science for its own special conclusions. The so-called syllogistic "proofs," demanded in the study of the *formal* aspect of inference, are merely exercises in the construction of syllogisms, and are intended simply to familiarize the student with the moods and figures. A few examples will illustrate this.

The student is, for instance, asked to *disprove* the statement that "*Not all rebellions are justifiable*". He does so by *proving* its contradictory, *viz.* that "*All rebellions are justifiable*". The only mood to prove this is *Barbara.* He sets down the scheme of *Barbara*, filling in all that is given, thus:—

M a P *All* *are justifiable;*
S a M *All rebellions are*
∴ *S a P* ∴ *All rebellions are justifiable.*

He has now to find a middle term which can be predicated of "*all rebellions*," and of which "*justifiable*" can be predicated,

No rule of logic can help him here: he must fall back on his own knowledge and ingenuity. In the example, perhaps "*suppressions of tyranny*" will suffice.

But this very example shows that the so-called "proof," here demanded, is not a *demonstration* (252) at all; for a demonstration requires not only that the conclusion follow from the premisses and be itself true, but *that the premisses also be true;* whereas here all three propositions are false. The premisses constitute what we may call a "formal proof" of the conclusion, *i.e.* they are such that *if they were true the conclusion would necessarily be true also.* So, in constructing "formal proofs" or "formally valid syllogisms," or in "finding combinations of premisses from which a given conclusion would necessarily follow," the student has not to take into account the truth or falsity of the propositions he deals with. He is, *ex hypothesi*, dealing not with the "material" or "truth" aspect, but only with the "formal" or "consistency" aspect, of the reasoning process. If, for instance, he were asked to construct a *formally valid* syllogism, having as its conclusion the proposition "*All men are mortal*," he might construct such a syllogism as the following:—

All horses are mortal,
All men are horses!
Therefore, *All men are mortal*

—which is an apt illustration of the maxim, *Ex falso sequitur quodlibet* (148).

WELTON, *Logic*, bk. iv., chap. iii. KEYNES, *Formal Logic*, pt. iii., chap. ii. JOSEPH, *Logic*, chap. xii. JOYCE, *Logic*, pp. 172 *sqq*. MELLONE, *Introd. Text-Book of Logic*, pp. 164 *sqq*.

CHAPTER IV.

THE DOCTRINE OF REDUCTION; ANALYSIS OF THE FIGURES.

168. THE NATURE AND AIM OF "REDUCTION": EXPLANATION OF THE MNEMONIC LINES.—We have now to examine the characteristics of each of the figures, their mutual relations, and the logical significance of the traditional doctrine on the "reduction" of the moods of the "imperfect" figures to those of the first or "perfect" figure. By the *Reduction* of a syllogism we mean, in general, *the process of so rearranging its premisses that the same conclusion still follows from them but now in a different mood*—whether of the same or of a different figure. But when we speak of reduction simply, we are understood to mean, with Aristotle and the Scholastics, reduction *from a mood of some other figure to a mood of the first.* The latter figure they regarded as the most perfect form of syllogistic inference, and in this they were right, for it is the form naturally assumed by the argument which *demonstrates* its conclusion by showing the *ratio essendi* of the latter. That it is the *only cogent* form of reasoning they did not maintain; nor would this be true, for reasonings in the other figures are equally cogent.

They do seem, however, to have taught that the cogency of the syllogistic inference *can be seen most clearly*, and *to the best advantage*, in the first figure, and to have concluded from this that the proper way to demonstrate the validity of a syllogism in any other figure was to "reduce" it to the first figure and apply the *Dictum de omni et nullo* to it when so reduced. Here they went too far: the cogency of syllogistic inference is not *always* seen most clearly in the first figure. There are, as we shall see, arguments which fall more naturally, some into the second, others into the third figure. Besides the method of "reduction," moreover, there are other and easier ways of testing the validity of syllogisms in figures other than the first: for instance, by

applying to them either the general rules of syllogism or the special canons of the figure in question.

Hence, reduction of the other figures to the first is not a *necessary* portion of the theory of syllogistic inference. And wherever arguments fall more naturally into other figures than the first, reduction to the latter figure is neither the most obvious way of testing the validity of such arguments, nor does it really establish the equivalence or unity of the different figures. We shall see that the first, second, and third figures are *specifically distinct* types or modes of what is *generically one* and the same sort of inference. Were there not a generic unity in the syllogistic process, as illustrated in all the figures and moods, these could not be shown to be "reducible" to one another. Reduction brings out this fundamental unity. It is, moreover, a useful logical exercise. We shall first explain the process, and then investigate more closely the relations between the various figures.

It was partly as an aid to the process of reduction that the mnemonic lines were constructed (162). We have seen already that the first three vowels in each word denote the quality and quantity of the propositions they respectively stand for—major, minor, and conclusion. Now as regards the consonants:—

(1) *B*, *C*, *D*, and *F*, the first four capital consonants of the alphabet, have been naturally chosen as the *initial* consonants of the words representing the moods of the first figure. *Whichever of these four consonants is the initial consonant of the name of a mood of any other figure, indicates that this mood is to be reduced to the mood whose name commences with the same consonant in the first figure.* Thus, *Darapti* is to be reduced to *Darii*, *Camestres* to *Celarent*, *Bramantip* to *Barbara*, *Felapton* to *Ferio*, etc.

(2) Of the consonants contained *in the body* of the words:

s denotes that the preceding proposition of the original syllogism is to be converted simply in the process of reduction.

p similarly indicates *conversio per accidens* of the preceding proposition of the original syllogism.

m signifies *metathesis*, or *mutatio praemissarum*, *i.e.* transposition of the premisses so that the major of the original becomes the minor of the new syllogism, and the minor of the original the major of the new one.

(3) Of the consonants occurring *at the end* of the words:—It will be noted that wherever *m* occurs in the body of a word, either *s* or *p* occurs at the end of it; and this signifies that the

conclusion *of the new syllogism* is to be converted—simply or *per accidens* as the case may be—in order to have the new conclusion exactly the same as that of the original syllogism. For, on account of the transposition of the original premisses, the conclusion of the new syllogism is in the form "P—S," *i.e.* it is drawn about *P* in terms of *S*: and this must be converted in order to get the original "S—P" conclusion.

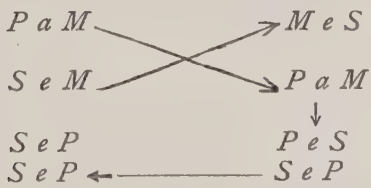
(4) The consonant *c*, in *Baroco* and *Bocardo*, signifies *conversio syllogismi* or change of syllogism: *i.e.* it signifies that arguments in these moods cannot be transposed *directly* into any mood of the first figure, and that their validity must be proved by an *indirect* process which will show by means of a syllogism in the first figure that if they were not valid, *impossible* or *absurd* consequences would follow (168). The letter *c* further implies that, in constructing the new syllogism which will show this impossible consequence, *the proposition immediately preceding c* in the original syllogism is the one *to be omitted* in the new.

But if we apply the processes of *obversion* and *contraposition*—avoided by the Scholastics on account of the negative terms involved—to the two moods in question, we may reduce these *directly*: *Baroco* to *Ferio*, and *Bocardo* to *Darii*. Hence we find substituted, for *Baroco* and *Bocardo*, the mnemonics *Faksoko* and *Doksamosk*, respectively: in which *k* denotes obversion, *ks* contraposition, and *sk* (at the end) simple conversion followed by obversion (of the *new conclusion*).

(5) The remaining consonants, *b*, *d*, *l*, *n*, *r*, *t*, have no significance in the mnemonic lines.

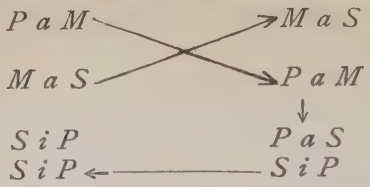
The following examples of direct reduction will suffice to show the application of the conventional regulations given above:—

(1) *Camestres* is thus reduced—to *Celarent* (*C*):



That is to say, we transpose the premisses (*m*), and convert *simply* (*s*) the original minor: this gives us a syllogism in *Celarent* with *P e S* for conclusion: this we convert simply (*s*) to obtain the original conclusion, *S e P*.

(2) *Bramantip* is thus reduced—to *Barbara* (*B*):



That is to say, we merely transpose the original premisses (*m*) in order to get a syllogism in Barbara with conclusion *PaS*, which must be converted *per accidens* (*p*) to give the original conclusion, *S i P*.

(3) *Fesapo* is thus reduced—to *Ferio* (*F*):

$$\begin{array}{ll} P\ e\ M & \longrightarrow M\ e\ P \\ M\ a\ S & \longrightarrow S\ i\ M \\ \hline S\ o\ P & \hline S\ o\ P \end{array}$$

That is, we convert the major simply (*s*), and the minor *per accidens* (*p*), in order to obtain a syllogism in the first figure.

(4) *Darapti* is thus reduced—to *Darii* (*D*):

$$\begin{array}{ll} M\ a\ P & \longrightarrow M\ a\ P \\ M\ a\ S & \longrightarrow S\ i\ M \\ \hline S\ i\ P & \hline S\ i\ P \end{array}$$

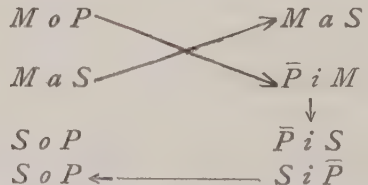
That is, we obtain the same conclusion by *Darii* if we convert *per accidens* (*p*) the minor premiss of *Darapti*.

(5) *Baroco* (*Faksoko*) is thus reduced *directly*—to *Ferio* (*F*):

$$\begin{array}{ll} P\ a\ M & \longrightarrow \bar{M}\ e\ P \\ S\ o\ M & \longrightarrow S\ i\ \bar{M} \\ \hline S\ o\ P & \hline S\ o\ P \end{array}$$

That is, we obtain the same conclusion in *Ferio* by contraposition (*ks*) of the major premiss and obversion (*k*) of the minor premiss of a syllogism in *Baroco* (*Faksoko*).

(6) *Bocardo* (*Doksamosk*) is thus reduced *directly*—to *Darii* (*D*):



That is, we transpose the premisses (*m*), contraposing (*ks*) the original major; this gives us a syllogism in *Darii* with conclusion

$\bar{P}$ *i* S, which we convert simply, and then obvert (*sk*), in order to obtain the original conclusion, S *o* P.

169. INDIRECT REDUCTION: EXTENSION OF THE DOCTRINE OF REDUCTION.—Besides the process of *Direct Reduction*, which has just been described, there is another method by which we may prove the validity of a syllogism belonging to a figure other than the first.

We may establish a given proposition by proving its contradictory to be false; and we may do this latter by proving that were this contradictory true something self-contradictory, and therefore *impossible*, would follow. Euclid often makes use of this method. It is called variously *Indirect Proof* (*cf. infra*, 254, *b*), *Reductio ad Impossibile* (ἀπαγωγὴ εἰς τὸ ἀδυνατον—Aristotle) *Reductio per Impossibile*, *Reductio ad Absurdum*, *Deductio ad Absurdum*. It always runs on these lines: "*The proposition P*—let us say—*is true; for, if not, then* $\bar{P}$ *is true; but if* $\bar{P}$ *be true, Q must be true; Q, however, cannot be true: we know it to be false; therefore* $\bar{P}$, *from which it follows, must be false; therefore its contradictory, P, must be true;—Q.E.D.*" We have referred more than once already (148; 156, *Cor.* 2) to the principle underlying this process: that *if the conclusion of a valid inference be false the premisses or antecedent from which it necessarily follows must also be false*—a principle which is involved in the very nature of logical inference.

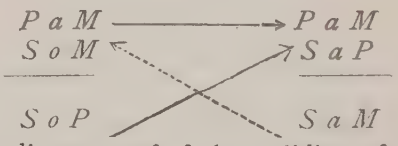
Now this principle, involved in the process of *Indirect Proof*, may be employed for the purpose of proving to a person *who admits the validity of the moods of the first figure only*, that the moods of the other figures are also valid. Utilizing this principle, we can force such a person, *by means of a syllogism in the first figure whose cogency he admits*, to admit also the validity of the moods of the other figures—*i.e.* that if their premisses be true their conclusions must be true—under pain of contradicting himself. It was in this way that the Scholastics, following Aristotle, proved the validity of the moods *Baroco* and *Bocardo*.

We may apply it to *Baroco* as follows:—

If it is true that *All P is M*, and that *Some S is not M*, then it is true that *Some S is not P*; for if this latter is not true, then, by the principle of excluded middle, its contradictory, *All S is P*, is true; and, *ex hypothesi*, the two original premisses, *All P is M*, and *Some S is not M*, are granted to be true. But if it be true that *All P is M*, and that *All S is P*, then it must be true that

All S is M—by the admittedly cogent reasoning of the first figure. But, by the principle of contradiction, *All S is M* cannot be true: for it is the contradictory of the other original premiss, *Some S is not M*, which is already admitted to be true; hence *All S is M* must be false; hence *either of the premisses from which it follows must be false.* But the premiss *All P is M* was originally given true; hence it must be the premiss, *All S is P*, that is false. Therefore its contradictory, *Some S is not P*, must be true. That is, the conclusion of the original syllogism (*Baroco*) has been proved—by a syllogism in the first figure (*Barbara*)—to be necessarily involved in the truth of its premisses, and to follow necessarily therefrom.

The original syllogism and the one that proves its validity are respectively:



Similarly, the indirect proof of the validity of *Bocardo* may be represented thus:

The process here illustrated is not "Reduction" in the same sense as when we speak of "Direct Reduction": for in the latter the premisses of the new syllogism are either identical with, or are eductions from, the premisses of the original syllogism; while in the former the new syllogism is quite a different syllogism from the original one. The new syllogism is, of course, involved in, or implied by, the original one; for *every valid syllogism involves two other valid syllogisms formed by combining the contradictory of the original conclusion with each of the original premisses successively, so as to yield for conclusion in each case the contradictory of the other original premiss.*[1] In indirect reduction we take the contradictory

[1] *Cf.* KEYNES, *Formal Logic*, p. 304, § 214, where it is proved "that if three propositions involving three terms (each of which occurs in two of the propositions) are together incompatible, then (*a*) each term is distributed at least once, and (*b*) one and only one of the propositions is negative"; and "that these rules are equivalent to the rules of the syllogism". Of three such propositions, any two are said to be *incompatible* with the third when from their truth *the contradictory of the third is a necessary consequence.* Three such propositions form what is called an *Antilogism* (*ibid.*,

of the conclusion that has been denied: this contradictory we combine with whichever premiss of the original syllogism will give us a syllogism *in the first figure*, having for its conclusion the contradictory of the omitted premiss.

The INDIRECT REDUCTION of a syllogism might therefore be defined as *the process of proving, by means of a syllogism in the first figure, that if the given syllogism were not valid, some self-contradictory consequence would follow.*

The process may be applied to the other moods of the second and third figures, as well as to *Baroco* and *Bocardo*, and its application to all the moods of these figures will be found to shed a very instructive light on the relations between the first three figures. It will be found, for instance, that if any given syllogism belongs to *any one of the first three figures*, each of the other syllogisms implied by it will be in *each of the remaining two* of those three figures.[1] From this it follows that "there must be an equal number of valid syllogisms in each of the first three figures, and that they may be arranged in sets of equivalent trios. These equivalent trios will be found to be as follows (sets containing strengthened premisses or weakened conclusions, being enclosed in square brackets);

(I)	(II)	(III)
Barbara,	Baroco,	Bocardo;
[A A I,	A E O,	Felapton;]
Celarent,	Festino,	Disamis;
[E A O,	E A O,	Darapti;]
Darii,	Camestres,	Ferison;
Ferio,	Cesare,	Datisi.

The corresponding antilogisms are A A O, [A A E,] E A I, [E A A,] A I E, E I A.

. . . Figure four is . . . self-contained in the sense that if we start with a syllogism in this figure, both the other syllogisms will be in the same figure. . . . It follows that in figure four the number of valid syllogisms must be some multiple of three. The number is, as we know, six. There are therefore two equivalent trios; and they will be found to be as follows;

[Bramantip, A E O, Fesapo;]
Camenes, Fresison, Dimaris."[2]

Since every valid syllogism in the second or third figure implies *two* others, only one of which is in the first figure, how are we to know with which premiss of the original we are to combine the contradictory of the conclusion, in order to obtain the syllogism in the first figure, needed in indirect reduction? The

p. 332). Any such trio, therefore, yields *three valid syllogisms*, each having a pair of the propositions as premisses and the contradictory of the third for its conclusion. From this it also follows that every valid syllogism implies *two other valid syllogisms*: each of these will take one of the original premisses, and the contradictory of the original conclusion, to prove the contradictory of the other original premiss.

[1] KEYNES, *op. cit.*, pp. 333-4. [2] *ibid.*, pp. 334-5.

rule given by the *c* in *Baroco* holds good for *all* the moods of the second figure: *it is the premiss preceding c* (*i.e.* the *minor*) *we are to omit.* And, similarly, *Bocardo* gives the rule for all the moods of the third figure: *i.e. it is the original major we are to omit.* These rules are embodied in the mnemonic lines given in some Scholastic treatises:

Majorem servat, variatque *Secunda Minorem.*
Tertia Majorem variat, servatque *Minorem.*[1]

By recognizing the processes of obversion and contraposition, we may extend the doctrine of reduction in various directions, beyond that of reducing the moods of the other figures to some mood of the first. We may, for instance, by obversion, reduce any given mood to another mood of different quality in the same figure. For example, *Darapti* may be thus reduced to *Felapton.*

$$\begin{array}{ll} M\ a\ P \longrightarrow & M\ e\ \bar{P} \\ \underline{M\ a\ S \longrightarrow} & \underline{M\ a\ S} \\ S\ i\ P & S\ o\ \bar{P} \\ & \downarrow \\ S\ i\ P \longleftarrow & S\ i\ P \end{array}$$

Again, we may show that every mood of the "imperfect" figures may be reduced not merely to *some one or other* of the moods of the first figure, but *to any mood we may choose* of this figure. This will be shown to be possible if we can show that the moods of the first figure are mutually reducible to one another. But *Barbara* may be thus reduced to *Celarent*:

$$\begin{array}{ll} M\ a\ P \longrightarrow & M\ e\ \bar{P} \\ \underline{S\ a\ M \longrightarrow} & \underline{S\ a\ M} \\ S\ a\ P & S\ e\ \bar{P} \\ & \downarrow \\ S\ a\ P \longleftarrow & S\ a\ P \end{array}$$

Similarly *Celarent* may be reduced to *Barbara.* So, also, *Darii* may be reduced to *Ferio;* and, *vice versa*, *Ferio* to *Darii.* Hence, we have only to show that *Celarent* is reducible to *Ferio* and *vice versa*, or that *Barbara* is reducible to *Darii* and *vice versa.* Some, indeed, maintain that it is not really necessary to reduce *Darii* to *Barbara*, or *Ferio* to *Celarent;* that *Darii* is really the same as *Barbara*, and *Celarent* the same as *Ferio*, since we know that the "Some S's" referred to in the conclusions of *Darii* and

[1] Zigliara, *op. cit.*, (35), x.

Ferio are exactly the same as the "Some S's" referred to in the minor premiss.[1] If direct reduction be attempted, it will be found to be impossible: the indirect process, or *reductio ad impossibile*, is the only one by which we can "reduce" *Barbara* to *Darii* (or *vice versa*), or *Celarent* to *Ferio* (or *vice versa*). Thus, we may *indirectly* prove the validity of a syllogism in *Barbara* by means of a syllogism in *Darii* as follows: If $S\ a\ P$ does not follow from the premisses $M\ a\ P$, $S\ a\ M$, then its contradictory, $S\ o\ P$, may follow from them as true. But if $S\ o\ P$ is true, so is its partial contrapositive $\overline{P}\ i\ S$; and $S\ a\ M$ is also given true. Combining the latter we have this syllogism in *Darii*:—

$$\begin{array}{l} S\ a\ M \\ \overline{P}\ i\ S \\ \hline \therefore \overline{P}\ i\ M = M\ i\ \overline{P} = M\ o\ P \end{array}$$

But $M\ o\ P$ cannot be true, for it contradicts $M\ a\ P$ which was given true; therefore the premiss $\overline{P}\ i\ S$ must be false; and therefore also $S\ o\ P$. Hence the original conclusion, $S\ a\ P$, must follow as true from its premisses.

Since all the moods of the first figure cannot be reduced *directly* to one another, it is not correct to say that the reasoning in any mood of any other figure than the first may be expressed in any mood of the first figure. What is true is this, that the validity of any valid mood of any figure other than the first (or of the first itself) may be proved by means of a syllogism in any mood of the first figure. Of course, a person who will not admit the validity of a syllogism in a mood of the *first* figure will not admit the validity of the *validating* syllogism that may be brought forward in any other mood of that figure. Indeed, even where there is question of the second and third figures, whenever an argument falls more naturally into one of these than into the first, its cogency may be seen more clearly in that figure than in the mood of the first figure to which the traditional doctrine would have us reduce it. That there are such arguments will appear presently from a closer study of each figure.

170. CHARACTERISTICS OF THE FIRST FIGURE.—The first figure (*a*) embodies the most usual and scientific form of syllogistic inference, *viz.* the application of some abstract, necessary truth, or general law, to concrete, particular cases subsumed under it: hence it was called by Aristotle the *perfect* figure of the syllogism. (*b*) It is the *only* figure which can prove *all four* forms of the categorial judgment, A, E, I, and O. (*c*) It is the only figure in which the *universal affirmative* (A) can be proved. This makes it all-important; for it is A propositions that all the sciences aim at

[1] *Cf.* KEYNES, pp. 336-7.

establishing. Hence, the proofs found in all the "exact," or "abstract," or "mathematical" sciences, run in the mood *Barbara*. (*d*) It is the *most natural* form of reasoning, inasmuch as the extremes suffer no inversion, occupying in the premisses the same position, as subject and predicate, which they occupy in the conclusion; whereas in the second figure the major extreme, in the third the minor extreme, and in the fourth (or first *indirect*) both extremes, suffer inversion of position in the passage from premisses to conclusion.

The *Dictum de omni et nullo* is directly applicable to this figure alone. The only difference between *Barbara* and *Darii*, and between *Celarent* and *Ferio*, is that in the two universal moods the principle laid down in the major is definitely applicable to all the members of the class *S*, whereas in the particular moods the extent of its application is left indefinite. And the only difference between the two affirmative and the two negative moods is that in the former case the general principle or law laid down is affirmative, while in the latter it is negative.

Dr. Keynes sums up the four moods in the following scheme: [1]—

"Rule . . . *All M is P* (or *is not P*),
Case . . . *All* (or *some*) *S is M*,
Result . . . therefore, *All* (or *some*) *S is P* (or *is not P*)."

With this it will be interesting to compare the *mixed hypothetical syllogism* (179):—

"*If anything is M it is P* (or *is not P*),
Certain subjects, S, are M,
therefore, *They are P* (or *are not P*)." [2]

This presentation of the first figure is practically the same as Kant's analysis of the syllogism: an analysis which he applied to hypothetical and disjunctive, as well as to categorical, syllogisms. The major premiss he interpreted as a rule connecting a predicate (P, $\bar{P}$) with a condition (M), the minor premiss as a subsumption of a subject fulfilling this condition, and the conclusion a determination of this subject by the predicate of the rule. The canon or axiom of the syllogism thus interpreted would be: "*Whatever satisfies the condition of a rule falls under the rule*".[3]

171. Characteristics of the Second Figure. — The second figure (*a*) proves *only negatives*. Hence it is appropriate for the "discovery or proof of the distinctions between things," [4] or for the disproof of some positive assertion. (*b*) It has been called the *Exclusive Figure*, because by means of it we may go on successively disproving or eliminating a number of supposi-

[1] *op. cit.*, p. 336, with other symbols. [2] *ibid.*, n. 2.
[3] *Cf.* Joseph, *op. cit.*, p. 286.
[4] Lambert, *Neues Organon*, ii., p. 139. *Cf.* Joseph, *op. cit.*, p. 192: "The conclusion states a fact of difference between two things, which the premisses prove but do not account for".

tions or hypotheses as to the nature or cause of a thing, until we happen to find the correct one. This gradual process of exclusion is called *abscissio infiniti*. It may be symbolized as follows:—

"*S either is or is not A;*
But, Every A is X,
and S is not X,
∴ S is not A.
If S is not A it either is or is not B;
But, Every B is Y,
and S is not Y,
∴ S is not B.

And so on, till we are left with only one possible conclusion —*S is P.*"[1]

For example, in the diagnosis of a disease the physician may proceed to reason thus: *Smallpox has certain symptoms; this disease does not show these symptoms; therefore it is not smallpox:* and so on. Or the botanist may reason thus: *Such an order of plants manifests such and such properties; this specimen has not these properties; therefore it does not belong to this order.*

The valid moods of this figure may be summed up in the scheme:
"Rule . . . *All P is M* (or *is not M*),
Denial of Result . . . *Some* (or *All*) *S is not M* (or *is M*),
Denial of Case . . . therefore, *Some* (or *All*) *S is not P;*"
which is otherwise expressed by the mixed hypothetical syllogism:
"*If anything is P it is M* (or *is not M*),
Certain subjects, S, are not M (or *are M*),
therefore, *they are not P.*"

While the corresponding syllogism in the first figure above, inferred from ground to consequence, the present one infers from denial of consequence to denial of ground.[2]

Is this a distinct form of inference from that exemplified in the first figure?[3]

[1] WELTON, *op. cit.*, p. 313. [2] KEYNES, *ibid.*, pp. 336-7.

[3] No doubt, the first figure does not always *demonstrate* or give the *causa essendi*, but sometimes only a sign or effect or index of the conclusion. But when the latter is negative, and the inference is from *the absence* of certain symptoms or marks in *S*, the argument runs more naturally in *Cesare* (or *Festino*) than in any mood of the first figure: "*All fish breathe through the gills*, and *Whales do not* ∴ *A whale is not a fish*". This is a natural argument (in *Camestres*). It is based on the absence of a certain feature in the whale. And if I want to put the argument into the mood which naturally corresponds to it in the first figure, I must make this "absence" of an attribute my middle term and reduce (by contraposition) to *Barbara*: "*What does not breathe through the gills is not a fish*, and *Whales do not* ∴ *The whale is not a fish*"; and not to *Cesare*, which gives a conclusion about fish, "whereas whales are really the subject of my thought" (JOSEPH, *op. cit.*, p. 292). Where the two extremes are disparate accidents, the second figure may not, perhaps, be quite as natural as the first, though indeed there is little to choose as between them: "Some ger-

It seems to be a distinct type: it shows us *that* a thing is not so, without showing us *why* it is not so:[1] and if we are asked to justify or validate our inference that "*S is not P*" we naturally do so, not by trying to throw the inference into the first figure, but by an appeal to the *reductio ad impossibile* (167): by arguing that "*if S were P it would be M, but it is not*" (or, as the case may be, that "*if S were P it would not be M, but it is*"): "The patient has not smallpox, because if he had he would reveal certain symptoms which he does not reveal"; "The Christmas rose is not a true rose, because if it were it would bloom in summer, but it does not," "Frozen meat contains no active bacilli, because if it did it would putrefy, but it does not". Those are all natural justifications of syllogisms in the second figure; and they are all indirect: they proceed by showing that if the conclusions were not true something contradictory of an admitted premiss would follow. And the force of this *reductio ad impossibile* is not apprehended as an afterthought by one who has argued in the second figure: it "is really a part of the thought grasped in the syllogism. . . . We see the validity of the conclusion by the contradiction that would be involved in denying it."[2] And this it is that differentiates the second figure from the first: the reasoning in the former is indirect, while in the latter it is direct. In the first figure the fact that *S* is something other than *P* (namely *M*) gives the mind an insight into the real cause or ground of its not being *P*; while in the second we merely see that we are forced under pain of contradiction to admit *that S* is not *P*, without seeing *why* it is not *P*. The perception of this contradiction is not *the reason of the truth of the conclusion*, the reason *why* the conclusion is true, but is only the reason why *we have to admit* the latter, the means whereby we realize the validity of the inference.

An axiom for the second figure, corresponding to the *Dictum de omni* for the first, and called the *Dictum de diverso*, has been formulated in the following, or similar, terms: *If an attribute* [*M*] *can be predicated affirmatively or negatively of every member of a class* [*P*], *any subject* [*S*] *of which it can not be so predicated does not belong to that class.*

An axiom more directly based on the intension of the terms, like the *Nota notae*, has been thus formulated by Mr. Joseph: "*no subject* [*S*] *can possess an attribute* [*P*] *which either excludes what it possesses or carries what it excludes* [*M*]".[3]

172. CHARACTERISTICS OF THE THIRD FIGURE.—The third figure (*a*) proves *only particulars*, proving each (I and O) in three different modes. It is, therefore, (*b*) appropriate for *disproving a general rule or statement* by establishing *some* exceptions to it—at least *one*. (*c*) It is the *only natural figure* for expressing argu-

aniums are not fragrant, because they are scarlet, and no fragrant flowers are scarlet," as compared with "Some geraniums are not fragrant, because they are scarlet and no scarlet flowers are fragrant" (*ibid.*, p. 290, n.)

[1] In answering this question we must bear in mind that "the same verbal form may be used where the thought in the speaker's mind is different. The character of an argument depends not on the verbal form, but on the thought behind it."—JOSEPH, *op. cit.*, p. 290.

[2] JOSEPH, *op. cit.*, p. 293.

[3] *op. cit.*, p. 295.

ments in which *the middle term is singular*, or entirely definite in quantity; more particularly if the other terms are general. The reason is, that in the third figure the middle term is twice subject, and the singular term is naturally subject of the proposition in which it occurs if the other term is general (81). For instance, it would be impossible to express the argument "*Socrates is wise*, *Socrates is a philosopher*, therefore, *Some philosophers are wise*" so appropriately in any other figure as in the figure (and mood, *Darapti*) in which it stands. (*d*) It has been called the *inductive* figure, because it expresses the mental process whereby we seek, by *adducing* or enumerating instances [of *M*], to establish some connexion between attributes or features [*S* and *P*] observed to be characteristic of these instances: and *Induction* is the general name of the process by which we establish universal truths from the facts of our experience. *S* and *P*, therefore, are general characteristics: in the premisses we either affirm both, or affirm one and deny the other, of the same *M's*: and thereby we seek, in the conclusion, to establish a connexion, affirmative or negative, between *S* and *P*: but, of course, the connexion cannot be a universal one: no mere enumeration of instances, in which *S* and *P* are (or are not) found together (in *M*), will warrant us in stating that *S must always* and *necessarily* be (or not be) *P*: all we can conclude is that *S may* be (or *need not* be) *P*: but this particular conclusion—in addition to disproving the universal opposite: "*S* cannot be (or must be) *P*"—has the merit of *suggesting*, as worthy of investigation, the *hypothesis* that the *universal* conclusion, "*S* must be (or cannot be) *P*," is perhaps true.

If the particular conclusion of the third figure, "Some S's are (or are not) P," be understood modally, as just suggested, *i.e.* as meaning that "S is (at least) compatible with the presence (or absence) of P," it matters not whether the middle term be *one single M*, or *some M's*, known to be the same in both premisses, whether the number be definite or indefinite, or *all M's* universally: the ground for arguing compatibility is equally strong in the case of one instance—properly observed, of course—as in the case of several. But if we seek to discover more than mere compatibility, if we suspect a *necessary connexion*, between *S* and *P* (or the absence of *P*), then the probability of such a connexion will, no doubt, depend upon the number of instances examined, though it will depend far more upon the nature of the instances, and of the attributes compared. The "case of the cow in my paddock" gives me the same ground for concluding that possessing horns, or cloven feet, is *compatible* with chewing the cud, as an indefinite number of such cases would furnish. But if there is question of collecting evidence for my suspicion that "all cloven-footed animals ruminate," fifty cows would

perhaps furnish somewhat more evidence than one cow ; but then, too, five instances *differing otherwise* from each other—a cow, a goat, a sheep, a deer, a camel—would furnish far stronger evidence than the former fifty : the nature of the instances is more important than their number. "Again, we might meet a Privy Councillor in a light suit, and yet not be led to regard the next man we met in a light suit as a Privy Councillor ; but if we met a guardsman in a breastplate, we should very likely suppose the next man in a breastplate to be a guardsman" : [1] the nature of the attributes here determines the difference. These considerations belong properly, as we shall see, to the logic of induction.

If we interpret the particular conclusion "Some *S*'*s* are (or are not) *P*" not modally, but assertorically, as referring to an unspecified portion of the class *S*, and ask ourselves of what portion of the class *S* is *P* affirmed (or denied), we must answer : of those things that were found in the premisses to be characterized not only by *S* but by *M* and by *P* (or the absence of *P*) ; and of course the extent of this class, or, in other words, the extension of the conclusion, will vary with the extent of the *M*'*s* referred to in both premisses. If I learned that all horned animals have cloven feet, and also that they all chew the cud, and if I infer from this that "some ruminants are cloven footed," I know the extent of the "some" in my conclusion : that it refers to all the ruminants that are horned ; and if I *keep this thought* in my conclusion, if my conclusion really is that "all ruminants *that have horns* have cloven feet" or that "all *horned animals* chew the cud and have cloven feet," I am merely adding up the information contained in the two premisses, and I have no syllogism at all. But if I *eliminate* the middle term ("horned") from my thought, and retain only the indefinite judgment that "some ruminants are cloven-footed," meaning thereby that *these attributes are compatible*, I have a genuine syllogism. Of course, in all such syllogisms the *ground* of the conclusion lies in the examined instances ; but the ground for a judgment forms no necessary part of the meaning or import of the judgment.

We have now to ask whether the third figure gives us a type of inference distinct from those already examined. And the answer will be in the affirmative. In the third figure our inference manifestly turns *on the instances* cited. It is because *S* and *P* are both present (or one present and the other not) *in the same instances*, that we have a right to connect *S* and *P* (affirmatively or negatively) in our conclusion. It matters not whether "all *M*'*s*" [or "most *M*'*s*"] are cited in both premisses, or "all" [or "most"] in one premiss and "some" on the other, our conclusion turns on the relations found to obtain between the extremes and *the same M*'*s* : even if "Some *M*'*s*" were cited in *both* premisses, we could still derive our conclusion did we know that *the same* "some" were cited in both premisses.[2] It is only upon those *M*'*s* that are compared both

[1] JOSEPH, *op. cit.*, p. 309.

[2] By giving the "Some M" a definite class-name, say "*N*" and substituting "All *N*" in both premisses, we reduce all the moods of the third figure to *Darapti* and *Felapton*. *Bocardo* may thus be directly reduced, through *Felapton*, to *Ferio*. Similarly *Baroco* of the second figure may be reduced, through *Camestres*, to *Cesare*.

with *S* and with *P*, our conclusion depends ; and if our conclusion, that " Some S is [or is not] P " or, which is the same, " S may be [or need not be] P " —if this conclusion be challenged, the *direct and natural way* of showing it to be justified is *by appealing to the instances* of *M's* that are both *S* and *P* [or, that are *S* and are not *P*].

This direct appeal to an instance was called by Aristotle ἔκθεσις, or *Exposition :* " if all *S* is both *P* and *R*, we may take some particular *S*, say *N;* this will be both *P* and *R*, so that there will be some *R* which is *P* ".[1] This is the real line of thought followed in the third figure : we accept the conclusion because we can cite instances of its truth. The instances need not be *individuals ;* they may be *kinds* or *species*. And, of course, they need not be produced physically, but only in our thought. They must, however, be produced in thought ; and it is upon them, as embodying the truth of our conclusion, that the latter is based. Hence, too, the middle term, as in the second figure, gives us only a *ratio cognoscendi* for our conclusion, not a *ratio essendi*. When we cite instances of *M* as we do in the third figure, for the purpose of drawing a conclusion about the relation of *S* to *P*, the instances are to us the *means of knowing* that such a relation exists, but they do not furnish us with the reason *why it exists*. When we argue that a cloven-footed animal may ruminate because we see horned animals that have both these attributes, we do not regard the possession of horns as *causing* the conjunction of the other two attributes in the horned animal. No doubt, if we know that " all horned animals ruminate," we may regard horns as a *sign* of rumination in animals possessed of horns ; we may then proceed to reflect that " some cloven-footed animals have horns " ; and because they have this sign of rumination we may conclude by a syllogism *in the first figure* (*Darii*) that " Some cloven-footed animals ruminate ". We *may* thus transpose our reasoning from the third to the first figure ; but it will be noticed that in doing so we really change our mode of reasoning ; we now no longer base our conclusion (about *S* and *P*) on instances (*M*) in which the asserted connexion between *S* and *P* is exemplified, but, rather, we argue that *P* is related to *S* because we find in *S* some characteristic, *M*, which we know to be a sign of the presence of *P*.

In the minor premiss of the third figure, *S* is predicate and is primarily regarded not as a class but as an attribute : indeed, it retains this aspect fundamentally even in the conclusion. Those, however, who regard it as a class in both positions have formulated the following axiom for the third figure :—

" *If anything* [*M*] *which is stated to belong to a certain class* [*S*] *is affirmed to possess, or to be devoid of, certain attributes* [*P*], *then those attributes may be predicated in like manner of some members of that class.*"

The third figure may be regarded as inferring merely the denial of a *necessary* connexion (negative or affirmative) between *S* and *P*. From this point of view all its moods might be summed up in the following scheme :—

" Denial of Result . . . *Some* (or *all*) *M is not P* (or *is P*),
Case . . . . . *All* (or *some*) *M is S*,
Denial of Rule . . . . therefore, *Some S is not P* (or *is P*)."[2]

[1] JOSEPH, *op. cit.*, p. 296 (from the *Anal. Pri.*, a. vi., 28^a, 24-26). *Cf.* KEYNES, *Formal Logic*, p. 323, n. 1.

[2] *Cf.* KEYNES, *op. cit.*, p. 337.

or, expressed hypothetically :—

Certain M's are (or *are not*) *P*,
These same M's are S,
therefore, *Though M is S it may be* (or *need not be*) *P*.[1]

173. The Fourth Figure and the Indirect Moods of the First Figure: Summary of Doctrine on the Figures. —Each of the three figures we have so far examined presents a distinct type of inference. The first figure applies the conditions of a general rule to cases subsumed under it. The second enables us to prove that cases do not fulfil the conditions of a rule, that they differ from those of a contemplated rule: without, however, showing us why they so differ. The third enables us to disprove a necessary rule or principle by an appeal to instances which refute it. In the latter the middle term is twice subject; in the second it is twice predicate; and in the first the major extreme is predicated of the middle term, while this in turn is predicated of the minor extreme. We have next to determine whether those syllogisms in which the minor is predicated of the middle, and this in turn of the major, exemplify any further new type of inference. If they do, there is something to be said for treating them as moods of a new and independent figure, the fourth; but if not, their erection into a new figure, although it may make for mere external symmetry in the treatment of the figures, will be really calculated rather to mislead than to give a true notion syllogistic inference. After a careful examination of its moods, Mr. Joseph concludes[2] that it "is not an independent type; its first three moods are merely moods of the first figure with the conclusion converted, as the process of reducing them assumes; its last two moods draw conclusions which are shown to be valid most naturally by reduction to the third [figure]". And this appears to be about the most accurate view to take of the matter.

We have seen that Aristotle did not recognize the fourth figure. Its first introduction into logic is attributed by Averroës, an Arabian philosopher of the twelfth century, to *Galen*, a Roman physician of the second century. Some logicians have not quite accurately described the relation of the fourth figure to the first by saying that it is merely the first figure with the conclusion converted.[3] It is true that if we regard the first premiss (P—M)

[1] *Cf.* Sigwart, *Logic*, i., pp. 355, 356. [2] *op. cit.*, p. 290.
[3] *Cf.* Bowen, *Logic*, p. 192; Thomson, *Laws of Thought*, p. 178; Clarke, *Logic*, p. 337.

as the minor, and the second (M—S) as the major, in *Camenes* and *Dimaris*, our conclusions, *P e S* and *P i S*, will be the converses of the conclusions of *Celarent* and *Darii;* and that, on the same hypothesis, the conclusion we get from the premisses of *Bramantip, viz. P a S, contains in it* the converse of the conclusion of *Barbara.* But it is not true that from the premisses of *Fesapo* or *Fresison* we can get, by the first figure, a conclusion which will be the converse of that of *Ferio:* we can, in fact, get *no conclusion at all in the first figure about P in terms of S* from the premisses of those two moods of the fourth figure; though the conclusion which we *can* get—about *S* in terms of *P*—was recognized by Aristotle in his treatment of the first figure.

Aristotle had a truer conception of the relation of what Galen afterwards called the fourth figure, to the first. Although he did not recognize the former *in name*, he recognized it implicitly *in fact;* and his immediate successor, Theophrastus, gave its five moods as supplementary, or *Indirect, Moods of the First Figure.*[1] We can best describe what an "indirect mood" of the first figure is by recalling the fact already referred to (148, 159), that it is the conclusion of the syllogism that determines which of the premisses is major and which is minor. If, therefore, we abstract from the conclusion and consider the premisses alone, we shall have *three figures* of syllogism instead of *four, viz.* (1) one in which *M* is subject in one premiss and predicate in the other, (2) one in which *M* is twice predicate, and (3) one in which *M* is twice subject. But in the first of these figures the extremes will either retain the same position in the conclusion as they had in the premisses, or the reverse. In the former case, the moods will be called the *Direct Moods of the First Figure;* in the latter, they will be called *Indirect Moods*, and the conclusions *Indirect Conclusions, of the First Figure.* We may, therefore, define an indirect mood of the first figure as *a mood in which the position of both extremes in the conclusion is the reverse of their position in the premisses.* We know that the direct moods of the first figure are four: that there are four ways of drawing a conclusion about the extreme which is subject in the premisses (*S*), in terms of the extreme which is predicate there (*P*). How many ways are there of drawing a conclusion about the extreme which is predicate in the premisses (*P*), in terms of that which is subject in the premisses (*S*)? We can do so in the case of A A (I), E A (E), and

[1] JOSEPH, *op. cit.*, pp. 258-60.

A I (I)—the first proposition, containing *P*, being now of course *minor* premiss; and the second, containing *S*, being *major* premiss. These three correspond to *Barbara*, *Celarent* and *Darii* respectively. From the premisses of *Ferio* (*M a P*, *S i M*), we can infer nothing about *P*. *Ferio*, therefore, has no indirect mood corresponding to it. On the other hand, the two pairs of premisses, "*M a P*, *S e M*" and "*M i P*, *S e M*," yield the indirect conclusion *P o S*, thus giving us two indirect moods, A E (O) and I E (O), of the first figure, which have no corresponding direct moods in that figure.

These five indirect moods of the first figure, A A I, E A E, A I I, A E O, I E O, correspond exactly to the five moods of the fourth figure—with this quite immaterial difference, that the mnemonics for the indirect moods give the minor premiss (containing *P*) first, and the major (containing *S*) second. The traditional mnemonics for the indirect moods of the first figure are: *Baralipton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesomorum*: corresponding to the later mnemonics for the fourth figure, *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.[1]

We have said that when a conclusion is drawn *indirectly* from premisses in the first figure, the first premiss in order, that containing *P*, is now minor; and the second, containing *S*, is major. But if we named the premisses not according to the position which the extremes *actually* occupy, but that which they *ought naturally* to occupy, in the conclusion (161), each premiss would retain the same name whether the conclusion be direct or indirect. The first figure (including direct and indirect moods) would then be the figure in which the major extreme is predicated of the middle term, and this in turn of the minor extreme, in the premisses, and in which the major is *directly* predicated of the minor, or the minor *indirectly* of the major, in the conclusion.

In the second and third figures, the position of the middle term does not reveal which extreme is major and which minor. Wherever the premisses

[1] One of the earliest forms of the mnemonic lines in Latin is that given by Petrus Hispanus, afterwards Pope John XXI. (d. 1277), in his *Summulae Logicales*, a work widely known in the mediaeval schools. They are also found in an unpublished work of William of Shyreswood, who died as Chancellor of Lincoln in 1249. They are as follows:—

Barbara, Celarent, Darii, Ferio, Baralipet,
Celantes, Dabitis, Fapesmo, Frisesmo, *Deinde*
Cesare, Camestres, Festino, Baroco, Darapti,
Felapton, Disamis, Datisi, Bocardo, Ferison.

Cf. JOYCE, *Principle of Logic*, p. 192; KEYNES, *op. cit.*, p. 329: who gives the more usual forms, *Baralipton*, *Frisesomorum*. There are many variants of the mnemonics to be found in mediaeval Latin treatises on logic. The variety is greatest in the case of the fourth figure, owing to the comparatively later date of its recognition. See Mansel's *Aldrich*, pp. 88, 89; KEYNES, *op. cit.*, p. 322, n.

permit either of these latter terms to be predicated of the other, a separate mood is explicitly recognized for each alternative : *Cesare* and *Camestres* in the second figure ; *Disamis* and *Datisi* in the third. *Darapti* combines in itself both alternatives : since the premisses agree both in quality and in quantity, their transposition does not give even the external semblance of a new mood. It is true that "in many and probably in most cases of syllogism in these figures it would be difficult to say which of the two terms was naturally major and which naturally minor, for they are not generally terms belonging to one series in a classification." [1] Whenever the extremes are so related that one is naturally minor and the other naturally major, and, nevertheless, it is the former that is predicated of the latter in the conclusion the mood may be regarded as an indirect mood. For example, this syllogism in *Festino* might be regarded as indirect ; "*No clergymen are members of Parliament ; Some professional men are members of Parliament ; therefore, Some professional men are not clergymen*".

The conclusions of the fourth figure, or of the indirect moods of the first, are indeed valid ; but their connexion with their premisses, or, in other words, the cogency of the sequence, is not easily apparent. By what sort of reflection or consideration do we persuade ourselves of its validity ? From the premisses (of *Bramantip* or *Baralipton*) : *All nitrogenous foods are flesh-forming ;* and *All grains are nitrogenous :* how do we justify the inference that *Some flesh-forming foods are grains ?* Apparently, by reflecting "either that from the given premisses it follows that *All grains are flesh-forming*, and our other conclusion follows by conversion from that : or else that if no flesh-forming foods were grains, no nitrogenous foods would be grains ; and that in that case grains could not all, or any, of them be nitrogenous" [2]—*i.e.* either by the typical reasoning of the first figure, or by the *reductio ad impossibile* which is characteristic of the second. The same is true of *Camenes* [*Celantes*] and *Dimaris* [*Dabitis*]. By the reduction of these three moods to the first figure, we "recognize in outward form as major and minor terms what we must acknowledge to be so in our thought".[3] Since, therefore, the simplest way of realizing their validity is by reasoning in the corresponding direct moods of the first figure and converting the conclusion, there is no ground for erecting those moods into an independent figure.

But *Fesapo* (or *Fapesmo*) and *Fresison* (*Frisesomorum*) have no corresponding direct moods of the first figure, by arguing in which we might see the validity of the former. Aristotle exhibited their validity by converting both premisses, and so reducing these moods to *Ferio*. This process does indeed reveal the natural movement of our thought whenever the premisses in these moods are *merely less natural expressions of those we obtain from them by conversion.* "*What tarnishes is not gold ;* and *Some things of gold are ancient ornaments ;* therefore *Some ancient ornaments do not tarnish* " : in this syllogism (in *Fresison*, or *Frisesomorum*) we are really arguing in the *first* figure, in *Ferio :* we validate our inference by reverting in thought to the *more natural* premisses : "*Gold does not tarnish ;* and *Some ancient ornaments are of gold*". But such cases are rare. More frequently, the reduction of *Fresison* (or *Frisesomorum*) to *Ferio* gives *unnatural* premisses. And the reduction of *Fesapo* (*Fapesmo*) to *Ferio* always

[1] JOSEPH, *op. cit.*, p. 259. [2] *ibid.*, p. 301. [3] *ibid.*, p. 304.

does so, for the *A* proposition cannot be regarded as the expression of a proposition which would more naturally be *I*. The really important syllogisms in the two moods in question are not those like the above "where the premisses are palpably in an unnatural form," but "those in which the position of the middle term, as predicate of the major premiss and subject of the minor, is the natural position". And "here conversion to the first figure produces a result as unnatural as there conversion to the fourth figure produced in the premisses of an argument naturally belonging to the first".[1] An example of such a syllogism (*Fesapo, or Fapesmo*) would be "*No mineral waters are alcoholic*, and *All alcohol is taxed*, therefore, *Some things taxed are not mineral waters*". Is there, then, any more natural way of showing the validity of *Fesapo* or *Fresison* than by converting both premisses, and arguing in *Ferio?* Two other ways at least are possible. We may show, as in the *second* figure, that the falsity of the conclusion would be incompatible with what is granted in the premisses, by the *reductio ad impossibile*, thus: "*If all things taxed were minerals, alcohol would be a mineral; but it cannot be, for no mineral is alcoholic*". Or, we may convert the first premiss to "*No alcohol is a mineral water*," and proceed by ἔκθεσις or exposition, as in the *third* figure, to point to an instance of alcohol as a proof that *a thing may be taxed and yet not be a mineral.* Which of these two processes reveals the line of thought really followed in *Fesapo* and *Fresison?* The cogency of the inference is so concealed in the actual form of the premisses that it is not easy to detect or lay bare the real line of thought. But it will help us if we ask: How could we *most naturally* prove this conclusion, that "*Some things taxed are not mineral waters*" by using the same middle term (*alcohol*)? Obviously, by asking ourselves: Is alcohol taxed? and is it a mineral water? In other words, we validate an argument in *Fesapo* or *Fresison most naturally* by having recourse to exposition—by reducing it to the *third* figure,—rather than by the *reductio ad impossibile*, or by reducing it to *Ferio* in the first.

We cannot better conclude the foregoing analysis, which is based on that of Mr. Joseph, than by quoting the latter's excellent summary of the results of his inquiry:[2] "There are three figures, each with a distinctive character, and the 'imperfect' figures are misrepresented by reduction to the first. The first is the chief, because the demonstrative, but not because the only figure. Arguments in it need not be demonstrative, but when they are, our thought is moving on a higher level of intelligence, though not of cogency, than in the other figures. In realizing the validity of the second figure, the inconsistency involved in denying the conclusion is a more prominent 'moment' in our thought than the necessity of admitting it. The third figure appeals not to relations of concepts, but to experience of the conjunction of attributes (or their disjunction) in the same subject, and from that argues the general possibility, under conditions unspecified, of what is exhibited in a given case. There is no fourth figure; but in the first three moods of the first figure we may also argue to the converse of their conclusions; and two moods may be added, with an universal negative minor premiss, in which, while the major term cannot be denied of the minor without fallacy, the minor can be denied of the major; though such a conclusion is only particular, and realized by the help of exposi-

[1] JOSEPH, *op. cit.*, p. 305. [2] *op. cit.*, pp. 306-7.

tion or of conversion or reduction *ad impossibile*. It must always be remembered that the character of an argument is determined not by the form into which it is thrown in words, but by that which it assumes in our thought. This is our justification for recognizing the figures as distinct types. In particular cases, a syllogism may not belong to the figure into which it has been verbally compelled ; in others it may be possible with the same terms to construct syllogisms in more than one figure ; but then there must be a real movement of thought in the process of conversion by which the change is effected. The theory of syllogisms ought not to be regarded as a lesson in the manipulation of symbols and the application of formulæ. What we have to look to is the character of the reasoning involved in it, and to that end we need to *realize* our symbols and see how the varying character of our terms, and of the relations between them in judgment, affects the inference. If our inquiry has done anything to bring this lesson home, its length and intricacy will not have been altogether vain."

WELTON, *Logic*, bk. iv., chaps. iii. and iv. KEYNES, *Formal Logic*, pt. iii., chap. iii. JOSEPH, *Logic*, chaps. xiii. and xiv. JOYCE, *Logic*, pp. 181 *sqq*. MELLONE, *op. cit.*, pp. 179 *sqq*.

CHAPTER V.

HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS.

174. THE PURE HYPOTHETICAL SYLLOGISM.—In classifying syllogisms in a previous chapter (149), we remarked that the rules of the pure categorical syllogism will be found to apply to the forms of reasoning called "pure hypothetical" and "pure disjunctive" syllogisms. The *pure hypothetical* (or *pure conditional*) *syllogism* is one *in which a hypothetical* (*or conditional*) *conclusion is inferred from two hypothetical* (*or conditional*) *premisses*. Here, instead of terms—subject and predicate—we have simple propositions—antecedent and consequent. These constituent factors must be three in number: the antecedent and consequent of the conclusion, and the factor which serves as "middle term". The typical form of such a syllogism, answering to *Barbara* in the first figure, is the following:—

If B then C;
If A then B;
∴ *If A then C;*

where *A*, *B*, and *C* stand for simple propositions, and where the "middle term," *viz.* the proposition *B*, gives the *reason* why the "minor term," *viz.* the proposition *A*, is asserted in the conclusion as the ground for asserting the "major term," *viz.* the proposition *C*. The reason why *A* is the ground of *C* is found in the comparison of both with *B*.

The general rules and corollaries which regulate the categorical syllogism are applicable here. This is most clearly seen when the premisses and conclusion are expressed in the *quantified* or *denotative*, rather than in the *abstract* or *modal*, form. The words "always," "never," "sometimes," "sometimes not," determine the quantity *of the antecedent* of the "*If*" proposition—just as the words "all," "none," "some," "some not," determine the quantity *of the subject* of the *categorical* proposition. The quantity of the *consequent* in each "*If*" proposition depends upon the

quality of the "*If*" proposition—just as in the case of categoricals the quantity of the predicate depends on the quality of the proposition. Thus, the consequent of the proposition "*If any S is M that S is always P*" is *undistributed*, because the proposition, being affirmative, refers indefinitely to "some" cases of *S* being *P*, and does not by any means tell us that the cases of *S* being *M* are *all* the cases of *S* being *P*: *S* may be *P* on other grounds and for other reasons also. Whereas, in the negative proposition "*Sometimes if an S is M that S is not P*," the consequent—*viz.* *S* being *P*[1]—is *distributed*; for, the *some* instances referred to of *S* being *M* are not *any* instances of *S* being *P*; they are excluded from *the whole class* of cases in which *S* is *P*.

Both premisses and conclusion may, however, be expressed in the modal, as well as in the quantified form: "*If B is true C is true; If A is true B is true;* therefore, *If A is true C is true*"; or "*If any S is M that S is always P; If any S is R that S is always M;* therefore, *If any S is R that S is always P*". But even though they be expressed in the modal or abstract form, the syllogistic rules of *quantity*, as well as of quality, apply to them: the consequent of the affirmative hypothetical is undistributed. Hence in these syllogisms we may have fallacies analogous to *undistributed middle* and *illicit process*. "*If Q then R; if P then not Q;* therefore, *if P then not R*"—is an example of *illicit major*.

The two syllogisms given above are examples of the mood *Barbara* of the first figure. Professor Welton gives the following material example:[2] "If any person is selfish, he is unhappy; if any child is spoilt, that child is selfish; therefore if any child is spoilt, he is unhappy". By recognizing the ordinary distinctions of quality, and of quantity [or modality (139)], in "*If*" propositions, we may express pure hypothetical reasonings in the same number of valid moods as we have for categorical syllogisms. Since, however, "*If*" propositions which are *particular* or *problematic* give us comparatively little information, it is only the moods whose premisses and conclusions are universal that are of any importance.

An example of *Cesare* in the second figure would be: "If any act is done from a sense of duty, it is never formally wrong; if

[1] In this rule—that the consequent of a negative hypothetical is distributed—the consequent itself must be taken *affirmatively*; the negative copula being understood to determine the *relation of consequent to antecedent*.

[2] *Logic*, I, p. 349.

any act is done from purely selfish motives, it is always formally wrong; therefore, if any act is done from purely selfish motives, it is not done from a sense of duty."[1]

The following example of *Bocardo* in the third figure, taken from the same context, illustrates the function of the problematic or particular hypothetical as *denying that something is necessarily connected with something else:* "If [*though*] a war is [*be*] just it is sometimes not [or *need not be*] successful; if any war is just it is always waged in defence of some right; therefore, if a war is [*though a war be*] waged in defence of some right, it is sometimes not [or *need not necessarily be*] successful".[2]

All the other moods and figures of the pure hypothetical syllogism may be reduced to the moods of the first figure by applying the ordinary rules for reduction. *Baroco* and *Bocardo* may be reduced *directly* to *Ferio* and *Darii* respectively, by following the rules of the mnemonics *Faksoko* and *Doksamosk.* Thus, the example of *Bocardo* given above may be reduced to *Darii* by contraposing (*ks*) the original major, transposing (*m*) the premisses, and finding the obverted converse (*sk*) of the new conclusion.

If any war is just it is always waged in defence of some right;

Though a war be unsuccessful it is sometimes just;

Therefore, *Though a war be unsuccessful it is sometimes waged in defence of some right;*

Therefore, *Though a war be waged in defence of some right it is sometimes unsuccessful;*

Therefore (original conclusion), *Though a war be waged in defence of some right it is sometimes not successful.*

175. THE MIXED HYPOTHETICAL SYLLOGISM.—The *mixed hypothetical*, or *hypothetico-categorical, syllogism, is a syllogism in which the major premiss is a hypothetical or a conditional proposition, the minor premiss categorically posits the antecedent or sublates the consequent of the major, and the conclusion accordingly posits the consequent or sublates the antecedent of the major.* There is no need to distinguish here between the *pure or abstract hypothetical*, and the *concrete conditional*, premiss (133); for the basis of the reasoning involved is the same in both cases. By stating the major premiss in the *conditional* or *denotative* form, the syllogistic character of the inference—as being the application, to

[1] *ibid.* [2] *ibid.*

a particular case, of a general rule—is made more explicit ; but, on the other hand, it is the abstract, hypothetical form that emphasizes the *necessary character* of the universal premiss involved : and it is on the necessary truth of this universal premiss that the cogency of the syllogism depends. The major premiss—*If S is M it is P*—lays down the general principle, the abstract, necessary law ; the minor—*This, these, some, all S's are M*—brings a case or class of cases under it ; the conclusion explicitly applies the law to those cases—*This, these, some, all S's are P.*

176. "MOODS" OF THE MIXED HYPOTHETICAL SYLLOGISM. —The major of the mixed hypothetical syllogism states that the antecedent is a "sufficient reason" for the consequent : that it expresses a truth with which the falsity of the consequent is incompatible (138). Therefore the assertion, in the minor premiss, of the truth of the antecedent, warrants the assertion, in the conclusion, of the truth of the consequent ; and, *vice versa*, the assertion, in the minor premiss, of the falsity of the consequent, warrants the assertion, in the conclusion, of the falsity of the antecedent (140). We have thus two "moods" of the mixed hypothetical syllogism : one, called the *Modus Ponens*—or *constructive* syllogism—in which the minor premiss *posits*, or asserts the truth of, the antecedent of the major premiss ; the other, called the *Modus Tollens*—or *destructive* syllogism—in which the minor premiss *sublates*, or asserts the falsity of, the consequent of the major premiss.

Of each of these moods there are four possible forms, according as the antecedent and the consequent of the major premiss are both affirmative, both negative, or one affirmative and the other negative. Thus, we have in the *Modus Ponens* :—

(1) The *Modus Ponendo Ponens : If A then C ; But A ; Therefore C ;*

(2) The *Modus Ponendo Tollens : If A then not C ; But A ; Therefore not C ;*

(3) The *Modus Tollendo Ponens : If not A then C ; But not A ; Therefore C ;*

(4) The *Modus Tollendo Tollens : If not A then not C ; But not A ; Therefore not C.*

Similarly, in the *Modus Tollens* we have :—

(1) The *Modus Tollendo Tollens : If A then C ; But not C ; Therefore not A ;*

(2) The *Modus Ponendo Tollens*: *If A then not C; But C; Therefore not A;*

(3) The *Modus Tollendo Ponens*: *If not A then C; But not C; Therefore A;*

(4) The *Modus Ponendo Ponens*: *If not A then not C; But C; Therefore A.*

It will be noticed that each subordinate form of the *Modus Ponens* is practically identical with a corresponding subordinate form of the *Modus Tollens*, and *vice versa*. Each can be got from the other by contraposing or converting the major (140). Thus, contraposing the major of the first form of the *Modus Ponens* we obtain "*If not C then not A; But A; Therefore C*"—which is the same as the fourth form of the *Modus Tollens*, except that the antecedent and consequent are transposed. Similarly, by converting the major of the second form of the *Modus Tollens* we obtain "*If C then not A; But C; Therefore not A*"—which is the second form of the *Modus Ponens*, with a similar transposition of antecedent and consequent.

In reasonings in the *Modus Tollens* care must be taken to infer *only the contradictory*, not the *contrary*, of the original antecedent. Whether in the minor we sublate the consequent by asserting its *contradictory*, or its *contrary*, we are warranted in inferring from this merely that the antecedent is *not true; i.e.* in inferring its *contradictory*. For instance, from the premisses: "*If there were no insane people, all lunatic asylums would be superfluous; but no lunatic asylums are superfluous*": we cannot infer that therefore "*all are insane*," (!) but only that therefore "*some are insane*".

177. Fallacies in the Mixed Hypothetical Syllogism. —We have already seen (138) why we cannot pass from the affirmation of the consequent to the affirmation of the antecedent, or from the denial of the antecedent to the denial of the consequent, of the hypothetical proposition; why, in other words, we can infer nothing from the premisses: "*If A then C; but C*"; or from the premisses: "*If A then C; but not A*". It may be well to recall the reason here. It is because the form of the hypothetical proposition does not guarantee, or imply in any way, that the antecedent (*A*) is the *indispensable* or *only possible* ground for the consequent (*C*), or that the latter could not be verified on other grounds and in other circumstances: it merely states that *A* is *a sufficient reason* for *C*, that *A* implies *C*, that wherever *A* is, *C* is,

but not *vice versa*. In other words, the hypothetical proposition is not *reciprocal* in virtue of its form. Of course, if we know from our acquaintance with the particular subject-matter of a given hypothetical proposition that it is *de facto* reciprocal, that *A* gives not merely *a sufficient*, but *the indispensable*, or *only possible*, reason for *C*, and that, therefore, *C* involves *A* just as *A* involves *C*, then we can infer from the presence or truth of *C* to the presence or truth of *A*, and from the absence or falsity of *A* to the absence or falsity of *C*: but our inference here is *material*, not *formal*.

The formal fallacy of inferring the falsity of *C* from the falsity of *A* is analogous to *illicit major* in the categorical syllogism; that of inferring the truth of *A* from the truth of *C* is analogous to *undistributed middle* in the categorical syllogism. This will be evident if we reduce the mixed hypothetical syllogism to the categorical (182).

178. THE REASONING IN THE MIXED HYPOTHETICAL SYLLOGISM IS MEDIATE REASONING.—Some logicians have, for various reasons, held that the reasoning in mixed hypothetical syllogisms is *immediate*, not mediate.[1] Of course, there is no middle *term* proper: it is not a term but a *proposition* which serves as a "middle" or "mediate" element, *by means of which we reach the conclusion*, and which *is itself dropped out* in the process. There is, therefore, in the process, a real link or bond which enables us to connect two elements of thought, and which serves that purpose alone. This we regard as sufficient and essential to constitute an inference "mediate," as distinct from "immediate".

Moreover, in the mixed hypothetical syllogism we infer from *two* propositions, neither of which is inferable from the other, and both of which are necessary to reach the conclusion. From the proposition "*If A is B, C is D*" we can infer *immediately* "*If C is not D, A is not B*"; but we cannot infer categorically that *A is not B*" unless we are also given, as in the mixed hypothetical syllogism, the premiss that "*C is not D*".

Furthermore, the mixed hypothetical is reducible to the categorical syllogism (182); and the latter is, admittedly, a *mediate* inference.

There seems, therefore, to be no valid reason why we should not regard the mixed hypothetical syllogism as a process of *mediate* inference. Of course, whenever the conjunction "*If*" is

[1] KEYNES, *op. cit.*, pp. 354, 357; JOSEPH, *op. cit.*, p. 316, n.

shown by the context to be equivalent to "*Since*" (134), then the hypothetical proposition—*If* (*Since*) *A is B, C is D*—is really the condensed expression of a mixed hypothetical syllogism, omitting the major premiss and combining in one single statement the minor premiss and the conclusion.

179. THE PURE DISJUNCTIVE (ALTERNATIVE) SYLLOGISM.—The form of syllogism in which *both* premisses are disjunctives—as distinct from the *mixed* disjunctive syllogism (180)—is so rare, if, indeed, it occurs at all, as scarcely to call for mention. It would appear to be at all events theoretically possible: since pure hypothetical syllogisms are possible, and their constituent propositions may be expressed as disjunctives. We know that the proposition, *If S is not P it is Q* (with its contrapositive, *If S is not Q it is P*) is the hypothetical expression of the disjunctive: *S is either P or Q* (146). Similarly, the hypothetical *If S is P it is Q* (with its contrapositive, *If S is not Q it is not P*), expresses the disjunctive, *S is either* $\overline{P}$ *or Q*.

But, by confining ourselves to alternative premisses we confine ourselves to *affirmative* premisses: for all alternatives are affirmatives. Hence, the syllogistic rules of *quality* have no application here. Professor Welton further states[1] that "we only secure a middle term when one of the alternatives in the minor premise negatives one of those in the major premise. From

S is either P or Q
S is either P or R

no conclusion can be drawn, except that *S is either P or Q or R* which simply sums up the premises. But from

S is either P or Q,
S is either $\overline{P}$ *or R,*

we can draw the conclusion *S is either Q or R*. This will perhaps be more clearly seen if each premise is expressed as a hypothetical proposition. We can write the premises in the form

If S is $\overline{P}$ *it is Q,*
If S is $\overline{R}$ *it is* $\overline{P}$*,*

whence it follows that *If S is* $\overline{R}$ *it is Q*, which expresses the disjunctive *S is either Q or R*."

Indeed it may be doubted whether we ever draw a conclusion from two such disjunctive premisses without thus mentally changing them into hypotheticals.

[1] *op. cit.*, p. 350.

180. THE MIXED DISJUNCTIVE (OR MIXED ALTERNATIVE) SYLLOGISM.—This form of syllogism is one *in which an inference is drawn from the alternative character of the major premiss by means of a categorical minor which denies one* (or *some*, but not *all*) *of the alternatives.* The disjunctive major—*Either X is true or Y is true ; S is either P or Q ; Either A is B, or C is D, or E is F*—states that either of a number of alternatives is true [and that possibly all are true (145)]. The negative categorical minor sublates or denies one or more—not all—of the alternatives given in the major : thus giving us the right to affirm the remaining alternatives in the conclusion—categorically if one only be left, alternately if more than one be left. For, given the alternative judgment "*Either X is true or Y is true*" we know that "*If X is not true, Y is true*," and that "*If Y is not true, X is true*" (146). Consequently, if in addition to the major we are also given that "*X is not true*," we can infer categorically that "*Y is true*" ; or if with the major we are also given that "*Y is not true*," we can infer that "*X is true*".

When the alternatives are negative—*either not-X or not-Y*—they are more usually expressed in the strictly disjunctive form —*Not both X and Y* (141). Hence, if either of the negative alternatives is asserted to be false,—*e.g.* if we sublate *not-X*, which by the *principle of the excluded middle* posits *X*—the other negative alternative, *not-Y*, is inferred to be true, *i.e. not-Y* is posited ; or, which is the same, *Y* is sublated. Hence, from the premisses "*No man can serve God and mammon ; Saints serve God*" we can infer that "*Saints do not serve mammon*". But, if for minor we took the proposition "*Spendthrifts do not serve God*," we could infer nothing as to whether they served mammon or not : for the original proposition states that a man must either not serve God or not serve mammon *or that he may possibly not serve either.* "*Either not-X or not-Y*" has been interpreted to mean "*Either not-X or not-Y, or possibly both not-X and not-Y*" (145).

Not every syllogism which contains a disjunctive (alternative) premiss is a mixed disjunctive syllogism. We have the latter only when the inference *is based on*, and *proceeds from*, the *disjunction or alternation* in the major. Such syllogisms, therefore, as "*M is either P or Q ; S is M ; therefore, S is either P or Q*" are *categorical* syllogisms in which one of the extremes happens to be an alternative term. They may occur in any figure.

181. VALID AND INVALID "MOODS" OF THE MIXED DIS-

JUNCTIVE SYLLOGISM.—The mood in which, by denying portion of the alternatives, we posit the remainder, is called the *Modus Tollendo Ponens.* From the nature of the alternative proposition it is evident that this mood is always valid. It will be noted that we need only *contradict* one alternative in order to posit the remainder: the *contrary* of any alternative is not only sufficient but superfluous. For example, from the premisses "*Either all A's are B or all C's are D; but some A's are not B*" we can infer the same conclusion as if the minor were "*No A's are B*," *viz.* the conclusion that "*Therefore all C's are D*". As in the case of mixed hypotheticals, we may distinguish four forms of the present mood according to the affirmative or negative character (*a*) of the alternative sublated in the minor, and (*b*) of the alternative posited in the conclusion. They are:—

(1) *Either X or Y, but not X, therefore Y;*
(2) *Either X or not Y, but not X, therefore not Y;*
(3) *Either not X or Y, but X, therefore Y;*
(4) *Either not X or not Y, but X, therefore not Y.*

These four forms will be seen to be equivalent to the four corresponding forms of the *Modus Ponens* of the mixed hypothetical syllogism (176) when the denial of the first alternative is taken as the antecedent of the hypothetical premiss, and to the four corresponding forms of the *Modus Tollens* when the denial of the second alternative is so taken. Thus, the first form "*Either X or Y, but not X, therefore Y*" will yield (1) "*If not X then Y, but not X, therefore Y*," which is the *Modus Tollendo Ponens* of the *Modus Ponens;* and (2) "*If not Y then X, but not X, therefore Y*," which is the *Modus Tollendo Ponens* of the *Modus Tollens.*

The only other possible "mood" of the mixed disjunctive syllogism is that in which, by positing in the minor a portion of the alternative major, we sublate the remainder of the latter in the conclusion. This is called the *Modus Ponendo Tollens;* and it may be expressed thus: "*Either X or Y, but X, therefore not Y*" This mood is *formally invalid*, because the correct formal interpretation of the alternative proposition is the *non-exclusive* interpretation: "*Either X or Y or possibly both*": which does not give us any right to sublate either alternative by positing the other (145). The fallacy corresponds to that referred to in the case of mixed hypotheticals (177).

Of course, if we know from the subject-matter in question

that X and Y are mutually exclusive, we are justified in inferring the denial of either alternative from the affirmation of the other. For instance, the inference "*He was either first or second in the race; but he was first; therefore he was not second*" is obviously valid; its validity, however, depends not on the alternation as such, but on the mutually exclusive character of the alternatives. If the major premiss were stated in the *strict disjunctive* form "*He could not be both first and second in the race*" (180), the reasoning might be called—and has been called[1]—the *Modus Ponendo Tollens* of the mixed "disjunctive" syllogism, taking "disjunctive" in its strict, etymological sense.

182. REDUCTION OF HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS TO CATEGORICAL FORM.—We have already seen that sometimes one and the same judgment may be expressed equally well either in the *categorical* or in the *conditional* form (135), and that the general principle under which the narrower case is subsumed in the first figure of syllogism may be expressed in either of these forms (170), thus showing the fundamental identity of the categorical, with the mixed hypothetical, syllogism. The two identical syllogisms would be:—

$$\left.\begin{array}{c}\textit{Whatever is } M \textit{ is } P;\\ S \textit{ is } M;\\ \text{Therefore, } S \textit{ is } P;\end{array}\right\} \text{ and } \left\{\begin{array}{c}\textit{If anything is } M \textit{ it is } P;\\ S \textit{ is } M;\\ \text{Therefore, } S \textit{ is } P.\end{array}\right.$$

The first of these is a categorical, the second a mixed hypothetical, syllogism.

Though we may hold that the pure hypothetical *proposition* cannot be reduced to the categorical, owing to an existential element implied in the latter and left doubtful in the former (134), we have to remember that *this element of doubt is removed by the categorical minor* in the case of the mixed hypothetical *syllogism*: so that there is no syllogism in the latter form which may not be reduced to the categorical. Such reduction is always possible, although it may often involve considerable circumlocution and verbal change. For instance, the syllogism:

If A is B, C is D;
But A is B;
therefore, *C is D;*

may be stated thus:—

All cases of A being B are cases of C being D.

[1] *Cf.* KEYNES, *op. cit.*, p. 362.

This is a case of A being B.
Therefore, this is a case of C being D.

Just as the *Modus Ponens* can thus be reduced to the first figure, so can the *Modus Tollens* be reduced to the second figure, of the categorical syllogism.

This process can be similarly applied to the *pure* hypothetical syllogism. Disjunctive syllogisms we have already seen to be reducible to hypothetical; and, through these latter, they may be reduced to categoricals. In all those cases, however, reduction only serves the general purpose of showing a generic kinship between naturally and specifically distinct forms of inference: it cannot totally eliminate the difference there is between these.

We have already alluded to Mr. Joseph's definition of the syllogism as excluding hypothetical arguments (148 n., 149). He also objects to the reduction of the latter to categorical (or syllogistic) form, to the use of the expression "middle term" in reference to an antecedent or consequent, and to the assimilation of the fallacies committed in hypothetical reasonings to "illicit process" or "undistributed middle".[1] The question is largely, though not entirely, one of nomenclature. It cannot be denied that many reasonings which fall naturally into the hypothetical form require "linguistic *tours de force*"[2] to reduce them to categorical syllogisms.

The *real* inherence of an attribute in a substance apparently underlies the *logical* relation of predicate to subject in the categorical judgment; and that inherence is different from the real production of an effect by a cause: this *real* dependence of effect on cause being apparently the main foundation for our conception of the *logical* dependence of consequent on antecedent in the hypothetical judgment. In neither case, however, is the parallelism between the real and the logical order complete: what is really the *effect*, for example, often stands logically as *antecedent* in the judgment: the order in which things *are known* often differs from the order in which they *are* or *happen*. Similarly, the logical subject of a categorical judgment is not always a substance of which the predicate is an attribute or quality. Mr. Joseph emphasizes the fact that the relation of logical dependence (of consequent on antecedent), expressed in the hypothetical proposition, is different from that of predicate to subject, expressed in the categorical. This, no doubt, is true; but, withal, it is not impossible to conceive the *production* of an effect, or the *dependence* of a consequent, as an *attribute* of a cause, or of an antecedent. If categorical and hypothetical judgments may be regarded as distinct modalities or sub-forms of the same *generic* form of thought, the judgment: it is permissible to regard hypothetical arguments as kinds or forms of syllogism. Though Aristotle did not call the arguments considered in the present chapter "hypothetical syllogisms,"[3] yet this nomenclature has become practically universal. That, however, does not alter the fact that the resemblance between them and categorical syllogisms is one which conceals important differences.

[1] *op. cit.*, pp. 312 *sqq.* [2] *ibid.*, p. 313. [3] *ibid.*, p. 316, n.

183. THE DILEMMA: ITS VARIOUS FORMS.—Logicians have designated by this title types of arguments more or less closely resembling one another, but not quite identical. This has led to a great variety of definitions. We shall select for special treatment the form of argument which seems the most important and most deserving of the title; referring afterwards to some other lines of treatment.

By the *Dilemma* (149) we mean *a syllogism in which one of the premisses is a conjunctive combination of two or more hypothetical propositions, whose antecedents are alternatively affirmed, or consequents alternatively denied, in the other premiss, which is a disjunctive or alternative proposition.*

The hypothetical premiss is conjunctive or copulative, *i.e.* it must contain at least two antecedents, or at least two consequents, so as to yield two distinct hypothetical propositions united conjunctively together. This premiss is usually referred to as the *major*. The disjunctive or alternative premiss—which is usually called the *minor*, though it is usually stated in the first place—either alternatively affirms the antecedents, or alternatively denies the consequents, of the major: thus giving us a right to affirm the consequent (categorically) or consequents (alternatively), or to deny the antecedent (categorically) or antecedents (alternatively) of the major, in the conclusion.

Strictly speaking, where there are *three* distinct hypotheticals and alternants, the argument should be called a *trilemma;* when there are *four* or *more*, a *tetralemma* or *polylemma;* but the name *dilemma* is used generically for all.

The dilemma differs from the mixed hypothetical syllogism only in having a *combination* of hypothetical propositions instead of a *single* one, for major premiss, and an *alternative* instead of a *categorical* minor. In other respects it resembles the mixed hypothetical syllogism. Like the latter it has two moods: it is *constructive* or *destructive* according as the minor posits the antecedents, or sublates the consequents, of the major.

In the *constructive* form the major must have (at least) two separate antecedents; and these may have the same consequent (which may be either categorical or disjunctive), or they may have different consequents. If they have the same consequent they form a *simple constructive* dilemma; if they have different consequents they form a *complex constructive* dilemma. Similarly, in the *destructive* form the major must have (at least) two separate

consequents; and these may follow either from the same antecedent (which may be either a simple categorical, a copulative, or an alternative proposition), or from different antecedents. If they follow from the same antecedent we have a *simple destructive* dilemma; if they follow from different antecedents we have a *complex destructive* dilemma.

We have thus four main forms, which will be better understood from the following symbolic illustrations. They are:—

(1) The *simple constructive*—
If A is B, E is F; and if C is D, E is F;
but, *either A is B, or C is D;*
therefore, *E is F.*[1]

(2) The *complex constructive*—
If A is B, E is F; and if C is D, G is H;
but, *either A is B, or C is D;*
therefore, *Either E is F or G is H.*

(3) The *simple destructive*—
If A is B, C is D; and if A is B, E is F;
but, *either C is not D, or E is not F;*
[or, *not both C is D, and E is F*];
therefore *A is not B.*[2]

[1] The "*simple constructive*" may yield an *alternative* conclusion, if *the single consequent* of the major be an *alternative*, *e.g.*:—
If A is B, either E is F or G is H; and if C is D, either E is F or G is H;
but, *either A is B, or C is D;*
therefore, *either E is F, or G is H.*
Compare the analogous syllogism given above (180): "*M is either P or Q; S is M;* therefore, *S is either P or Q*": which is *categorical* even though it has a *disjunctive* conclusion.

So, too, by prefixing a condition "*If X is Y*," for example, to the alternative premise "*either A is B or C is D*," we should get for conclusion "*if X is Y, then either E is F or G is H*". This condition makes the whole dilemma hypothetical, but does not change in any way the character of the reasoning: it is a mere accidental variation.

[2] The same conclusion may be reached by substituting for the denial contained in the alternative or disjunctive minor, the more complete denial contained in the *remotive* minor "*neither C is D or E is F*". This would give us the two distinct mixed hypothetical syllogisms:—

(*a*) *If A is B, C is D;*
but *C is not D;*
therefore, *A is not B.*

(*b*) *If A is B, E is F;*
but *E is not F;*
therefore, *A is not B.*

But, though these reach the same conclusion as the dilemma, the reasoning in them must not be confounded with the peculiar alternative character of dilemmatic reasoning.

The following are simple, not complex, destructive dilemmas (*cf.* preceding note):—

(4) The *complex destructive*—

If A is B, E is F; and if C is D, G is H;
but, *either E is not F, or G is not H* [or, *not both E is F and G is H*];
therefore, *either A is not B, or C is not D* [or, *not both A is B and C is D*].

It will be noticed that in the *simple destructive* dilemma *both* consequents must follow (conjointly) from the antecedent of the major: it would not suffice that *either* follow (alternatively): from the major "*If A is B, either C is D or E is F*," we can derive nothing by means of the alternative minor "*Either C is not D or E is not F*"; for *the latter does not deny or sublate the consequent of the hypothetical*, "*Either C is D or E is F*": the two propositions are compatible. To get a conclusion from the given major, we should need as minor the *remotive* proposition "*Neither C is D nor E is F*: which would give us not a dilemma, but a mixed hypothetical syllogism in the *Modus Tollens*.[1]

Similarly in regard to the *complex destructive* dilemma, we could not have as major the proposition "*If W then Y or Z; and if X then Y or Z*" instead of "*If W then Y; and if X then Z*"—for, in order to get a conclusion from the former major by the *destructive* way of sublating consequents we should need as minor "*Neither Y nor Z*"; which would give us not a dilemma but a compound or double mixed hypothetical syllogism in the *Modus Tollens* with conclusion "*Neither W nor X*".

The following simple examples will help to familiarize the learner with those forms of argument:—

(1) The *simple constructive*: "If I tell the truth or if I tell a lie, I shall get into trouble; I must either tell the truth or tell a lie; therefore, I must get into trouble".

(2) The *complex constructive*: "If Aeschines joined in the public rejoicings, he is inconsistent; if he did not, he is un-

(*a*) *If both A is B and C is D, then E is F; and in the same hypothesis G is H;*
but, *either C is not D or G is not H* [or, *not both C is D and G is H*];
therefore, *Either A is not B or C is not D* [or, *not both A is B and C is D*].

(*b*) *If either A is B or C is D, then E is F; and in the same hypothesis G is H;*
but *either C is not D or G is not H* [or, *not both C is D and G is H*];
therefore, *Neither A is B nor C is D.*

[1] Mr. Joseph recognizes this form of argument as a simple destructive dilemma, citing as an example one of Zeno's arguments against the reality of motion (*op. cit.*, p. 332):—

"If a body moves it must either move in the place where it is, or in the place where it is not;

"But it can neither move in the place where it is nor in the place where it is not;

"Therefore, it cannot move."

Probably most people would be inclined to regard such an argument as a dilemma, because it has a hypothetical premiss and offers alternatives; but it offers them only to sublate them *totally*; hence it does not fall within the definition we have adopted (*cf.* 185).

patriotic: but either he did or he did not; therefore, he is either inconsistent or unpatriotic" (Demosthenes, *On the Crown*).

(3) The *simple destructive:* "If I am to carry out my plan, I must keep my pupils and write my book; but either I must drop my pupils or I must drop my book; therefore I cannot carry out my plan".

(4) The *complex destructive:* "If you are rich, you are able to give me five pounds, and if you are kind you are willing to do so; but you are either unable or unwilling; therefore, you are either not rich, or not kind".

184. Reduction and Validity of the Various Forms of Dilemma.—As in the mixed hypothetical syllogism, so in the dilemma, the constructive and destructive forms are reducible to one another by taking the obverted or full contrapositives of the majors. Thus, the *simple constructive*, given in symbols in the preceding section, may be reduced to the following:—

> "*If E is not F, A is not B; and if E is not F, C is not D;*
> But *Either A is B or C is D;*
> Therefore *E is F:*"

—which is the *simple destructive* form. As a concrete example, the simple destructive dilemma—"If I am to regain health I must give up work and take a sea trip; but I cannot do both; therefore I cannot regain my health"—may be put in the simple constructive form "If I either continue to work, or abandon the sea trip, I cannot regain my health; but I must do either; therefore I cannot regain my health".[1]

A dilemma is *formally* valid—*i.e.* its conclusion follows necessarily from its premisses, and is necessarily true *if* they are true—provided its conclusion is drawn in the way explained, *viz.* by alternatively positing antecedents, or sublating consequents, of a compound hypothetical major. The principle on which the reasoning is based is that involved in all inferences from hypotheticals: that the antecedent is a "sufficient reason" for the consequent.

A *conclusive* or *demonstrative* dilemma must, however, not only be formally valid: it must, in addition, be *materially* valid, *i.e.* its premisses must be *true*. Defect of truth in the premisses is a *material* fault; but the conditions required for truth, in the peculiar premisses of the dilemma, are such that defect of truth

[1] *apud* Welton, *Logic*, i., p. 881.

in them is easily mistaken for a *formal* fallacy, whereas it is in fact a *material* one.

Firstly, the disjunctive or alternative minor must be *exhaustive of the denotation of its subject* (143), *and of all the possible alternatives that can arise* in the kind of predication made about that subject; or, if the premiss gives a choice between judgments of independent import and with different subjects (144), *all the possible alternative judgments* that can be admitted in reference to the given subject-matter *must be enumerated.* In other words, unless the alternation is complete the premiss must be regarded as false. Whether it is complete or not, must be determined from the subject-matter in each individual case. And it is precisely because it is so easy to allow a possible alternative to pass unnoticed and unrecorded, that so many dilemmas are inconclusive. To detect such an alternative is described as *escaping between the horns of the dilemma.*

Then, secondly, the conclusiveness of the reasoning will be dependent on the truth of the hypothetical premiss. This must, of course, obey the same rule in regard to complete enumeration of hypotheses as the disjunctive premiss does: for it takes up each alternative and connects this with a certain antecedent or consequent. But, furthermore, the *connexion which it establishes* in each case between antecedent and consequent *must be one of really necessary and universal dependence.* In other words, the hypotheticals must be taken as necessary, apodeictic, universal, and must as such be true. Here, again, we must determine whether they are true or not, in each separate case, by our acquaintance with their subject-matter. To lay bare the falsity of any portion of the hypothetical premiss, and thus to show that the alleged unpleasant conclusion of the dilemma is not necessarily true—is the achievement known as *taking the dilemma by the horns.*[1]

A third way of escaping from a dilemma is by "rebutting" it, *i.e.* by retorting with another dilemma which will appear to prove the contradictory of the previous one. Of course, only really defective dilemmas can be successfully rebutted—and not even all these. Very frequently, the conclusion reached by the rebutting dilemma is only an *apparent* contradictory of the original conclusion. The rebutting dilemma may be composed of entirely

[1] Perhaps the most satisfactory way of dealing with Zeno's argument against motion, as given above.

new and original premisses (supposed to be admitted by one's adversary); but it is usually constructed, in the case of *complex* dilemmas, by *transposing* and *denying* the consequents (in a constructive), or the antecedents (in a destructive dilemma), of the original argument.

A few examples will illustrate the force of these rules :—

(1) *If emigrants are useless they are a burden to the colonies; if they are useful they are a loss to the mother country;*

But they are either useless or useful;

Therefore emigration is either a burden to the colonies or a loss to the mother country.

Here the hypotheticals are not true unless we insert "*to the colonies*" after "*useless*" and "*at home*" after "*useful*". But if we read the terms *as thus qualified*, in the minor, only two out of four possible alternatives are enumerated there. So we ensure the truth of one premiss only by showing the falsity of the other. Thus we "escape between the horns".

Or, we might attempt to "rebut" the dilemma thus :—

If emigrants are useless they are no loss to the mother country : if they are useful they are no burden to the colonies.

But they are either useless or useful.

Therefore, emigration is either no loss to the mother country or no burden to the colonies.

This conclusion, however, is quite compatible with the previous one: both alike are incomplete or non-exhaustive: and both dilemmas are equally defective. But the "rebutting" dilemma is sufficiently plausible to disconcert one's adversary by thus arriving at an *apparently* opposite conclusion to his.

(2) The Athenian mother tried, thus, to dissuade her son from entering public life :—

"*If you say what is just men will hate you ; and if you say what is unjust the gods will hate you.*

"*But you must say either the one or the other.*

"*Therefore you will be hated.*"

To which the son replied :—

"*If I say what is just, the gods will love me ; and if I say what is unjust, men will love me.*

"*But I must say either.*

"*Therefore I shall be loved.*"

Again, the conclusions are compatible : the hatred of the gods and the love of men follow from the one alternative (saying what

is just); the love of the gods and the hatred of men, from the other. Granting the truth of the two majors, the two dilemmas are valid: which shows that if we attempt to "rebut" a conclusive dilemma, the "rebutting" dilemma will be either inconclusive itself, or it will reach a conclusion not *really* incompatible with the original conclusion. But a rebutting dilemma is really successful only when it conclusively establishes a proposition incompatible with the original conclusion.

The rebutting of a complex constructive dilemma by transposition and negation of consequents may be represented thus:—

If P then R, and if Q then S;
But either P or Q;
Therefore, either R or S;

which is rebutted by—

If P then not S, and if Q then not R;
But either P or Q;
Therefore, either not S or not R.

There is no theoretical reason why the complex destructive dilemma could not be similarly rebutted. Thus the dilemma:—

If P then R, and if Q then S;
But either not R or not S;
Therefore, either not P or not Q;

may be rebutted by—

If not P then S, and if not Q then R;
But either not S or not R;
Therefore, either P or Q.

But, practically, it is impossible to form new hypotheticals with any plausible show of truth in them by combining the negative of each original antecedent with the other original consequent. The *complex destructive* must, therefore, be reduced to the complex constructive before being rebutted. *Simple* dilemmas cannot be thus rebutted, because a transposition of antecedents and consequents cannot be effected.

(3) The dilemma has often a *practical* aspect: being used to show that something ought, or ought not, to be, or to have been, done, *i.e.* to inculcate a line of conduct. An apocryphal story represents the Caliph Omar as thus justifying the destruction of the famous Alexandrian library:—

If the books are in conformity with the Koran they are superfluous; if not they are pernicious.

Either they are or they are not in such conformity.

Therefore, being either superfluous or pernicious, they ought to be destroyed.

This dilemma, though formally valid, is not demonstrative,

because the major is false: it assumes that the Koran contains everything worth knowing and nothing that is pernicious.

(4) A most plausible form of inconclusive dilemma is that known as the *Ignava Ratio*, or *Lazy Argument*. It is based on the unwarranted and untrue assumption that a certain eventuality will be independent of, and uninfluenced by, a certain line of conduct from which it aims at having us abstain:—

It is certain that you are destined either to pass or to fail at the coming examinations;

But if you are destined to pass, you will pass without studying; and if you are destined to fail, study will be labour in vain.

Therefore do not study for them.

The incompleteness of the alternatives given, and the dependence of *all* the various alternatives on the fact of study or idleness, are sufficiently obvious here: though the question may still perplex us whether the *present truth* of an alternative, or of a hypothetical, judgment, involves the *present truth* of one definite alternative, or consequent, to the exclusion of other alternatives, or of the contradictory of the given consequent.[1]

(5) Of the same sort is the inconclusive argument against virtue, based upon God's infallible foreknowledge:—

Either God foresees that you will be saved, or He foresees that you will be lost.

In the former case you will infallibly be saved, no matter how vicious your life may be; in the latter you will infallibly be lost, no matter how virtuous it be.

Therefore you need not live virtuously.

But God likewise foresees that our being saved and lost *depends upon the sort of life we lead*—virtuous or vicious. The assumption that He foresees the result as independent of virtue or vice, or foresees it to be independent of the latter, is groundless and untrue.

185. Other Views of the Dilemma.—It will be seen, from the various examples given, that the dilemma usually leads to an alternative between two equally unsatisfactory positions—though the conclusion is not always and necessarily an alternative. The awkward character of the position into which it forces an adversary is indicated by the saying that he is "on the horns of a dilemma". It is a vigorous and drastic dialectical device; but

[1] *Cf.* Joseph, *op. cit.*, pp. 166, 168.

it is proportionately dangerous; and it may be retorted with very disconcerting results if not skilfully used.

UEBERWEG [1] emphasizes "*choice of alternatives, all leading to the same conclusion*" as the essential character of the dilemma. This would exclude the *complex* forms altogether, while it would include some forms of argument which are really mixed hypothetical or mixed disjunctive syllogisms.

HAMILTON [2] and LOTZE [3] give as the dilemma a form of argument which is really the *Modus Tollens* of the mixed hypothetical syllogism. It may be expressed as follows:—

If X then either Y or Z;
But neither Y nor Z;
Therefore not X.

This, evidently, gives no choice of alternatives; but the dilemma is supposed to give such a choice: and it cannot do so unless there be a disjunctive premiss.

THOMSON'S definition [4] of a dilemma as "a syllogism with a conditional premiss, in which either the antecedent or the consequent is disjunctive"—is altogether too wide. It does not demand an alternative minor, and so it includes many forms of mixed hypothetical syllogism, such as that given above.

Mr. JOSEPH defines the dilemma as "a hypothetical argument offering alternatives and proving something against an opponent in either case". This makes room for the form illustrated by the example of Zeno's argument, as a simple destructive dilemma.

MANSEL, WHATELY, JEVONS, and CLARKE reject the *simple destructive* form of the dilemma, apparently because the same conclusion may be reached by *totally* denying the consequents of the major, as by alternatively denying them, *i.e.* by the *Modus Tollens* of the mixed hypothetical syllogism. This is true, but it is no reason why we should not recognize both ways of reaching the conclusion as distinct. Moreover, the same is true of the simple constructive form: we can get the same conclusion by conjunctively (copulatively) affirming the antecedents of the major, as by alternatively affirming them, *i.e.* by the *Modus Ponens* of the mixed hypothetical syllogism. Hence, to be consistent, those logicians should reject the simple constructive, as well as the simple destructive, dilemma. Besides, these are mutually reducible; hence, they must stand or fall together.

The view of the dilemma set forth above (183) is that propounded by WELTON, KEYNES, FOWLER, and STOCK, among others. It attaches the name *dilemma* to a form of argument sufficiently distinct from those previously set forth in the present chapter. It seems, therefore, preferable, and more conducive to clearness than any of the other views just mentioned.

WELTON, *op. cit.*, bk. iv., chap. v. KEYNES, *Formal Logic*, pp. 354 *sqq.* JOSEPH, *Logic*, p. 166 *sqq.*, 312 *sqq.*

[1] *Logic* (Eng. tr.), p. 455.
[2] *Lectures on Logic*, i., p. 350.
[3] *Logic* (Eng. tr.), i., p. 127.
[4] *Laws of Thought*, p. 203.

CHAPTER VI.

ABRIDGED AND CONJOINED SYLLOGISMS.

186. THE ENTHYMEME.—In the present chapter we shall examine abridged and conjoined syllogisms. These are irregular in the sense that they are not fully and explicitly stated in the recognized logical form of major, minor, and conclusion. It is these, nevertheless, rather than the logically stereotyped forms, that we usually find in any ordinary piece of argumentative discourse. The most common of them, perhaps, is *the syllogism in which one of the constituent propositions is omitted for the sake of brevity.* To this abridged form of syllogism modern logicians have given a name which Aristotle used in quite a different sense to denote a sort of probable argument, the *Enthymeme.* A weak, or merely probable, argument is often given an appearance of strength by suppressing or keeping in the background the weaker premiss. This fact, together with a mistaken etymology—tracing the name to the *suppressed* premiss (ἐν θυμῷ),—may account for the employment of the Aristotelean term to denote any syllogism in which one of the propositions is suppressed.

A very common form of enthymeme is that in which the conclusion is stated first, and one of the premisses joined on as a reason for it by the words "*because*," "*for*," or "*since*".

Logicians distinguish *three orders* of enthymeme:—

The First Order—in which the *major* premiss is omitted;
The Second Order—in which the *minor* premiss is omitted;
The Third Order—in which the *conclusion* is omitted.

A simple example in the first order would be "*John is intelligent, for he is successful*"; in the second, "*John is intelligent, for all successful men are intelligent*"; in the third, "*John is successful and all successful men are intelligent*".

When an enthymeme belongs to the *third* order, the position of the middle term shows at once whether the syllogism belongs to the second or to the third figure. If the middle term be once

subject and once predicate the syllogism will belong to the first figure, or to the fourth [or indirect form of first (173)], according to the extreme which is made subject or predicate of the conclusion.

When an enthymeme belongs to the *first* or the *second* order, we may without much difficulty determine its figure and mood, and so fill in the missing premiss. If the given premiss and the conclusion have the same predicate, the argument belongs to the first or to the third figure; if they have the same subject, the argument is in the first or in the second figure; if the predicate of the conclusion is subject of the given premiss the argument is either in the second or in the fourth figure; finally, if the subject of the conclusion is predicate of the given premiss the argument is either in the third or in the fourth figure.

We may, furthermore, have pure hypothetical enthymemes (174). If, however, we get an enthymeme containing a minor premiss and a conclusion, with no term common to them, we must conclude that the syllogism is a mixed one: a mixed hypothetical or a mixed disjunctive. The enthymeme "*C is D because A is B*" belongs to the *first* order, and may have for major either the proposition "*If A is B, C is D*" or the proposition "*Either A is not B or C is D*".

187. THE POLYSYLLOGISM.—This is the name given to *a chain of reasoning consisting of a number of syllogisms each of which proves one of the premisses of one of the two immediately adjacent syllogisms.* If we select any pair of successive syllogistic links in such a chain, we shall find that one of them has for its conclusion a premiss of the other. The former in relation to the latter is called a *prosyllogism;* the latter in relation to the former is called an *episyllogism.* Every intermediate link in the whole chain is therefore both a prosyllogism to the link on one side of it, and an episyllogism to the link on the other side.

Now, a chain of syllogistic reasoning may proceed either *from prosyllogism to episyllogism,* or in the opposite direction, *from episyllogism to prosyllogism.* A chain of reasoning which proceeds onward from prosyllogism to episyllogism, *i.e.* each syllogism of which proves, or has for its conclusion, the major or the minor premiss of the succeeding syllogism, is called a PROGRESSIVE, or SYNTHETIC, or EPISYLLOGISTIC, chain of reasoning. A reasoning process which proceeds backward—each succeeding step (syllogism) proving, or giving a reason for, one of the premisses of

the preceding step—is called a REGRESSIVE, or ANALYTIC, or PROSYLLOGISTIC, chain of reasoning.

The former, or *Progressive Polysyllogism*, is commonly met with in the *Synthetic* or *Deductive* sciences, such as Geometry and Mathematics, constituting what is known as the *Synthetic* or *Deductive Method.* When such a polysyllogism is condensed by the omission of some of its constituent propositions it is called a SORITES.

The latter, or *Regressive Polysyllogism*, is met with in the *Analytic* or *Inductive* sciences, such as Physics, constituting what is known as the *Analytic* or *Inductive* method. When it is similarly condensed it is sometimes called an EPICHEIREMA.[1]

With these two abridged chains of reasoning, the progressive and the regressive, we shall now briefly deal.

188. THE SORITES.—A *sorites* is *a progressive polysyllogism in which all the conclusions are omitted except the final one, and all the major or minor premisses are omitted except the initial one.* Every sorites is a *progressive* polysyllogism, *i.e.* it proceeds from prosyllogism to episyllogism. In such a process, each syllogism may prove either the major or the minor premiss of the succeeding syllogism. Hence, there are two possible kinds or forms of sorites: one in which each constituent syllogism proves the major premiss of the subsequent one (the minors being assumed as true or otherwise proved): the other in which each constituent syllogism proves the minor premiss of the subsequent one (the majors being assumed as true, or otherwise proved). The following are examples of each sort:—

(*a*)		(*b*)	
Major . .	*Every Z is P*	*Every S is X* . .	*Minor.*
Minor . (1)	*Every Y is Z*	*Every X is Y* . .	*Major.*
[*Conclusion and Major*]	[∴ *Every Y is P*]	[∴ *Every S is Y*] .	[*Conclusion and Minor*].
Minor . .	*Every X is Y* (2)	*Every Y is Z* (2)	*Major.*
[*Conclusion and Major*]	[∴ *Every X is P*]	[∴ *Every S is Z*] .	[*Conclusion and Minor*].
Minor . (3)	*Every S is X*	*Every Z is P* . .	*Major.*
Conclusion	∴ *Every S is P*	∴ *Every S is P* . .	*Conclusion.*

In these examples the propositions that are usually omitted are supplied in square brackets.

In the first form, (*a*), it will be noted that all the *majors* are

[1] Aristotle used this title to designate the syllogism by which a disputant attacked the thesis defended by the respondent in a dialectical discussion. *Cf.* JOSEPH, *op. cit.*, p. 325, n. 3.

suppressed *except the first*, and all the conclusions (which are identical with these majors) are suppressed *except the last.* This form, in which each syllogism proves the (suppressed) *major* of the succeeding one, and in which the first expressed proposition is a *major* premiss and all the remaining expressed propositions (except the last) are *minor* premisses, is called the GOCLENIAN SORITES.

In the second form, (*b*), it will be noted that all the *minor* premisses are suppressed *except the first*, and all the conclusions (which are identical with these minors) are suppressed *except the last.* In this form, too, the minor premisses are written before the majors; but this is an immaterial point (148). This second form, in which each syllogism proves the (suppressed) *minor* of the succeeding one, and in which the first expressed proposition is a *minor* premiss and all the remaining expressed propositions (except the last) are *major* premisses, is called the ARISTOTELEAN (or ORDINARY) SORITES.

The *Goclenian* sorites is so called from *Goclenius* (1547-1628), a professor of Marburg, who first drew attention to it in a commentary on Aristotle's *Organon.* The *Aristotelean* is called after Aristotle, though it is nowhere treated by him. Nor was the term *Sorites* used by Aristotle. He refers but vaguely to the form of reasoning we are at present considering (*An. Post. a.* xiv. 79ª, 20, xx.-xxiii.). It was first treated expressly by the Stoics, and first called the *Sorites* by Cicero.[1] But it was not till long afterwards that this name came to be generally used in its present sense. The term *sorites* (σωρος, a heap) was, indeed, used, but in quite a different sense, by the Greek philosophers. With them it denoted a certain form of fallacy based on the difficulty of assigning the exact limits of a concept. For example, "Does one grain of corn make a *heap?*" "No." "Do two?" "No." "Do three?" "No" . . . "Do three thousand?" . . . "But the addition of any one grain does not change into a heap what was not a heap?" . . . "Therefore either three thousand grains do not, or *one* grain *does*, make a heap?" It was called the *Calvus* (bald) in this example: "Does pulling one, . . . two, . . . three, etc. . . . hairs from a man's head make him bald?" A similar fallacy arises from such questions as: "On what day does a lamb become a sheep?" It is sometimes confounded with a somewhat different fallacy called the *Fallacia plurium interrogationum*, or *Fallacy of Many Questions* (274, *f.*).

From the analysis given above it will be seen that the sorites is a series of enthymemes; that the first of these is of the third order, that the last is of the first (in the *Goclenian*) or second (in the *Aristotelean*), and that the intermediate ones are represented each by one proposition only—a *minor* in the *Goclenian*, a *major*

[1] *Cf.* KEYNES, *op. cit.*, p. 371, n.

in the *Aristotelean*, form. In the *Goclenian sorites*, the *first* premiss is a *major* and the *remaining* premisses *minors;* in the Aristotelean the *first* is a *minor* and the *remaining* ones *majors.* This is the best practical test for discerning the form to which any given example belongs. Other features of each form have suggested other definitions for the sorites, and other means of distinguishing between the two forms. Omitting the bracketed propositions from the examples given above, we have (1) the Goclenian and (2) the Aristotelean forms expressed thus :—

(1)	(2)
Every Z is P	*Every S is X*
Every Y is Z	*Every X is Y*
Every X is Y	*Every Y is Z*
Every S is X	*Every Z is P*
∴ *Every S is P.*	∴ *Every S is P.*

In both cases the premisses are the same, but the order is reversed. Although the order of the premisses in the Goclenian approaches nearer to the usual order of premisses in the simple syllogism, still the Aristotelean form, in which each new term appears first as predicate and then as subject, is the more commonly recognized form.

In either form all the constituent propositions may be hypotheticals ; and such *pure hypothetical* chains of reasoning are not uncommon.

We may also derive a *categorical* conclusion from such *hypothetical* sorites by having the concluding syllogism a *mixed hypothetical.*

In the Goclenian form the last premiss and the conclusion may be categorical ; for example (where X, Y, Z, etc., stand for propositions) :—

If Z then C
If Y then Z
If X then Y
If A then X
But A [or, *But not C*]
∴ *C* [or, *not A*].

In the Aristotelean form a categorical minor must be added to the final hypothetical [major] premiss, and be taken in conjunction with the suppressed conclusion from this latter, in order to yield a categorical conclusion. Thus :—

If A then X
If X then Y
If Y then Z
If Z then C
[∴ *If A then C*]
But A [or, *But not C*]
∴ *C* [or, *not A*].

The following may be taken as an example of the mixed hypothetical Goclenian sorites: "If the human soul can form abstract, universal concepts, it has an activity beyond the power of all material agencies; if it has such an activity, its being must be superior to that of matter; if its mode of being is above matter, it is spiritual; if it is spiritual, its existence is independent of its union with the body; if this be so, it will continue to exist after the dissolution of the body; if it continues so to exist, then it must be immortal; but the soul can form abstract, universal concepts; therefore the soul is immortal".

189. FIGURES, MOODS, AND SPECIAL RULES OF THE SORITES.—The forms of sorites we have been dealing with so far, yield constituent syllogisms *in the first figure*. The special rules for their validity will, therefore, be mere applications of the special rules of the first figure, and will be grounded on the same general rules of syllogism as these were (161).

Taking the *Aristotelean* sorites, we see that the first premiss is a *minor*, that the others are *majors*, that the suppressed conclusions are *minors*. Now the minors, whether expressed or understood, must be all affirmative; for if any of them were negative the major would have to be affirmative, leaving its predicate *undistributed*, and the immediate conclusion would have to be negative, *distributing* that same predicate: hence *illicit major*. Furthermore, only the last [major] premiss can be negative; for were any preceding [major] premiss negative it would yield a negative conclusion to be combined as *minor* premiss with the succeeding major; and thus some constituent syllogism would have two negative premisses. Hence the rule of *quality*:—

(1) *Only one premiss, and that the last, can be negative.*

Since, then, all the premisses except the last must be affirmative, it follows that all of them, except the first, must be universal; for, were any, except the first, particular, the succeeding syllogism (in the first figure) would have a particular major, thus involving *undistributed middle*. Hence the rule of *quantity*:—

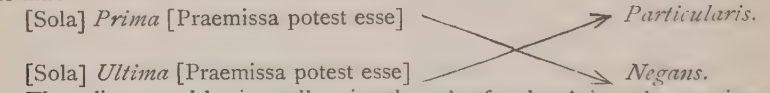
(2) *Only one premiss, and that the first, can be particular.*

Precisely similar reasoning reveals two corresponding rules for the *Goclenian* sorites in the first figure :—

(1) *Only one premiss, and that the first, can be negative.*

(2) *Only one premiss, and that the last, can be particular.*

The student should try to reason out both pairs of rules from those of the first figure, rather than rely on memory for the distinction between each pair. Remembering that the *Aristotelean* sorites comes naturally before the *Goclenian* (both historically and alphabetically), and that "*p*" commences the words "*prima*" and "*particularis*," he may get some assistance from the mnemonic :—



These lines, read horizontally, give the rules for the Aristotelean sorites. Read in the direction of the arrow-heads, they give the rules for the Goclenian sorites.

We have next to inquire whether it is possible to have a sorites in any other figure than the first, *i.e.* all the constituent syllogisms of which are in some figure other than the first. This question was not satisfactorily answered until Dr. Keynes constructed examples of sorites in the second and third figures,[1] —the constituent syllogisms being in the moods *Baroco* and *Bocardo* respectively.

The following is an example of *Baroco* in the second figure (with the suppressed propositions supplied) :—

Minor . . . . . *Some S is not X.*
Major . . . . . *Every Y is X,*
[*Conclusion* and *Minor*]. . [∴ *Some S is not Y*].
Major . . . . . *Every Z is Y.*
[*Conclusion* and *Minor*] . [∴ *Some S is not Z*]
Major . . . . . *Every P is Z*
Conclusion . . . . . ∴ *Some S is not P.*

We may have a sorites in *Camestres*, as well as in *Baroco*, of the second figure. *Only* the *first* syllogism of a sorites in this figure can be in *Cesare* or *Festino*.

It will be observed, from the example given, that the sorites in the second figure is analogous to the *Aristotelean* sorites in the first figure : a minor premiss comes first ; the other expressed propositions are majors ; the suppressed conclusions are minors.

Logicians have not expressly formulated rules for the present

[1] *op. cit.*, pp. 373 *sqq.*

form, or for the form in the third figure, given below. For the sorites in the second figure we must derive them from the special rules of the second figure, applied to the successive constituent syllogisms. The following will be found to result from the rules of the second figure :—

(1) *Either the first or the second premiss must be negative, and no other (expressed) premiss can be negative.*

(2) *Either the first or the second premiss may be particular, and no other (expressed) premiss can be particular.*

The following is an example in *Bocardo* of the third figure (with the suppressed propositions supplied) :—

Major . . . . . *Some Z is not P*
Minor . . . . . *All Z is Y*
[*Conclusion* and *Major*] . [∴ *Some Y is not P*]
Minor . . . . . *All Y is X*
[*Conclusion* and *Major*] . [∴ *Some X is not P*]
Minor . . . . . *All X is S*
Conclusion . . . . . ∴ *Some S is not P.*

We may have a sorites in *Disamis* as well as in *Bocardo* in the third figure. *Only* the *first* syllogism can be in any of the remaining moods of this figure.

The sorites in the third figure is analogous to the *Goclenian* sorites in the first: a major premiss comes first; the other expressed propositions are minors; the suppressed conclusions are majors.

The following rules will be found to secure the validity of the sorites in the third figure :—

(1) *Only one premiss, and that the first, can be negative.*

(2) *Either the first or the second, but not both, and not any other (expressed) premiss, can be particular.*

On account of the multiplicity of middle terms intervening between the first premiss and the ultimate conclusion, the sorites is a form of argument peculiarly liable to the fallacy arising from the employment of an ambiguous middle. Individual variations of shades of meaning may be separately almost imperceptible, but the cumulative effect of such slight variations inevitably leads to considerable deviation from truth. Even a single enthymeme may conceal a fallacy: a long chain of enthymemes will do so much more effectively.

190. THE EPICHEIREMA.—The *Epicheirema* is a *Regressive Polysyllogism abridged by the omission of one of the premisses of*

each prosyllogism. Every syllogism may have a reason assigned for one, or for both, of its premisses. These reasons may be stated as enthymemes. If a reason be thus assigned in support of *both* premisses of the original episyllogism, we have a *Double Epicheirema.* For example:—

Every M is P, because *every Q is P ;*
Every S is M, because *every S is R ;*
∴ *Every S is P.*

If only one premiss of the original episyllogism be thus supported, we have a *Single Epicheirema.* For example:—

All P's are M's because *they are Q's ;*
No S's are M's ;
∴ *No S's are P's.*

In the first of these examples, the enthymeme proving the major is of the second order, for it omits its minor (*Every M is Q*); that proving the minor is of the first order, for it omits its major (*Every R is M*). The reason given for the major in the second example is also an enthymeme of the first order, omitting its major (*All Q's are M's*).

Conceivably, the premiss of each enthymeme alleged in support of either premiss of the original syllogism might be itself supported by a new syllogism; and each premiss of these similarly supported; and so on, the syllogisms multiplying in geometrical progression. We rarely, however, find concrete examples in which the supporting process is carried farther than one or two steps backward.

WELTON, *op. cit.*, bk. iv., chap. vi. KEYNES, *op. cit.*, pt. iii., chap. vii.

CHAPTER VII.

NATURE AND CHARACTERISTICS OF INFERENCE.

191. ARE THERE OTHER MEDIATE INFERENCES BESIDES THE SYLLOGISM?—The answer to this question will evidently depend on the definitions we accept of *mediate inference*, and of the *syllogism* (147); and, ultimately, on the interpretation we give to the act of judgment (83, 99, 132, 141). The essential feature of mediate reasoning, as opposed to mere immediate inference, seems to be that it leads us *mediately* to a relation between two concepts or objects of thought, *i.e.* by the introduction of a third concept with which the two former are successively compared. Were we, therefore, to define the syllogism as "any combination of two judgments from which a third necessarily follows" (147), this wide definition would include all possible kinds of mediate inference. For the only reason why a third judgment necessarily follows from two others is because in the latter the mind establishes two mental relations in which it sees the third relation to be necessarily involved. This implies that in all mediate inference there must be three mutually related elements or objects of thought; and we have, therefore, to decide whether it is not possible for the mind to establish, between three such elements, relations of a different kind from those with which the syllogism deals, and by an inferential process which is not reducible to the syllogism.

Now, if we examine all the arguments which have been treated as categorical[1] syllogisms in former chapters, we shall find that each and every one of them not merely (1) relates two objects of thought with a third; but, furthermore, that (2) these relations are all *relations of extension and intension* between the three objective concepts compared; that in every such inference (3) one of the antecedent comparisons *reveals a universal and neces-*

[1] The remarks that follow in the text apply to hypothetical and disjunctive arguments not directly, but only in so far as they may be reduced to categorical form (182).

sary relation; so that (4) every such inference is *from the general to the particular*, in this sense, that it is the application—in some form or other, according to the *Dictum de omni*, or a corresponding axiom for each of the other figures—of some general rule or principle to some less general group of cases, or to some single case, brought under the former (170-3).

These, therefore, we gather to be the essential characteristics of the categorical syllogism. The *first* of them is common to all mediate inferences. How far the *third* and *fourth* are also essential to mediate inference, or, at least, how far all mediate inference is dependent on the intuition of some universal and necessary truth regarding the subject-matter of that inference, or whether it is possible to have mediate inference from *particular* or *indefinite* (93) propositions,[1] without the tacit intervention of *any* universal principle—we shall determine below (194). The *second* characteristic seems to be peculiar to the syllogism, so that forms of mediate inference not possessing it would be non-syllogistic. Are there any such?

In referring to the *Axioms of Identity and Diversity* (157)—which are certainly presupposed by all syllogistic reasoning—we pointed out that the "identity" and "diversity" in question were not to be understood as a mathematical "equality" or "inequality" of two magnitudes or multitudes. It is a relation of "identity" or "diversity," of two *objective concepts*, in regard to their *extension* and *intension*, that we have seen to underlie the whole theory of the categorical syllogism. The judgments entering into it connect two objects of thought, by means of the logical copula "*is* (not)," "*are* (not)," as *subject* and *attribute*, as *logical*, *extensive whole* and *logical part*, or as *intensive whole* and *intensive part*, according to the predicative, extensive, or intensive interpretations, given to these judgments.

Now, it is conceivable at all events, that the mind may establish, between its concepts, relations other than that of "*subject* to *attribute*". And we have in fact encountered some such already, *e.g. ground* and *consequent* in the hypothetical judgment. These relations may reveal others, by way of mediate inference, in virtue of certain principles distinct from, but just as immediately and intuitively evident as, the axioms that govern the various

[1] Care must be taken not to confound these with definite, individual or singular propositions, which formulate what are usually called "particular" (*i.e.* individual) facts.

figures of the categorical syllogism. The question is not whether these relations, expressed in the individual propositions, can or cannot be reduced to the *subject-attribute* relation, and be interpreted accordingly. Theoretically at least, all relations expressed in a judgment may be so reduced: ordinary logic assumes the possibility of such reduction (82, 99, 118). But the question rather is this, whether there are combinations of those other relations, which, as they stand, yield mediate inferences, while if they were reduced to the *subject-attribute* relation they would yield no conclusion according to the nature and laws of the syllogism. Take, for instance, this simple example, which is typical of a very wide class of mathematical inferences :—

(I) *A is greater than B ;*
B is greater than C ;
∴ *A is greater than C.*

This is certainly a mediate inference: it is certainly valid: its constituent propositions can certainly be reduced to the "*subject-attribute*" form ("*A* is a subject endowed with the attribute of being *greater than B*" : "*A* '*is*' *greater than B*" ; etc.) ; and, nevertheless, when its propositions *are* read in this way—as they must be in order to get a connexion of two objects of thought by the *logica* copula, "*is*,"—the given argument is just as certainly *not a syllogism.* For it has *four* terms, *viz.* (1) "*A*," (2) "*greater than B*," (3) "*B*," (4) "*greater than C*".

The truth is that when the mind goes through this simple inference it has *three* objects of thought before it, *viz. A*, *B*, and *C ;* that in the three constituent judgments it relates these terms (in pairs), *not* by the logical copula "*is*," which would express *subject-attribute* identity, but by a copula which expresses a directly and intuitively apprehended *relation of magnitude* between the terms of each pair—by the copula "*is greater than*". And the mediate axiom on which the inference is based is this self-evident mathematical axiom : *A magnitude which is greater than another is greater than all magnitudes than which this latter is itself greater.*[1] This mediate axiom underlies the argument in question in precisely the same manner as the *Dictum de omni* underlies the syllogism :—

(II) *Man is mortal ;*
Socrates is a man ;
∴ *Socrates is mortal.*

[1] *Cf.* KEYNES, *op. cit.*, p. 385.

Various attempts have been made to reduce the former argument, (I), to a syllogism. Of these the following alone calls for some notice :—

(I, *a*) "*Whatever is greater than a greater than B is greater than B; A is greater than a greater than B;* therefore, *A is greater than B.*"

This is certainly a syllogism, and a valid one; but, just as certainly, it *is not an expression of the former argument*, (I): for it has not the same terms. It does bring out, however, this important fact, to which we shall presently return (194), that the argument in question (and the same is true of *all mediate inferences whatsoever*) *involves, and depends for its validity on, the apprehension of some intuitively evident, abstract and universal truth or principle*, of which it is an application. The application of the principle involved in the first inference, (I), is stated in the major premiss of (I, *a*): "*Whatever is greater than a greater than B is greater than B*". This is a particular case of the general mathematical axiom which we formulated above for the argument (I) under examination.

In a precisely similar way, we may show that the *syllogism*, (II), given above depends on the *Dictum de omni*, by means of this other syllogism, having for major the required application of the *Dictum.*

(II, *a*) *Whoever belongs to a class of beings that are mortal is himself mortal; Socrates belongs to a class of beings that are mortal;* therefore, *Socrates is mortal.*

The major of this syllogism (II, *a*) simply states the narrower application, employed in the previous syllogism (II), of the *Dictum de omni.* Yet it would hardly be correct to say that (II) *is reduced to* (II, *a*), or that the latter is another or equivalent expression of the former; but rather that (II) *involves* (II, *a*) just as (I) involves (I, *a*).

192. Some Classes of such Inferences.—(A) The *Syllogism*, of which (II) is an example, is, therefore, a mediate inference *from judgments which express each a relation of subject to attribute.*

(B) The class of arguments of which (I) is an example, includes all mediate inferences *from judgments which express each a relation of degree between two measurable magnitudes.* All *a fortiori* arguments belong to this class; *e.g.* "*A is hotter than B; B is hotter than C; therefore, a fortiori, A is hotter than C*".

To this class we would refer the arguments given by Father Joyce in his *Principles of Logic* (p. 199):—

"The triangle ABC is equal to the triangle DEF.
The triangle GHI is equal to the triangle DEF.
∴ The triangle GHI is equal to the triangle ABC.

and

$12 = 7 + 5$; $12 = 20 - 8$; ∴ $7 + 5 = 20 - 8$".

These arguments are based on the *mathematical* axiom "Things which are equal to the same thing are equal to one another," not on the *logical* "*Dictum de omni*". Father Joyce believes this view to be erroneous; and he gives two reasons: "In the first place the data most certainly gives us a subject-attribute relation: for this is inseparable from judgment". This we admit; *but it is not from the combination of these subject-attribute relations that we reach the respective conclusions.* No doubt, as we pointed out above, we can reach each of the conclusions *by a syllogism* the major of which embodies the very mathematical axiom on which the arguments are directly based. Father Joyce gives such a syllogism:

"Any two quantities, each of which is equal to the same third quantity, are equal to each other.

"ABC and GHI (as being equal to DEF) are two quantities, each of which is equal to the same third quantity.

∴ ABC and GHI are equal to each other." But this syllogism can hardly be claimed to be an equivalent expression of the original argument. Nor can it be fairly denied that when people do actually reason as in the two arguments given above, the copula in their minds is not "*is*," but "*is equal to*": "the word *equals* is a *copula* in thought and not a *notion attached to a predicate*".[1]

"Secondly," continues Father Joyce, "it is impossible that the axiom 'Things which are equal to the same thing, etc., etc.,' should be a principle of inference. It is a truth relating to the real order, not to the conceptual. It is necessary to the inference, but it is not a canon governing the inferential process itself. A canon of inference must have explicit reference to the conceptual order."

We cannot allow that because the axiom in question relates "to the real order," therefore it cannot "be a principle of inference". The *Dictum de omni* relates to the real order: it is a self-evident intuition of the mind about the nature of the real order, and not, as a Kantist might perhaps contend, an empty, subjective form, revealing nought but the nature of thought itself: and yet we recognize in the *Dictum de omni* a principle of inference. And why? Because it is *conceptual as well as real: i.e.* because it formulates a law based on certain characteristics (intension and extension) of our concepts—characteristics *which these concepts derive from the nature of the reality which forms their objects.* But so is the other axiom in question *conceptual as well as real.* It is because we *intellectually conceive* reality as constituted of *classes* of things, and these *things* as endowed with *attributes*, that we are able to formulate subject-attribute judgments, and to lay down the *Dictum de omni* and the other *dicta* of syllogistic inference. Similarly, it is because we *intellectually conceive* reality as embodying *magnitudes* and *multitudes*, and these as related *in degree* to one another, that we can form judgments having a quantitative or mathematical copula (symbolized by $=$, $>$, $<$, etc.), and lay down distinct

[1] De Morgan, *Syllabus*, pp. 31, 32;—*apud* Keynes, *op. cit.*, p. 386, note.

self-evident principles of inference concerning magnitude and multitude. And precisely the same may be said, in anticipation, about the principles underlying the inferences of the classes (C), and (D), below. The intellect conceives, as holding good in *reality*, certain groups of relations—whether of origin (C), or of location in space or time (D)—which enable it to compare or relate certain objects otherwise than by the logical copula of subject and attribute, and to lay down concerning these objects certain self-evident truths on which inferences about these objects are seen to be based.

All such axioms of inference must be *both* conceptual *and* real. Father Joyce admits that those other axioms besides the *Dictum de omni* are "necessary to . . . inference". They are therefore "conceptual". They are even explicitly conceptual: they do not, of course, refer to the "extension" aspect of the concept, or regard it as a "logical whole" resolvable into "logical parts": but this is not the only aspect of the concept, nor is the *Dictum de omni* the only axiom that can claim to be "conceptual" merely because it is the axiom that does especially deal with this particular side of the concept.

Neither can it be objected that the *Dictum* is the only axiom that is conceptual in the sense that it alone regards the concept as a "*secunda intentio mentis*". The "extension" aspect of the concept belongs, of course, to the latter only in so far as the latter is considered to be a "*secunda intentio mentis*". The abstract concept is universalized only in and by the mind: the "*intentio universalitatis*" results from the mental comparison of the abstract type with the individuals from which it was abstracted, and thus makes the abstract type a "*second intention*". But precisely the same is true of the other relations, superadded to the direct concept by the mind, in the case of the other axioms. The mental act of relating one objective concept to others as an *equal*, *greater*, or *less*, magnitude than others, or as related in *time* or *space* to others, also makes such concept a *secunda intentio mentis*. And it is because there is a *foundation in reality* for *all* those mental relations, which we establish between our concepts, that our *conceptual* processes of inference are also *real*—that the application of the *secunda intentio* to the *prima intentio* is legitimate.

(C) Another class of mediate inferences embraces all those derived *from judgments which establish between pairs of objects relations of such a character that these involve further similar relations*, which are thus inferred from the former. For example, "*A is the brother of B, B is the sister of C, therefore A is the brother of C*". Not all relations, of course, are of this kind. For each class of mediate inferences from those which evidently *are* of this kind, we can frame some mediate axiom like this: "Whoever is the brother (or sister) of any individual is the brother (or sister) of all the brothers and sisters of that individual": and this axiom we can make the major premiss of a *syllogism* which is involved in each particular inference of the class.

(D) A still more familiar class of mediate inferences includes those *from judgments which express each a "time" or "space" relation between two objects of thought.* For example, "*Dublin is*

north of Cork; Belfast is north of Dublin; therefore Belfast is north of Cork"; "A died five years before B; B died the same year as C; therefore A died five years before C". These inferences, too, are based, each upon some intuitively evident, abstract, universal truth concerning the nature of the relation in question—*i.e.* concerning relations of space or of time—some principle, of which every valid inference of that particular kind is merely an application.

The three classes of inferences, (B), (C), and (D), do not by any means exhaust all the types of mediate inference actually employed by the mind, but they are the most familiar and important classes. In each of them the copula employed in the constituent judgments is not the logical copula, "*is, are,*" but, rather, some such connecting link as "*is greater* or *less than*, or *equal to*"; "*is related in some special way to*"; "*is prior* or *posterior in time to*, or *simultaneous with*"; "*is related in space to,*" etc. The investigation and classification of all such logical relations, other than that expressed by the logical copula, "*is, are,*" belongs to a department of research known as the *Logic of Relatives.*

Many logicians contend that all the above inferences—and in fact all *mediate* inferences—are *syllogistic;* but these logicians either explicitly or implicitly so widen the definition of syllogism that, in this wider acceptation, it does include all of them (*cf.* 147, 148).

Professor Welton,[1] who holds that they are not syllogistic, points out, further, that "neither are they deductive; for in them [there] is no subordination of a special case under a general principle, but an inference of co-ordination from particular to particular". Apparently, he here uses the term "particular" not in the technical sense of "indefinite," but rather as meaning a "singular" or "individual" judgment. The examples he gives are composed of singular judgments. He himself ably opposes Mill's contention that in the syllogism we reason from "particulars". We shall see presently (193) that there can be no mediate reasoning from particulars, whether definite or indefinite, *without the aid of a universal.* And Professor Welton admits this, for he continues: "No doubt, the validity of the inferences rests upon material considerations of degree, time, space, etc., which are *universally applicable;* but these considerations stand in the same relation to the special arguments as the *dicta* of the four figures do to the syllogisms in those figures; and are not, therefore, the implied major premisses of the arguments. The syllogism remains, then, as the one type of deductive reasoning, and should not be discarded on account of the existence of these other valid inferences, whose scope is not very great, and whose want of generality must always make them of but little importance." [2]

[1] *op. cit.*, i., p. 411.

[2] *ibid.* (italics ours). *Cf.* JOSEPH, *Logic*, p. 319, n. 2, where the author evidently

The question has been raised whether those inferences, whose classification as syllogistic or non-syllogistic is open to dispute, are *formal* or *material* inferences.

If by a *formal* inference we mean one in which the truth of the conclusion follows *necessarily* from the premisses *on the assumption that these premisses are true*, then they are just as formal as the syllogism, *i.e.* their conclusions follow by the same hypothetical necessity from their premisses as in the case of the syllogism. They contain, of course, a material or "truth" aspect, as well as a formal or "consistency" aspect. But so, too, does the syllogism. In this, just as in those, the conformity of the whole mental process throughout, with the *reality* which it interprets, depends not merely on the cogency of each such form of inference,[1] but on the truth of the judgments which constitute its antecedents or premisses.

193. The Possibility of Mediate Inference from Particular Judgments.—We have seen that a certain small number of *immediate* inferences of minor importance can be derived from particular or indefinite judgments (116-21, 140.). Even in those cases, however, the mental process does not take place without the aid of certain self-evident, necessary, *universal* truths, which are embodied, as "laws of thought," in all our judgments (11). The syllogism, as we saw, cannot be valid unless it has at least one *universal* premiss (156). Moreover, the recognition of a syllogism as formally valid is merely the recognition of an individual case or application of some self-evident *universal* axiom, which the mind apprehends as embodied in the special case before it. And the same is true of all forms of mediate inference (192): all alike derive their cogency as forms of reasoning from

takes "deductive" inference as wider than "syllogistic" inference (of which latter he takes the narrower view, excluding hypothetical and disjunctive arguments; *cf.* 148, 174), and as including those forms of inference (referred to above) which are based upon intuitions of necessary relations seen to hold universally in the domains of time, space, magnitude, multitude, etc. Such principles as these cannot, he thinks, be reduced to "logical" principles, nor *vice versa*: "There are some who have represented logic as at bottom a branch of mathematics; and others seem inclined to suppose that mathematics can be reduced to formal logic. . . . I ought perhaps to say that I do not understand how either theory can be true" (*ibid.*, p. 512). If we take "logical" inference as resting on self-evident intuitions of *Being*, then "mathematical" inference will be "logical" and *something more*, inasmuch as it demands as basis not merely self-evident intuitions about Being, but also intuitions about a special category of Being, *viz. Quantity*.

[1] Or, in other words, on the truth of the self-evident axiom of the form of inference in question.

the mental recognition, and acceptance as true, of some *necessary and universal intuition upon which we ground the possibility and legitimacy of the passage from antecedent to consequent.* Did we not, for instance, recognize as evidently, *necessarily*, and *universally* true, the principles laid down in the preceding section—in reference to relations of *magnitude*, *multitude*, *identity*, *space*, *time*, etc.—as axioms of the corresponding classes of inferences, we should have no logical ground for passing from premisses to conclusion in any of them.

Hence, we may draw the general conclusion that no logical inference from *particular* or indefinite judgments is possible without the mental intervention and assistance of *universal* truths.

This teaching may seem at first sight to contradict much of our ordinary experience: for we are commonly said to "draw inferences" from "particular" facts, to "reason" by *analogy* from some "particular" fact to some other similar "particular" fact, etc. Here, however, the term "particular" is used in the sense of "individual," "singular". And we shall find, moreover, on closer analysis of those processes by which we argue from individual facts to other individual facts by *analogy*, or to general truths by *induction*, that there is, in all such cases, a tacit *universal axiom* in the mind, underlying the mental process, and guaranteeing the validity of the passage from antecedent to consequent.[1]

Inference by analogy is a process in which it is claimed that we reason directly *from particulars to particulars.* Mill, contending that all inference is *from particulars*, writes:[2] "It is not only the village matron who, when called to a consultation upon the case of a neighbour's child, pronounces on the evil and its remedy simply on the recollection and authority of what she accounts the similar case of her Lucy". But is the reasoning here directly from one individual case to another, *without any universal* principle? *Apparently*, it is. "This child is affected like Lucy; therefore what cured Lucy will cure it also." *Really*, however, the universal must and does intervene. For, why does the village matron account this case "similar" to that of Lucy? Because it reveals the same symptoms? And, therefore, the same *kind of*

[1] This will be illustrated in connexion with the doctrine of *Induction*, when we come to analyse the various forms and phases of the process by which we ascend mentally from the concrete, individual facts of sense experience, to the apprehension general intellectual truths.

[2] *Logic*, ii., ch. iii., § 3.

disease? And what cured a *certain kind of disease* once ought to be expected to do so again? This is undoubtedly the line of reasoning followed—though so rapidly that in this, as in many similar cases, the intermediate links are scarcely conscious, and leave the impression of only a single step from the one particular case to the other. It is not from the particular instance of the disease that the matron argues, but from *the general principle*, thus:

The kind of disease from which Lucy suffered *can be cured by certain remedies*—those administered in her case;—

But this is an instance of that kind of disease;

Therefore it will be cured by those same remedies.

Again, in arguing from circumstantial evidence, we may be said to be "inferring from particulars," though *we are aided throughout by universals.* "When a jury," writes Father Joyce,[1] "after weighing a mass of evidence, acquit or condemn a man accused of burglary, they undoubtedly infer: but they do not employ syllogistic reasoning. They form a critical estimate of what certain particular facts involve. They decide that these facts are compatible or incompatible, as the case may be, with the man's innocence. The evidence taken as a whole may be sufficient to produce certitude: but no sane man would endeavour to state it in the form of a general law. Cardinal Newman has discussed this form of inference at length in his *Grammar of Assent.* But the difference between the two forms of reasoning was familiar to St. Thomas and was carefully noted by him more than once." [2]

There is, of course, a process by which mental states are associated so as to suggest one another. But this, whether in man or in animals, is not a *logical* process—a passage from what is apprehended by reason as a *rational ground* to what is apprehended as a *consequent* of that ground. Whatever may have been St. Thomas's view about the manner in which we *intellectually* apprehend *particular facts*, and *reason* about *concrete matters*—and his doctrine on the *vis cogitativa* is not very clear,—it is fairly certain, at all events, that if we do *reason consciously* and *logically* about such facts, and pass logically from one to another by rational inference, we do so by the aid of *universal* intellectual truths which reveal a rational connexion between those facts. The difference referred to by St. Thomas between those inferences and inferences from abstract, universal truths, does not consist in the entire absence from the former of the universal element present and prominent in the latter, but rather in the fact that the sentient activity is conjoined with the intellectual activity of the mind in the former inferences—in the formation of "singular" concepts—and not in the latter.

Circumstantial evidence is the cumulation or addition of a number of distinct inferences, each pointing with more or less probability towards some concrete, individual conclusion, *e.g.* "*A.B. committed the murder in question*".

[1] *Logic*, p. 200.

[2] "St. Thomas holds that as these inferences are solely concerned with particular facts, they are effected by the *vis cogitativa* in which sense and intellect meet. See *Summa Theol.*, II., ii., Q. 2, Art. 1; *De Pot.*, Q. 14, Art. 1."—*ibid.*, n.

Of course, we can lay down no principle "in the form of a general law" to regulate the cumulative effect of these inferences upon the mind that makes them. But every single inference—connecting each separate "circumstance" with the suspected conclusion—may be shown *to have a universal axiom underlying it.* Furthermore, these inferences appear to be for the most part *syllogistic;* to be a series of deductions from the hypothesis that the accused is guilty.[1] For example: "If A.B. were guilty he would leave footprints of a certain size and shape in the vicinity of the crime; there are such footprints there; hence he is probably guilty". And so of every other circumstance. The force of each circumstance may, likewise, be expressed by an Aristotelean enthymeme in the second figure. For example: "There was evidently a struggle in which the murderer's clothes would have been blood-stained. A.B.'s clothes are blood-stained. Therefore probably, etc." [2]

194. ERRORS ON THE FUNCTION OF THE UNIVERSAL JUDGMENT IN THE PROCESS OF INFERENCE: THE "PARADOX" OF INFERENCE.—The contention that all mediate reasoning involves *universal* truths, suggests a number of questions about the origin and function of these latter as *axioms* and *premisses* in mediate reasoning. We reach our knowledge of conclusions through and from our knowledge of premisses. Where and how do we get our knowledge of these premisses, especially of *universal* premisses, and of the universal *axioms* of mediate reasoning? Do we not *derive* these somehow or other from our knowledge of the *particular facts* of our sense experience? Does not *all* our knowledge *ultimately* originate in sense experience—according to the maxim: *Nihil est in intellectu quod prius non fuerit in sensu?* And does not this experience bring us into contact with particular facts alone, not with universal truths? In which case, all our reasoning processes would inevitably *originate* in a knowledge of "particular" facts, from which, then, our "universal" truths would be "inferred"?

These questions, which will be answered in due course, mark the transition of our investigation from the more *formal* to the more *material* aspect of our thinking processes. We suggest them here only in order to call attention to certain erroneous views of the empiricist school of philosophers on the nature of the *universal judgment,* its *origin,* its *function* in reasoning, and, consequently, on *the nature of the mental process of reasoning itself.* Those errors, originating with Locke and Hume, were disseminated widely by John Stuart Mill in his well-known work on Logic: more particularly in a chapter on *The Functions and Logical Value of the Syllogism,*[3] where he writes as follows:—

[1] *Cf.* WELTON, *Logic,* ii., p. 84.
[2] *Cf. infra,* 262, *b.*
[3] *Logic* bk. ii., chap. iii., 4.

"All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulæ for making more: The major premise of a syllogism, consequently, is a formula of this description: and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according* to the formula: the real logical antecedent, or premise, being the particular facts from which the general proposition was collected by induction. Those facts . . . may have been forgotten: but a record remains. . . . According to the indications of this record we draw our conclusion. . . . For this it is essential that we should read the record correctly: and the rules of the syllogism are a set of precautions to ensure our doing so."

The view here expressed—that the universal premiss of a syllogism is a mere register of an enumeration or collection of instances, and that the conclusion is inferred, not from the universal, but from the particulars from which the latter was "collected by induction" (199)—raises the question as to where exactly, in the whole mental movement from the sense perception of particular facts to the proved universal conclusions of science, the special step from known to unknown, which we call "logical inference," comes in. And an analysis of this inferential step from the known to the unknown reveals, in turn, a peculiar difficulty which has been styled the *Paradox of Inference.* Finally, the consideration of this paradox will bring us face to face with the time-worn objection to the syllogism: that the latter is not a valid process of inference at all, but always and necessarily involves the fallacy of assuming what it purports to prove: the fallacy known as *Begging the Question* (*Petitio Principii*).

We will deal first with the paradox of inference.

Logical inference must be a *conscious* passage from one or more judgments to some new judgment which was implied in the former (79, 82, 116, 147). The passage of thought, too, must be recognized to be valid: to have in the antecedent a sufficient ground for the consequent. Having distinguished between the *meaning* and the *implications* of a given propositional form (82), we saw that, owing to the difficulty of determining where mere verbal change ended and change of meaning commenced, it might be doubted whether certain processes of immediate inference really deserved the name of *inference* at all (109, 116). Hence arises the general question, applicable to all forms of logical inference, mediate and immediate: What sort or what degree of difference must there be between premisses and conclusion, in order to constitute the passage from the former to the latter a *logical inference?* The signi-

ficance of this question will be better grasped when it is pointed out that all inference involves this paradox: On the one hand, in all inference we advance to something "new," something hitherto "unknown," some "new truth," some item of knowledge "additional" to what is given us in the premisses: otherwise there would be no progress, no advance of thought at all, no inference. But on the other hand, since in all logical inference the conclusion *follows necessarily* from the premisses, it must be somehow or other already contained in those premisses: otherwise we could not get it out of them. Now, these two characteristics of inference—the "novelty" of the conclusion as compared with the premisses, and the "necessity" with which it follows from them—seem to be mutually incompatible.

If we interpret the former characteristic to mean that the conclusion must be something *entirely* "new" and "additional to" the premisses, do we not deny that all those inferences which we describe as "formally or logically necessary" are inferences at all: seeing that in these the conclusion is recognized to be somehow contained or involved in the premisses, so as to follow necessarily from the latter? And if we interpret the second characteristic to mean that the conclusion must *so* necessarily follow from the premisses that it must be seen to be actually and explicitly contained in them, do we not render all such "necessary" inferences absolutely worthless as means of making progress in knowledge: seeing that in all such inferences the conclusion cannot be anything "new," but must be merely a part of what was already contained in our knowledge of the premisses?

J. S. Mill, who is an exponent of the Empirical school of philosophy, seems to have adopted the extreme interpretations just suggested. For he teaches, as we have seen, that the *formal* process of thought, by which we pass *necessarily* from the premisses to the conclusion of the syllogism, is not a process of inference at all, but a mere record of an inference already made: that the real inference consisted in "collecting" the universal premiss of the syllogism "by induction" from previous particular facts: that the syllogism itself, if regarded as a process of inference, is always a *petitio principii*, since the conclusion which it purports to prove is already assumed, being contained in the universal premiss.

While we, on the other hand, admit, with Mill, that the first source and starting point of all our knowledge is the particular

fact of sense experience, we dissent entirely from his views on the nature of the universal judgment, on the nature of the inductive process by which we "ascend" to it from particular facts, on the function it discharges in the "descending," deductive, syllogistic process, of reaching scientific conclusions; and also as to the part of the whole process which deserves the name of "logical inference": he bestowing this title on the "ascending" or "inductive" stage alone; we, on certain steps in the "ascending" or "inductive" stage, and on the whole "descending or "deductive" stage.[1]

Beginning, then, with an examination of the paradox, we must endeavour to answer the question by which we introduced it: What sort or what degree of difference must there be between premisses and conclusion, in order to constitute the passage from the former to the latter a *logical inference?* A mere verbal difference is certainly not sufficient to constitute a logical inference. There must be a difference in *meaning*, *i.e.* the conclusion must contain as (part, at least, of) its *meaning*, some element which was no part of the *meaning* of the premiss or premisses, but which was, nevertheless, an *implication* of the latter. Now, each of these elements of the judgment—meaning and implication—is, in the first place, *mental* or *subjective*, but is also *real* or *objective*, inasmuch as the judgment is a subjective representation or interpretation of some objective reality. It is, however, an *inadequate* representation of the latter; and it is just precisely because this is so that inference is possible (82, 116). It is because we can distinguish between the subjective, conventional, and objective intension (31) of the concepts which enter into our judgments, that we can also distinguish between what we may call the *individual psychological content*, the *current logical meaning*, and the *sum-total of the logical implications* of these judgments themselves.[2]

Now, there can be no doubt that the mental formulation of a given judgment or judgments brings into consciousness a certain psychological content which varies from one individual to another; that out of this a certain definite portion is selected and fixed so as to constitute the "*meaning*" of those judgments; and that, finally, an individual may be in conscious possession of

[1] These divergences of view will be gradually illustrated and justified in the course of our treatment of *Induction*.

[2] *Cf.* KEYNES, *op. cit.*, p. 421.

this meaning, and of the knowledge it conveys to him about reality, *without being in conscious possession of a further fund of knowledge*, which is nevertheless *obtainable* from what he already possesses about that reality, if he only brings the analytic and synthetic activity of his reason to bear on what he already possesses, and on the reality it reveals to him: if he only analyses further the extension and intension of the concepts which constitute the judgments he has already formed: if he only compares and relates mentally the objects—or aspects of reality—which those various concepts reveal to him: if he only reverts again and again to the data which his external and internal senses have offered to his intellect about the reality: these data furnishing the activity of his intellect with grounds and foundations for the establishment of *new* intellectual relations between the concepts which he has already formed and already compared with one another in *antecedent* judgments, in other words, with grounds for *new judgments* about these data. And, this being undoubtedly the case, we can see that the possibility, or rather the fact, of inference, though it presents at first sight an appearance of contradiction for which it has been called a "paradox," contains nothing that is really contradictory.

For, when we say that the conclusion, compared with the "known" premisses, contains a "new," "additional," "unknown" truth, we do not at all mean that this latter is so totally independent of, and disconnected with, the former, as to be utterly underivable from it by any activity of the mind: if it were, all advance in knowledge by way of inference would be impossible. We mean simply that it is "new," "additional," "unknown," etc., in so far as it is not yet *actually* possessed in consciousness, as actual knowledge, by the mind. This, however, is in no way incompatible with its being *potentially*, *virtually*, latent in our knowledge of the premisses, *i.e.* contained in them in such a way that, by the activity of the mind, by the exercise of our mental energy ("*virtute*" *mentis*), on these premisses, it can be drawn out from its latent, potential state, into the form of explicit, actual knowledge.

And, similarly, when we say that in the process of inference the conclusion must *follow necessarily* from the premisses, we do not by any means commit ourselves to the view that before the process of inference takes place, before that mental work in which the process of inference consists, has commenced, the conclusion

must be *actually* in the premisses, *i.e.* actually seen by the mind which knows the premisses. Of course, the conclusion must be "in" the premisses *somehow;* otherwise we could not get it out of them. But *how* must it be in them? It must not be in them *actually:* if it were, inference would be superfluous. Therefore, it must be in them *latently*, *virtually*, as a *necessary implication*, *i.e.* in such a way that it will be necessarily and inevitably brought to light, discovered, made an item of actual knowledge, by the mind which concentrates its energies long enough and keenly enough on the *partial* view of reality which it already possesses in its knowlege of those premisses.

But it is here precisely—in this scrutinizing, comparing, arranging, and analysing, of the various judgments which embody the objects of our knowledge, so as to discover and establish new and fertile and instructive relations between these objects, in other words, so as to enunciate *new judgments*, and discover *new truths*—that the whole difficulty of making progress in knowledge lies, and that the genius of the master-mind will reveal itself by making remarkable headway.

We must not be misled, by the trite and easy examples [199, (12)] of inference with which text-books of logic abound, into believing that real, first-hand inference, is a trivial factor in the growth of knowledge;[1] nor must we forget that there is no comparison between the student's effort to assimilate *already accomplished results*, in his study of the various sciences, and the long and arduous labours by which these results were for the first time achieved. It is easy to follow when the path is broken. For the millions of minds that are capable of assimilating all the known truths of geometry and mathematics on being taught these, how few, comparatively, are the minds that in the course of the world's history gradually accumulated that vast treasure of knowledge, by discovering for the first time the individual truths which compose it?

[1] "If our reasoning processes were carried on with the continuity and intricacy displayed in mathematics we should soon have obvious proof over what a distance we may have advanced by a succession of such apparently insignificant steps. Everyone who has studied mathematics must have experienced a feeling of surprise at times in finding how far he has been carried on in this way. He starts with a premise which it may take some trouble to distinguish from a pure identity, and finds that, starting from this, he may be imperceptibly led on by intuitively obvious advances into some profound and far-reaching algebraical formula."—VENN, *Empirical Logic*, p. 377.

We are now in a position to examine explicitly a difficulty which is at least as old as Sextus Empiricus (A.D. 200), but which owes its currency in modern times to John Stuart Mill: the objection that every syllogism, if regarded as a means of proof, involves the fallacy of *Petitio Principii*.

195. THE SYLLOGISM AND THE "PETITIO PRINCIPII".—The fallacy known as *Petitio Principii*, or *Begging the Question*, or *Arguing in a Circle* (the "*Circulus Vitiosus*"), is a fallacy that is liable to be committed in attempting to *prove* the truth of a given judgment. It consists in assuming, in one or other of many possible ways, in the premisses of our demonstrative syllogism, and utilizing for our proof, a knowledge either of the conclusion itself which we want to prove, or of some other judgment whose truth will be admitted only through a knowledge of this conclusion, and as an inference from the latter. The way in which the fallacy is alleged to be committed in the syllogism—the only form of the fallacy which concerns us here—is the following: Not only is the truth of the conclusion really and *objectively* involved in the truth of the premisses, in the sense that the conclusion would not be true unless the premisses were true; this is the case in every formally valid syllogism: if the conclusion were false the premisses would be false (148): this, however, does not involve any fallacy whatsoever; but it is further alleged that *in no case can we know the premisses to be true unless we already know that the conclusion is true, i.e.* unless the conclusion is *subjectively* in the premisses; or, *in no case can we establish or prove the premisses unless the conclusion has been already established or proved;* or, *in no case can we establish the universal premiss without simultaneously, and from exactly the same source, establishing the conclusion: so that the syllogism never actually* proves *its conclusion, but is a mere reminder that the latter was already established in the process of establishing the universal premiss.*

We have now to inquire whether these allegations are always and necessarily true of all syllogisms. Whether a given syllogism involves the fallacy in question or not, will evidently depend on the source from which we derive our knowledge of the premisses—especially of the universal premiss, which, therefore, is the only one we need consider here. If we cannot obtain this knowledge independently of the conclusion we commit the fallacy; otherwise we do not.

Mill's contention is that we can *never* reach a knowledge of

the universal premiss without having encountered and examined the conclusion in that very process.

Now *if that were so*, if it were always necessary to meet and examine every individual case under a universal principle before formulating this principle and assenting to it with certitude, then, undoubtedly, every syllogism *would be* a *petitio principii*. Having *such a "universal"* as its major premiss, and some particular case under this "*universal*" as conclusion, it would clearly be a *petitio principii*, if put forward as a proof of this conclusion. In fact, such a syllogism would not be a real inference at all.

There *are* cases in which "universal" judgments are formed in the way indicated, by a complete enumeration of instances; and it will be freely admitted that wherever our universal premiss *is* a mere *collective* universal (92, *a*, 1) comprising a concrete, limited class, and is reached by an *enumeration* of all the particular facts or instances that constitute it, while our conclusion refers to one or more of these facts or instances, though we have indeed the form of a syllogism, we have neither *proof* nor *inference*.[1] Take the following examples of apparent syllogisms with *enumerative* universals as major premisses: "*All the apostles were Jews; Judas was an Apostle; therefore he was a Jew*". "*All the books on this shelf treat of logic; this is one of them; therefore it treats of logic*". "*All the known planets move in elliptical orbits around the sun; the earth is one of the known planets; therefore it moves in an elliptical orbit around the sun*". "*All the days of the week are called after pagan deities; Monday is a day of the week; therefore it is called after a pagan deity*". These have the outward form of syllogisms. They are in fact not inferences at all; and, if advanced as *proofs* of their respective conclusions, they *do* involve the fallacy of *petitio principii*: the enumerative majors could not be known to be true unless the conclusions were antecedently or simultaneously known to be true: for in each case it was by an examination of all the individual instances (including the conclusion) that the truth about the whole concrete collection was reached.

[1] A collective universal major may afford a real inference to one who has learned its truth otherwise than by an enumeration of its members. The minor, in such a case, gives real information not contained in his knowledge of the major: "If I learn that the vessel XY was lost at sea with all on board, and learn subsequently, or by some other means, that my friend AB was a passenger on that vessel, then there is no doubt that the conclusion is 'something new,' although the major states a mere collective fact, which (for those who know, but not for me) already contains the conclusion."—MELLONE, *Logic*, p. 231.

From a consideration of those examples we shall be able to determine whether the so called "syllogism" with a collective major is typical of the ordinary syllogism. Is the concrete, collective, enumerative judgment, typical of the ordinary universal? Or, is the only way of establishing a universal judgment, and of reaching certitude about its truth, the method of complete enumeration of all the instances that constitute its extension? Mill's attitude would suggest an affirmative answer to these questions; but, surely, the correct answer to them is the negative one. The concrete, enumerative universal, is *not* the real, genuine universal judgment at all; for this latter is *abstract* and *necessary*, and is grounded not on any enumeration of instances, but on an analysis of the nature and properties of the object which it represents in thought. The examples given above are *not typical* of the *real syllogism;* for this latter must contain as one of its premisses, not a mere collective judgment reached by an actual and exhaustive enumeration of instances, but a judgment which announces some *abstract* principle, or law, or truth, which has been reached otherwise than by enumeration, and which is seen to apply *necessarily*, and therefore always, to all actual and possible instances under it.[1] In regard to such *genuine* syllogisms, the question, therefore, is, whether the abstract, universal premiss can be established, assented to, and formulated with certitude, independently of any enumeration of instances. Now, *Mill's fundamental error*—an error common to the whole Empirical school of philosophers—*was to assume that we can never be certain of the universal unless we are already certain of all the individual instances under it:* mistaking the unimportant collective judgment for the genuine abstract universal.

"When we say, he writes,[2] *All men are mortal, Socrates is a man, therefore Socrates is mortal;* it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition, *Socrates is mortal* is presupposed in the more general assumption, *All men are mortal:* that *we cannot be assured of the mortality of all men, unless we are already certain of the mortality of every individual man.*"

[1] The universal premiss of the syllogism would therefore be more suitably expressed in the abstract form "*P as such is Q*," or "*It is the nature of P to be Q*," or "*Whatever is P is Q*," or in the conditional form "*If anything is P it is Q*," rather than in the enumerative categorical form "*All P's are Q's*".—KEYNES, p. 427, and n. 1; *cf. supra*, 161.

[2] *Logic*, ii., c. 3, § 2 (italics ours).

But the objection is not unanswerable. The simple answer to it is that we *can* "be assured of the mortality of all men" *without* being "already certain of the mortality of every individual man" —including Socrates. And if we can, the objection evidently falls to the ground. The proposition "*All men are mortal*" does not express a mere collective judgment: it is the *denotative* or *extensive* expression of the strictly abstract judgment, "Man, *as such*, is mortal". This judgment embodies a necessary truth: that the *nature* of man, *being composite*, is *subject to dissolution* of its component parts, *i.e* subject to death, *mortal* (87). And we attain to certitude about this abstract truth, not by any inference *from* the particular instances, but by an analytic examination of the nature of man: for which, of course, the study of *some* instances is necessary. Then, having reached the abstract truth that "Man *as such*, is, *of his nature*, mortal," we immediately universalize it into "*All men* [actual and possible, past, present, and future] *are mortal*"; and we do so quite independently of any information about unexamined individuals.

196. SOME CLASSES OF SYLLOGISMS EXAMINED.—A syllogism may, of course, commit the fallacy of *Petitio Principii* in many other ways than by having a collective major: rarely, however, so openly as in this example given by Dr. Keynes:[1] *All M is P* (for *all S is P* and *all M is S*); and *all S is M*; therefore *all S is P*. But in the same context the author states that we have the fallacy of *petitio principii* whenever "the major premiss is an analytic proposition". If the "analytic proposition" be understood in the sense in which we have accepted it (85-88), *i.e.* as a proposition *in materia necessaria*, this statement is not true: on the contrary, it is only in so far as the universal premiss is *abstract* and *necessary*, and in proportion to the degree of that necessity (*metaphysical*, *physical*, or *moral*), that the syllogism will conclude *cogently*, and be exempt from danger of invalidity through the possible occurrence of an exception to the general rule in the case or cases to which the syllogism seeks to apply the rule. And, similarly, our certitude about the conclusion will never be higher than our certitude about the universal premiss. We may distinguish three classes of cases. (*a*) "The major premiss may itself be accepted as axiomatic, or it may be deducible (without the aid of the conclusion) from more ultimate principles that are accepted as axiomatic."[2] These premisses we would de-

[1] *op. cit.*, p. 426. [2] *ibid.*, p. 427.

scribe as *analytic*, or *in materia necessaria;* they are common in mathematics; and they do not involve the reasonings dependent on them in any *petitio principii*. They give us *absolute* or *metaphysical* certitude about their conclusions. (*b*) "The major premiss may be an imperfect (*i.e.* scientific) induction, based on evidence that does not include the conclusion"[1] The general laws or truths which we reach by scientific induction (wrongly called "imperfect"—*cf.* 207) can become major premisses of syllogisms which apply these truths to explain particular facts brought under them. There is here no *petitio principii:* the minor and conclusion disclose these facts for the first time to the mind: hence they could not have been the basis of our knowledge of the major. Such syllogisms give us not metaphysical or absolute, but *physical* certitude, *conditional* on the stability and uniformity of the natural phenomena with which they deal. (*c*) "The major premiss may be based on authority, or may be accepted on testimony; or it may be the expression of a civil law, or of a command, or of a rule of conduct; and in none of these cases can it be in any degree grounded on the conclusion."[2] In the wide domains of the social, political, and economic sciences, of history and jurisprudence, of religion, natural and supernatural, the reasoning employed is, for the most part, syllogistic inference from general principles, maxims, laws, etc., which are either deliverances of authority, or generalizations of *moral* universality and necessity—from the observed course of human conduct. These are not *collective*, but *abstract*, universals. Having to do with phenomena within the domain of free will, they are, of course, less stable and more liable to exception than in the preceding cases: and since it is on their stability that the conclusions derived from them are based, we can have only *moral* certitude about the latter.

Were we to understand by an analytic proposition one in which the predicate gives us the whole or part of the definition or connotation of the subject-term (85), we might inquire whether a syllogism containing such a proposition as major premiss is necessarily a *petitio principii*. Dr. Keynes says it is: "For if *M* by *definition* includes *P* amongst its properties, I am not justified in saying of *S* that it is *M* unless I have already satisfied myself that it is *P*. The following is an example: *All triangles have three sides; the figure ABC is a triangle; therefore it has three sides*".[3] Here there is apparently no escape from the *petitio principii:* I cannot know that *ABC* is a triangle unless by

[1] *op. cit.*, p. 427. [2] *ibid.*, p. 428. *Cf.* MERCIER, *Logique*, pp. 196-200.
[3] *ibid.*, p. 426.

knowing that it has three sides: it must be three-sided in order to be a triangle. The fallacy is here committed *not in the major, but in the minor;* and it is committed when the major is a merely *verbal* proposition, a proposition which gives some attribute that belongs to the connotation of its subject-term (*M*) and in the absence of which from any object (*S*) this latter would not receive the class name (*M*) or be admitted into the class. We have no other way of determining the truth of the minor—"*S is M*"—than by seeing whether it has all the attributes (*including P*, therefore) the possession of which entitle it to be called *M*.

If, then, our major is a mere nominal definition (complete or partial) of our middle term, we commit the fallacy in the minor premiss. If, however, *P* is a property, or an accident, of *M*, so that we can know that *S is M* without knowing whether *S is P* or not, we escape the fallacy. It is sometimes urged that there is no real subsumption, or bringing of *S* under the condition of a rule, as the first figure demands (168), whenever the major "states a connexion seen to be necessary between *P* and *M* as such . . . [because] in this case no one can judge that *S* is *M* without *eo ipso* recognizing it to be *P* as well."[1] This indeed, is so, and it is in this recognition precisely that the force of the syllogism lies. And no one accuses the syllogism of *petitio principii* on this score. But neither can we deny that there is a real subsumption of a case under a rule. It matters not that the rule is here apprehended *as embodied in the case*, instead of being applied *as an extrinsic and independent rule*. This point will come up again in connexion with *Demonstration* (258).

We must next inquire whether the fallacy is committed in the syllogism—much more commonly met with—in which the major premiss is the *converse of a definition*, or of part of a definition; the middle term being, not the subject defined (the *res definienda*), but either the whole or part of the *definition* of the latter? For example, is this syllogism a *petitio principii*:—

All three-sided figures are triangles;
ABC is a three-sided figure;
therefore, *ABC is a triangle?*

It is not a *petitio principii:* for the major is the expression of a strict, abstract, universal judgment, of this general form: "Whatever is conceived to have certain attributes is a thing of a certain nature or kind or class"; and the minor is discovered or known independently of the conclusion. It must, however, be admitted that this type of syllogism, considered in itself, expresses an inferential step which is exceedingly small in comparison with the mental labour needed to establish the minor premiss. An example of the following kind will illustrate this better:—

All substances which have modes of operation and a mode of existence independent of, and higher than, those of matter, are spiritual substances;
The human soul has such modes of operation and existence;
Therefore, the human soul is a spiritual substance.

This syllogism is typical of quite an extensive class of arguments familiar to students of philosophy. It would be childish, of course, to put forward such a syllogism as embodying in itself a complete proof of the spirituality of the human soul. It merely gives the one final descending (inferential) step to the desired conclusion, from the commanding position we

[1] JOSEPH, *op. cit.*, p. 307—with other symbols.

are supposed to have reached by having established the minor premiss. None of those condensed syllogistic proofs, which we meet in philosophy, purport to give us anything more than this final step. The previous work of establishing the minor premiss is the most laborious part of the whole process. And it is from an impartial estimate of the value of the evidence brought forward in support of such minor premisses that we must weigh the worth of such proofs.

The consideration of a major premiss which is a *definition*, or its converse, will at once show the importance of a question already raised in connexion with *Definition:* Does a definition necessarily imply the *existence* of the objects denoted by the term defined? (54). What, for instance, would be the attitude of one who denied or doubted the spirituality of the soul, towards the major premiss of the syllogism given above? He would, presumably, admit and accept it as an intensive *nominal* definition of what himself and his opponent understood by "spiritual substance," and would then immediately question *its existential import; i.e.* he would not admit (without proof) that there are *de facto* any such things in existence at all as "spiritual substances" —any objects to correspond to this concept. Or, possibly, admitting that there are substances endowed with such properties, he might deny the minor —that the human soul is endowed with these properties.

197. THE APPREHENSION OF THE UNIVERSAL JUDGMENT AND ITS APPLICATION.—It will be opportune here to compare the mental process by which we "ascend" from particular facts to universal truths with the process by which we "apply" the latter to cases brought under them, and draw out by inference other truths involved in them. The mental labour will be found to lie not exclusively on the "ascending" or "inductive" side.

The truth embodied in the universal premiss "All men are mortal," to which we referred above, is one which lies on the border line (85) between those more abstract and evident analytic truths which are got by a simple process of intuitive abstraction and universalization, such as, for example, "The whole is greater than its part," "Virtue is praiseworthy," "Whatever happens has a cause,"—and the generalizations which we reach only after long and laborious processes of observation, experiment, hypothesis, and verification, processes comprised under the general title of *Induction*,—as, for example, "Water reaches its maximum density at four degrees centigrade," "All bodies in the universe tend to move towards one another with an acceleration which varies directly as the product of their masses and inversely as the squares of their respective distances asunder".

The nature of the *Induction*, by which so many of our general truths are established, will occupy our attention later on (*cf.* 212). It is a process by which we start from particular facts and

reach general laws. The whole process, though it involves deductive, hypothetical, and disjunctive reasoning, is not itself a logical inference "from particular to general". It is essentially a process of *abstraction* and *generalization*, *i.e.* of *conception* and *judgment*—aided by complex and laborious subsidiary processes of observation and experiment.[1] Although, therefore, we admit, with Mill, that our knowledge of the general truths established by induction, as indeed our knowledge of all general truths whatsoever, starts from the particular facts of sense experience, we deny that this ascent from particular facts to general truths can be properly described as a logical inference (198). We admit freely, however, that in most cases the inductive ascent from the particular facts, by way of observation, hypothesis, and experiment, to the abstract law, is much more laborious than the mere formal generalization of this abstract law, and the subsequent application of it, by way of syllogistic inference, to new instances brought under it.[2]

It is not true, however, to say that once the abstract, general law or truth is established by induction, all the real mental work is over. It would be, indeed, if the law in question were a mere collective judgment, for then all the individual instances *would have been already known*, and the minor premisses which would bring them under the collective majors, would be mere records of work already done. This, in fact, is a difficulty against Mill's view of the syllogism, and one he has not succeeded in answering: that, in his view, the minor premiss would be useless and superfluous;[3] whereas in every real syllogism it is not only useful but necessary. And why? Because when the universal major is established, all the mental work is *not* over: the general

[1] *Cf.* JOYCE, *Logic*, p. 217. "This process is not ratiocinative. We do not argue from premisses to conclusion." *Cf. infra*, 212.

[2] *Cf.* VENN, *Empirical Logic*, p. 377. "There is a great deal of labour and insight required perhaps for the acquirement of our major premise in the form in which we can employ it, and then there is a single almost instantaneous step of inappreciably small advance."

[3] Mill's reply, that the major indicates individuals only by marks and that the function of the minor is to identify an individual by comparison with those marks, only shows the impossibility of his own interpretation of the major. If the latter is a judgment about a *collection*, what right have we to assert it until we have examined the minor and put it into the collection? What right have we to *assert* what we do not *know*? "I do not say that a person who affirmed, before the Duke of Wellington was born, that all men are mortal, *knew* that the Duke of Wellington was mortal; but I do say that he *asserted* it" (*Logic*, bk. ii., chap. iii., § 2, note). The "assertion" of an *unknown fact* is not an assertion of a judgment at all.

principle announced in the major is abstract: "*M as such is P*". It has then to be *formally* universalized, to "*All M's are P*,"—a process which is easy enough, being an almost spontaneous mental consequence of the formation of the abstract judgment "*M as such is P*". But, furthermore, it has to be universalized *materially*,[1] by discovering among the actual and possible data of human knowledge all the things or events which are instances of that law, all the *S*'s that partake of the nature of *M*, and that can, therefore, be brought into the class *M*. Now, this very process of recognizing in something (*S*), beneath a whole mass of irrelevant or differentiating details, the essential attributes which make it *M*, and the possession of which enables us to bring it into the class *M*, and thereby to predicate *P* of it also—this very process is one which, no less than the establishment of the abstract law itself, calls for mental energy, ability, and often genius of the highest order (197). The world had to wait a long time for Franklin to enrich it with this piece of reasoning, which, like all great things, appears so simple: "The sort of spark produced by the electrical machine is due to electricity; a flash of lightning perhaps is a spark of the same sort. Therefore lightning is, perhaps, due to electricity." So, too, the general abstract truth that "The force of expanding steam (*M*) is capable of causing motion (*P*)," was long enough known before the genius of Papin and Watt led to the invention of a special instance of the force of expanding steam, namely, the steam-engine (*S*), for the purpose of obtaining motion (*P*) from it.[2]

Again, the general elementary axioms of the mathematical sciences—the sciences of *magnitude* and *multitude*—are the common endowment of all ordinary minds; and yet, what prolonged and intense application it has cost the keenest of minds to bring to light the more and more complex thought-objects—the new "*S's*"—involved in the original, elementary concepts: and to establish, between these, *new* relations, which, nevertheless, were *virtually* contained in the simpler primary ones.

We must be careful, then, not to confound two distinct states or conditions in which we may regard the premisses of any mediate inference. We may look at them in the *remoter* state of *unformed materials*—concepts lying unanalysed and uncompared in a mind that holds them passively. These materials,

[1] *Cf.* MAHER, *Psychology*, pp. 295-6, note 4 (4th edition).
[2] *Cf.* MERCIER, *Logique*, p. 193.

while in this state, do not *actually* contain the conclusion among them; they contain it *virtually:* which means simply that the mind has the *power,* whether it chooses to exert it or not, of so analysing, arranging, comparing these materials, that, of themselves, and without calling in the aid of any extraneous factor, they will *necessarily* yield the conclusion (197). It is in doing this that the labour and the progress of inference lie. Let us suppose this work done. The materials are now in an *ordered* or *formed* state, more *proximate* to the desired conclusion than in the previous state; for now they are held together in consciousness in the form of *two judgments of such a character that the mind* which consciously formulates them *cannot help seeing a third judgment involved in them.* In this state they contain the conclusion not virtually but *actually;* but when this state is reached, *the mind has done its work, the act of inference is already completed.*

It is quite possible, therefore, for the mind to be in possession of the materials which yield the conclusion, without knowing the conclusion itself. No one will deny this possibility unless he fails to distinguish between the objective reality itself, the objective things, facts, events, on the one hand, and the mind's subjective insight into them, on the other. In the objective reality itself, the truth of the conclusion is simultaneous with, and just as actual as, the truth of the premisses; all the truth which the mind can gather about the objective reality is there in it, whether the mind gathers it or not; the facts are all there together: but they need not, therefore, be all *known* together: our mental insight into the whole objective reality may be, and is, only partial; and it can therefore grow, and does grow, by our analysing, examining, modifying, the partial knowledge, the undeveloped materials—concepts and judgments,—which our minds already possess. It is only the failure to grasp this easy distinction between *subjective* knowledge and *objective* reality that can have permitted some authors to deny the possibility of our knowing the logical premisses of a conclusion without knowing the conclusion itself.[1]

198. Logical Grounds and Ultimate Sources of Inferred Conclusions.—Since we have admitted, with Mill (194), that all our knowledge, and, therefore, all the conclusions of all our syllogisms, have their *ultimate* origin in our knowledge of the particular facts of sense experience, it may, perhaps, still be

[1] *Cf.* Venn, *op. cit.*, p. 375.

objected that in every syllogistic reasoning the *real ground* of our conclusion, the *real evidence* by which it is supported, is not to be found in the universal premiss of the syllogism, but lies rather in those particular facts of sense experience which are the source from which the universal premiss was derived : so that, after all, it might fairly be contended that the conclusion of the syllogism is established not so much *by means of* the universal, as *in* and *with* the latter, and by means of the self-same facts from which we derive the latter : this being, therefore, rather a record of an inference completed, than a starting-point for an inference that is still to be accomplished ?

In answer to this, let us *first* point out that it does not involve the syllogism in any *petitio principii* to hold that the particular facts of sense experience are the *ultimate source* both of its premisses and of its conclusions, provided we insist that the universal premiss is not a mere enumeration of those facts, and the conclusion itself necessarily one of them. Then, *secondly*, the process by which the universal is got from the particulars is not a process of *logical inference* at all, but a process of intellectual abstraction and generalization which is sometimes a simple intuition, as in the case of self-evident mathematical axioms, while it is sometimes much more complex and laborious, as in the discovery of laws of physical causation by means of induction (197).[1] *Thirdly*, it is proper to distinguish between the *logical grounds* or *evidence* for the conclusion of a logical inference, and the ultimate *psychological* and *metaphysical motives* for our accepting and assenting to those logical grounds themselves.[2] And, while insisting that the *logical* evidence for the conclusion of a syllogism lies partly in its major and partly in its minor premiss, we may freely admit that the mind could never have come into possession of either premiss without facts of sense-experience from which to abstract its concepts. In this sense, particular facts form the *ultimate sources*,—not, however, the *logical grounds* or *evidence*,—of the conclusions of our syllogisms :[3] but in this passage from particular fact to

[1] Induction, of course, involves inference in various ways. Although the *apprehension* of an abstract law of causation, by way of hypothesis, is not an inference, yet the process of *verifying* it by eliminating all alternative hypotheses, may be set forth in the form of a mixed alternative syllogism. *Cf. infra*, 209, 212 ; JOSEPH, *op. cit.*, pp. 405 *sqq.*

[2] *Cf.* VENN, *Empirical Logic*, pp. 124-9, where he discusses the grounds of our belief in the *Uniformity of Nature.*

[3] *ibid.*, pp. 377-9, where the author discusses this whole question from the point of view of induction.

particular conclusion there always intervenes some universal principle which is not merely the *record* or *register* of a *past* inference, but rather the *logical ground* of an *actual* one.

Behind these universal axioms and premisses, which form real starting points of our conscious, logical inferences, the *Logic of Inference* has neither the right nor the duty to go, as Mill does, "in order to inquire into their grounds: the customary logical process of taking them for granted, and starting from them as the origin of our reasoning, is quite consistent . . . no general proposition can be a true *ultimate* starting point; . . . [but] a theory of the syllogism which requires for its explanation and justification that the full account of it should, so to say, straggle over our whole life, if we are to find scope for both its premises and its conclusion, is surely unfitted for the purpose of logic. . . . Mill's explanation . . . seems . . . to be a transgression into the province of psychology; an attempt to determine the *ultimate* sources of knowledge."[1] An unsuccessful and misleading attempt it is too; for, though Mill is right when he places the ultimate source of all our knowledge in sense experience, he entirely misinterprets the nature and function of the higher cognitive faculty of the mind—the intellect or reason—when he teaches that it can merely sum up the individual experiences of sense into collective judgments.[2]

199. SUMMARY OF CHAPTER.—We may now conveniently summarize the main conclusions we have reached in the foregoing discussions.

(1) If we understand by the "syllogism" that form of inference which is usually analysed into moods, figures, etc., in text-books of logic, it would appear that there are other forms of mediate inference which cannot be reduced to it.

(2) These, no less than the syllogism, have their *formal* and their *material* sides; and also their own mediate axioms, which are both *conceptual* and *real*.

(3) The process by which the conclusion is reached, by the application of the axiom, may *in all cases* be expressed in the form of a syllogism proper.

(4) Syllogistic inference is described as "deductive," because it compares particular cases with a general rule or principle, in order to see if they fulfil the conditions of that rule or principle.

(5) It is not possible to reason *syllogistically* from two indefinite premisses, or even from premisses one or both of which are universal, *without the aid of some underlying universal axiom*, of which the given syllogism is a partial embodiment.

(6) It is not possible to reason *mediately* from one, or from a definite number, or from an indefinite multitude, of individual

[1] *Cf.* VENN, *op. cit.*, pp. 377-9. [2] *Cf.* MELLONE, *op. cit.*, p. 385.

facts—embodied in indefinite (particular) or definite (singular or general) judgments—to another or other facts, or to universal conclusions embodying those facts, *without invoking mentally some universal principle which connects all the facts*, and *some self-evident axiom*, of which the inference in question is an application.

(7) The universal judgment, which serves as axiom of a mediate reasoning, or as universal premiss of a syllogism, is not a mere "concrete" or "collective" universal which sums up facts of sense experience.

(8) The syllogism is itself a real process of inference: not a mere record of the mental process by which its universal premiss was reached.

(9) This latter process is not simply a *logical inference;* it is a process of *abstraction* and *generalization*, *i.e.* the formation of abstract and universal *concepts* and *judgments;* sometimes subserved by observation, experiment, analysis, hypothesis and inference: in which cases it is called "induction".

(10) The conclusion of every real inference must be a *new* truth, *other* than the premisses; and yet it must be *necessarily involved* in those premisses. This may be called a paradox, but the two statements are not really incompatible.

(11) *Objectively*, the reality revealed in the truth of the conclusion is not posterior to, and consequent on, that revealed in the truth of the premisses, but coexistent with, and implied in, the latter. *Subjectively*, however, the *knowledge* of the conclusion is not *actually* present in the knowledge of the premisses, but only *virtually*—so as to be necessarily derivable from this latter by the exercise of the mental activity called "reasoning".

(12) The mistaken notion that the step from premisses to conclusion, in a mediate inference, is trivial and unimportant, is fostered by the simple and self-evident illustrations we meet with in the logical treatment of the syllogism; by forgetfulness of the fact that all the mathematical and purely deductive sciences have been built up by such inferences; and of the fact that it is much easier for us to follow a line of reasoning along which others conduct us than it was for them to discover it for the first time.

(13) The mistaken notion that the syllogism can *prove* nothing, but is always and necessarily a *petitio principii*, rests upon a deeper error about the *nature* and *origin* of the universal premiss: the error, namely, that this premiss is a *collective* judgment, and is

reached *only through an actual enumeration of instances, including the conclusion.*

(14) While our knowledge of the universal premiss of the syllogism is always *ultimately* based on sense experience of particular facts, this premiss itself is a *proximate, logical* ground of the conclusion; nor is the derivation of the latter from it a *petitio principii* so long as "it is admitted that the conclusion does not itself constitute any of the data [or *facts*] from which the [universal] premiss is obtained".[1]

(15) The universal premiss of the syllogism may be either a *metaphysically*, a *physically*, or a *morally* "necessary," proposition: it may be gathered from the individual data of sense experience, not by an enumeration of the latter (7), but in a variety of other ways, for example, (*a*) by direct intellectual *Intuition*, as in axioms, (*b*) by *Induction*, as in physical laws and in generalizations from human conduct in the social sciences, (*c*) by *Authority*, human or divine, as in the sciences of law and theology.

WELTON, *op. cit.*, bk. iv., chap. vii. KEYNES, *op. cit.*, pp. 385-8, 413-24. JOSEPH, *op. cit.*, pp. 225 *sqq.*, 278 *sqq.* JOYCE, *Logic*, pp. 190 *sqq.* MILL, *Logic*, ii., chap. iii. MELLONE, *op. cit.*, pp. 226-39. VENN, *Empirical Logic*, chap. xv. MERCIER, *Logique*, pp. 188-204.

[1] KEYNES, *op. cit.*, p. 429.

QUESTIONS AND EXERCISES.

INTRODUCTION.

CHAP. I.—Enumerate man's faculties of knowledge. How do acts of the senses differ from acts of the intellect? Enumerate the latter. What are the respective characteristics of an object of sense and an object of intellect? Is the object of the universal idea contained in the things of sense? In what state is it conceived by the intellect? Why is it true to say "John is a *man*," and false to say "John is *human nature*"? Do our universal ideas represent things truly and adequately? What do you understand by the "*Problem of the Universals*"? Give an outline of some of the principal attempts to solve it.

CHAP. II.—What is the aim of logic? its formal object? its material object? Is logic a science or an art? The "*Ars Artium*"? *Logica Docens*? *Logica Utens*? In what senses has the distinction between "formal" and "material" logic been understood? Distinguish between conception, judgment, and reasoning. Explain the main divisions of logical science. Enumerate and explain the "Laws of Thought". Whence do they derive their necessity and universality? Explain the connexion between logic and criteriology.

CHAP. III.—With what sciences is logic most closely connected? What do you understand by *ens rationis*? *prima intentio mentis*? *secunda intentio mentis*? genetic laws, and normative laws, of thought? What are the functions of language? Define categorematic and syncategorematic words, logical term, name, *suppositio materialis*. How does logic classify the parts of speech? State and discuss some definitions of logic. What special advantages are claimed for the study of logic? Name Aristotle's logical treatises. What have been the main directions of the development of logic in mediaeval and in modern times?

PART I.

CHAP. I.—In what sense does logic treat of names and terms, and of things? To what does the logical term refer—concepts or things? Give examples of equivocal terms. Explain and illustrate the *analogical* use of terms. Distinguish between it and the *univocal* use. Explain the division of terms into general and singular. How are the latter subdivided? What is a proper name? Are collective terms general or singular? Distinguish between the distributive and the collective use of unitary names. Classify "substantial" terms. Explain the two kinds of

meaning—*intensive* and *extensive*—involved in concepts and terms. Distinguish and name three possible degrees of *intension.* Define *connotation.* How far is it conventional? Is it purely arbitrary? Is it absolutely fixed? Distinguish between "implication" (*direct* and *indirect*) and "suggestion"; illustrate the difference. Whether is *connotation* or *denotation* fixed first? Define and illustrate *extension* or *denotation.* What do you understand by the *Realm* or *Sphere of Denotation?* by the *Universe of Discourse?* Distinguish between the *denotation* and the *application* of a term. Explain and illustrate the relative variations of intension and extension. Distinguish between an *abstract* and a *concrete* term. Are all concepts the result of abstraction? Are all *potentially* universal? How is such a concept made to represent one single individual? Distinguish between "thing" and "attribute". Is the division of terms into abstract and concrete an exhaustive division? Is it absolute, or may the same term be abstract or concrete according to the context? Have *all* terms extension? Is the division into general and singular exhaustive? Discuss extension with reference to abstract terms. Discuss intension with reference (*a*) to abstract terms, (*b*) to proper names. What are *Incompatible Terms?* Explain *material* contradiction in concepts and terms. How are terms with negative prefixes or affixes related to the original positive terms from which they were formed? What are the characteristics of formally negative terms? Compare formal with material contradictories (*a*) in connotation, (*b*) in denotation. Discuss the import of formally negative terms (*a*) apart from judgment, (*b*) in judgment. Are such terms spurious or meaningless? Does the form of negation belong properly to the concept, or to the judgment? What are contrary terms? Have all terms contraries? Compare contrary with contradictory opposition of concepts. When are concepts said to be mutually *repugnant?* What are *privative* terms? Define *relative* term. Is there a sense in which all concepts and terms might be called relative? What is a *fundamentum relationis?* What is an *absolute* concept or term? Give the logical characteristics (i.e. state whether singular or general, connotative or non-connotative, abstract or concrete, positive or negative, absolute or relative, etc.) of the following terms: "The house," "God," "Sun," "Space," "The Andes," "Iron," "Fairy," "Institution," "Parliament," "Nationhood," "A Daniel come to Judgment," "Lough Derg," "Pius X.," "Shameful," "Shameless," "Invaluable," "Lame," "Creator," "Logic".

CHAP. II.—What are the predicables? Are they *logical* or *real* entities? What are *Secundae Intentiones Mentis?* Explain Aristotle's fourfold scheme of predicables, and compare it with the fivefold scheme of Porphyry. Work out the latter scheme by examining (*a*) the intension, (*b*) the extension, of the concepts compared; and prove that the fivefold scheme is exhaustive. Compare the terms *essence* and *nature* with the term *connotation.* Distinguish between the *specific* essence and the *individual* essence. Does the name we give a thing determine its essence? Is our knowledge a knowledge of "nominal essences" merely? Define and compare *Genus* and *Species.* Explain *Summum Genus*, *Infima Species*, *Subaltern Genera and Species*, *Predicamental Line*, *Cognate Species*, *Cognate Genera.* Of what is *Porphyry's Tree* an example? Distinguish between the logical and the

biological meanings of the terms *genus* and *species*. Are there any *fixed species infimae* in the logical sense of the word *species?* In the biological sense? Does the logical subdivision of the traditional *species infimae* imply that our knowledge is limited to names and does not extend to things? Can the same *species* have various *differentiae* and various *genera?* Distinguish between generic, specific, and individual *differentiae* and *propria*. Define *specific proprium* in the stricter sense. Are *propria* and *differentiae* interchangeable? Explain the nature of the "necessary" connexion that must exist between the *proprium* and the *essence*. Is the "necessity" in question purely "formal," or is it also "real"? And, if "real," is it "physical" or "metaphysical"? Are the so-called "physical properties" of bodies, at a knowledge of which we arrive by physical induction, properties in the strict, logical sense of the word? Explain and illustrate the distinction between separable and inseparable accidents (*a*) of the individual, (*b*) of the species. Distinguish between the latter and the specific *proprium*.

CHAP. III.—When are our ideas said to be *clear* and *distinct?* What logical processes contribute to make them so? How did the Scholastics define *Definition?* What are the "*tres modi sciendi*"? Has *definition* any other function than that of making our ideas distinct? Is *definition* concerned with *names*, with *ideas*, or with *things?* How, therefore, would you propose to define *Definition?* Does definition aim at conveying *full* knowledge of the thing defined, or only knowledge sufficient to identify the latter? How far back does definition carry analysis of connotation? Why must the *genus* assigned in definition be the *genus proximum?* Can the same thing have different *genera proxima?* and consequently different definitions? Does formal logic recognize any *right* to definition, or does it rather regard the demand for a definition as *exceptional?* What influences lead to change in definition? What classes of terms, thoughts, or things cannot be defined? Are the judgments by which we *identify*, or *name*, or *classify* things, definitions? What is *Definition by Type?* Distinguish between the definitions: "A triangle *means* a three-angled figure" and "A triangle *is* a three-sided figure". Explain the various distinctions suggested by logicians between "nominal" and "real" definitions. What is the Scholastic view of the function of "nominal" definition? May the distinction be retained? What do you understand by "Verbal Disputes"? What, as a rule, is their origin? What are *etymological* definitions? *synonymous* definitions? *private* or *technical* definitions? *genetic* definitions? *distinctive explanations? descriptions?* State the *Rules of Definition*. Explain, with illustrations, the effects of violating each.

CHAP. IV.—Define logical division, enumeration, *Fundamentum Divisionis*, co-division, subdivision. What is the aim of logical division? Compare it with definition, with metaphysical analysis, with physical division, with verbal division. Why must the basis of division be always a *separable accident* of the class to be divided? Is division a *formal* or a *material* process? What is *Dichotomy?* Is it purely formal? How is it applied in symbolic logic? State the *Rules of Logical Division*. How are mutually exclusive and collectively exhaustive results secured? What is cross-division? disparate division? State the advantages and disadvantages of dichotomy. Define classification. How is it involved in the use of language? State the

problem of classification. What is "Aggregation"? On what will the formation of intermediate classes depend? Can logic suggest the proper attributes to select as grounds of classification? On what will the selection depend? Distinguish between two kinds of aim or purpose in classification, and two resulting kinds of classification. Discuss the designations "natural" and "artificial". Illustrate classification for "special purposes". Is classification the arrangement of *objects?* What is *Diagnosis?* To what end is artificial classification usually a means? How do the intermediate classes of an artificial classification compare with those of a natural classification of the same objects? What is the aim of classification for general purposes? What test or tests have been suggested for determining which attributes are "most important," as grounds of division, in a natural classification? Do we discover "resemblances" between members of a class after we have arrived at that class on the basis of a previously known "important attribute"? or do we, *vice versa*, discover the "importance" of the attribute after we have arrived at the class on the basis of "resemblances"? Can the "importance" of an attribute be determined by the number of others involved in it? Is "degree of resemblance" a sound basis for the aggregation of individuals into groups, and of these into wider groups? Is it wrong to "judge by appearances"? Why is classification on general resemblances called "Natural"? Compare the difference between natural, with that between artificial, classes. What sense do botanists and biologists attach to the terms "species" and "genus"? What do they regard as the *origin*, and what as the *test*, of a species? How has advance in knowledge modified their views? Do "natural" groups in the organic world sometimes fade away into one another? In doubt about the relative importance of points of resemblance, what will influence our decision? How has the evolution hypothesis affected the interpretation of "affinity" and "species," and the selection of "important" attributes? Define scientific nomenclature. Describe two systems. Why are the names of botanic species and varieties not definitions? What is scientific terminology? Enumerate the characteristics of a good terminology.

CHAP. V.—What are axioms or principles? What bearing have their predicates on all our knowledge? Is the classification of all possible predicates a logical or a metaphysical problem? What is the nature of Aristotle's classification of the "Categories"? Define "Category," "*Praedicamentum*". Compare the *praedicamenta* with the *praedicabilia*. Are the former a classification of *relations?* Distinguish between *substantia prima* and *substantia secunda;* between the latter and *accidens* or *attribute*. How would you reason out the tenfold scheme of Aristotle? Enumerate the categories, giving subdivisions of each. Discuss the different interpretations of the doubtful categories. Distinguish between *Quantitas*, *Ubi*, and *Quando;* between *Ubi* and *Situs*. To which category does "habit" belong? Are the categories based on the parts of speech? Refer to their appropriate categories the *substantive*, the *adjective*, the *verb*. Connect the pronoun, the adverb, the preposition, the conjunction, with the categories. Show, by an example, how the categories exhaust all the possible heads of inquiry about an individual. Find a place for mental states in the categories. Do language-roots indicate individual subjects, or abstract predicates? Are the categories classes of

things? Are they *subjective* or *objective?* a classification of the *forms*, or of the *materials* of thought? Did the Scholastics regard Aristotle's categories as an enumeration of the *suprema genera* or highest classes of existing or possible things? Are Aristotle's nine *accidentia* subordinate to *accidens* as *genus supremum*, or are they co-ordinate with *substantia?* Give Hamilton's modification of Aristotle's scheme. Explain briefly Mill's understanding of the scope of the categories. Explain the process by which he reduced them all to "Feeling". What did Kant understand by the categories? Explain his distinction between the *matter* and the *form* of thought. How many "categories" of the understanding did he enumerate? How did he arrive at his results? Indicate some of the defects of his scheme.

PART II.

CHAP. I.—Distinguish between a logical proposition, a significant term or phrase, and a grammatical sentence. Define judgment and proposition. Distinguish between subject and predicate. Explain the unity and plurality of the judgment. Compare judgment with conception, and with reasoning, in regard to truth. What does "interpretation" mean? How is the process carried on? How does "naming" involve judgment? Are concepts or terms true or false? What is meant by the "objective reference" of the judgment? Give some definitions of judgment that emphasize this reference. What is a *Criterion of Truth?* In what sense is a judgment, even about a particular matter of fact, universal? Does the truth of judgment depend on *time?* Is truth dependent on our wishes? What do you understand by the statement that logic considers the judgment and the proposition *in the abstract?* What is *formulation?* Distinguish between the import or meaning, and the implications, of a proposition. On what does the meaning of a proposition depend? Is it purely conventional? How far does logic allow us to change the wording of a proposition?

CHAP. II.—Indicate three distinct classes of problems regarding the import of judgments and propositions. From what different standpoints may judgments be classified? Indicate four main grounds of division, with the members in each class. Define each kind of judgment enumerated. Explain the distinction based on *Relation.* Enumerate the synonyms for *essential* and *accidental* propositions. Explain the nature of the distinction as understood in scholastic philosophy. Are all propositions "in necessary matter" *per se notae?* Illustrate different "modi dicendi *per se*". In what sense are these judgments "*a priori*"? Does the distinction depend on the knowledge of the individual judging? May it be interpreted to turn on the connotation of the concepts? Is it a fixed distinction? Explain the terms "Verbal" and "Real," "Explicative" and "Instructive" or "Ampliative," "*A priori*" and "*A posteriori*," "Analytic" and "Synthetic". Why did Kant describe certain judgments as "synthetic *a priori*"? What judgments are here referred to? Explain his use of the terms employed. How do Kantists and Scholastics, respectively, account for the characteristics of these judgments? Account for the terms "metaphysical" and "physical" as applied to these judgments. What do you understand by *Modality* in categorical judgments? Distinguish between *Dictum* and *Modus*, between

formal and material modality. Enumerate and define the various kinds of modal judgment. Is modality subjective or objective? What are the respective modal forms of judgments *in materia necessaria*, and judgments *in materia contingenti?* Is there an element of meaning in the judgment expressed assertorically, which is absent from the same judgment expressed modally, and *vice versa?* In what sense may the force of the *apodeictic* be described as affirmative, and that of the *problematic* as negative? Are we to interpret as *apodeictic* all categorical statements of necessity consequent on the operation of law? Explain the subjective view of modality. Can the modal forms satisfactorily express degrees of assent? Can the assertoric form be regarded as a member of any modal division? Discuss the meaning of the problematic modal proposition.

CHAP. III.—Distinguish between the *formal* and the *material* elements of the proposition. When is a term said to be "distributed," and when "undistributed," in a proposition? On the quantity of which term does the quantity of the proposition depend? Explain the fourfold scheme of propositions with their respective symbols. State, prove, and illustrate the rules for the distribution of the predicate. Define universal, general, singular, particular, collective propositions. How is the quantity of a collective determined? Enumerate all the quantitative signs of the general proposition, discussing ambiguities. Distinguish between two classes of general propositions. Discuss the modality, the origin, the scientific value, of each class. Name the judgments of each class. Can they be distinguished formally? How is the universal negative expressed? What is the essential difference between a universal and a particular? How would you classify the singular proposition? Where the subject of the *definite* singular proposition is a connotative singular term, is the *defining* word or phrase regarded as "constituting" the subject, or as "quantifying" the general term regarded as subject? Discuss the ordinary and the logical meanings of the word "some". What are the sources, and what the special functions, of the particular proposition? What, exactly, is the *limitation* implied in knowledge expressed by the form "Some S's are P," in the form "S may be P"? Determine the meaning of "most," "few," "a few". What are plurative propositions? numerically definite propositions? How are these interpreted as regards quantity? Distinguish between a "complex" and a "compound" proposition. What is an exponible proposition? How many kinds are there? Are these compound propositions? State, explain, and justify the rules for determining the quantity of an indesignate proposition. Is negation a form of the term, or of the proposition? Are "infinite" judgments distinct from both affirmative and negative judgments? Is the whole connotation of the predicate denied of the subject in the way in which it is affirmed of the latter? Is a negative judgment meaningless unless subject and predicate agree in a proximate genus? In what sense does denial presuppose affirmation? Does affirmation involve negation? Which is the more fundamental form of predication? What is the function of negation? What are its grounds? Does mere denial give information?

CHAP. IV.—Why is it possible to interpret the same form of proposition in different ways? Enumerate the principal ways. Which comes nearest to the ordinary meaning of the mental act of judgment? Which is followed in the fourfold scheme? Interpret "*All is not gold that glitters*". Explain

how the connotative interpretation gives what is most fundamental in predication. Discuss Mill's advocacy of this reading, especially in regard to contingent propositions. Can the comprehensive interpretation be applied to synthetic judgments? Compare the class-inclusion view with the predicative view. Is any one of these views correct, to the exclusion of all the others? Is the predicate always, or ever, quantified in thought? What are Euler's circles? Explain the formation of Hamilton's eightfold scheme. Is Hamilton's postulate admissible? Does it justify his interpretation of the judgment? Can we have intelligible predication about indefinite classes? Can this sort of predication be expressed in propositional forms that are interpreted as relating definite classes to each other? Show that if the Hamiltonian forms deal with definite classes the scheme is both defective and redundant. Show that if "some" means "some only," A is equivalent to η, Y to O, and ω to U, thus yielding a fivefold scheme. Show that I is ambiguous in this view. What is "Integration"? Why cannot "some," interpreted as "some only," yield a scheme of simple, irreducible propositions? Interpreting "some" in the traditional, indefinite sense, draw the five combinations of Euler's circles: represent them by the numerals 1, 2, 3, 4, 5: set down the symbols for the eight Hamiltonian propositions in a vertical column: and place after each the numeral or numerals which indicate the diagrams compatible with it. Give examples of the U proposition, with their equivalents in the predicative scheme; also of the Y proposition; also of the η proposition. Examine the ω proposition. What is a logical equation? Are our ordinary judgments equations? Are Hamilton's forms equational? Suggest an equational method of expressing the four predicative forms.

CHAP. V.—Define logical opposition of propositions. Are all "opposites" incompatible? Explain the "square of opposition". State and prove the laws of subalternation. Define contradictory opposition. State and prove its laws. Why is contradiction the most perfect sort of logical opposition? What are its characteristics? Has every proposition a contradictory? Has any proposition more than one contradictory? Why is there no mean between two contradictories? Define contrary opposition. State and prove its laws. Compare it with contradictory opposition. There are two ways of disproving a universal: which is the easier and safer? Have all propositions contraries? What is material opposition? Define subcontrary opposition. State and prove its laws. Apply the doctrine of opposition to singulars; to modals. Summarize the inferences by opposition.

CHAP. VI.—Define immediate inference, eduction. Show how, from the form *S-P*, seven other forms are derivable. Name the four processes of eduction. Which are fundamental? Define accurately each process. State the rule for obversion. Connect it with the laws of thought. Does it deserve to be called an inference? Give some synonyms for obversion. Is "material" obversion an inference? Distinguish the geometrical from the logical converse. State the rules of conversion. Apply and prove them. Distinguish *simple* conversion from conversion *per accidens*. Does A ever admit of *simple* conversion? Prove that E and I convert simply. Why has O no converse? Show that conversion involves real progress of thought. Is the validity of logical conversion self-evident? Distinguish the partial from the full contrapositive. State the rule for contraposition. Why has I

no contrapositive? How is the reciprocal of *S a P*, *viz. P a S*, established in inductive inquiries? State the rules for inversion. Why have particulars no inverses? On what assumptions, as to the existence of the classes *S*, *P*, $\overline{S}$, and $\overline{P}$, in the sphere of reference of the proposition, are those various inferences valid? What is inference by added determinants? Is it *formal* or *material?* Give some valid, and some invalid, examples. Define inference by complex conception, and immediate inference by converse relation. Derive all the immediate inferences you can, both by way of opposition and by way of eduction, from the following propositions: (1) All logicians are wise; (2) Discretion is the better part of valour; (3) Any fool can ask the question; (4) Not every wise man can answer the question; (5) No boy was ever an old man; (6) Few men live a century; (7) All solids contain carbon; (8) Not to all men is it given to be philosophers; (9) The soul does not think always; (10) All trees are not pines; (11) Not all whom we admire deserve our admiration; (12) All cannot receive this saying; (13) Two straight lines cannot enclose a space; (14) Many rules of grammar overload the memory; (15) Nemo mortalium omnibus horis sapit.

CHAP. VII.—In what sense does every *judgment* imply the existence of its "subject"? Is this reference to an "objective sphere" implied in the intelligible use of every *term?* Does the use of a term in a proposition not only imply the existence of some sphere of denotation, but also the actual existence of the object denoted, in that sphere? Is this a question for logic to deal with, and why? What assumptions as to existence justify the traditional doctrine on *Opposition* and *Eduction?* How is the "universe of discourse," to which a judgment refers, to be determined? Does logic decide the nature of these spheres of reference, or of "existence" in these spheres? State the question of existential import. Does it admit of alternative answers? What considerations should guide us in the choice of an interpretation? Discuss fully the influence of four distinct suppositions: (*a*) on *Opposition;* (*b*) on *Eduction.* What does ordinary usage tell us about the existential import (*a*) of universals; (*b*) of particulars? Give some reasons for adopting the view that particulars imply, and universals do not imply, the existence of their subjects. Do modal propositions usually imply the existence of their subjects? Can we infer the assertoric from the modal, or *vice versa?* Discuss the relation between the proposition *All S is P* and *All not-S is P.* Can you derive a valid inverse from the proposition, *All human actions are foreseen by the Deity?* On the assumption made in 129, examine the validity of the following inferences: (*a*) *All S is P* and *Some R is not S*, therefore *Some not-S is not P;* (*b*) *All S is P* and *Some R is not P*, therefore *Some not-S is not P.*

CHAP. VIII.—What are the two parts of an "*If*" proposition called? What is the function of such propositions? On what does their truth or falsity depend? Show how every "*If*" judgment rests on an underlying categorical affirmation or denial. Distinguish two classes of "*If*" judgments, giving the main characteristics, with examples, of each class. Is the difference between them fundamental? Can all "*If*" judgments be reduced to the form, "*If S is M it is P*"? How does this form show the relation of the "*If*" judgment to the categorical judgment? Can the proposition "*If this S is M it is P*" be expressed categorically? Whether does "*If*" ex-

press *doubt* or *inference?* What is the doubtful element in the "*If*" proposition? Is a compound judgment which contains no doubt about the occurrence of its antecedents appropriately introduced by "*If*"? Is a doubt about the occurrence of the antecedent exactly identical with a doubt as to whether a given case, or cases, belong to the class of the antecedent? Can the conditional proposition be always expressed in categorical form? What is such transformation called? Is the conditional judgment usually modal, or usually assertoric? On what do the quantity and the quality of the conditional proposition depend? Distinguish between the contradictory and the contrary of "*If A is B, C is D*"? Construct the modal square of opposition for conditionals. What is an *Adversative* or *Discretive* proposition? What is the *quantity* of the first example given in the present chapter? What are the most important eductions from conditionals? Illustrate these. May the *modal* (categorical or conditional) be inferred from the assertoric, or *vice versa?* Are pure hypotheticals to be interpreted *assertorically*, or *modally?* Does the modal interpretation imply that in the apodeictic form *A* gives the full and adequate ground for *C?* Or that it gives the *only possible* ground for *C?* What is a reciprocal hypothetical? Why are not all hypotheticals reciprocal? How could they, conceivably, be made reciprocal? Is this ideal a practicable one? Have hypotheticals distinctions of *Quality?* of *Quantity?* of *Modality?* Define the hypothetical. Compare the propositions, *If A then C*, and *If A then not C*, in point of opposition. What are the chief eductions from hypotheticals? What are the more common fallacies here? Discuss *quality*, *quantity*, *modality* in the following: (*a*) "If a man is unfortunate, he is not, therefore, to be despised"; (*b*) "If a man is honest, he will not deceive"; (*c*) "If the summer be dry, the harvest will be good"; (*d*) "If A is B, C is D"; (*e*) "If the sun moves round the earth, some astronomers are fallible"; (*f*) "If he persists in his extravagance, he will be ruined"; (*g*) "If a man is good, he is wise".

CHAP. IX.—Distinguish between *Alternative* and *Disjunctive* forms of proposition. Define the alternative proposition. How is a judgment affected by an alternative subject? by an alternative predicate? How does a judgment with an alternative predicate differ from a categorical? Can an alternative judgment be negative? Interpret the propositional forms, "*All* [or *Some*] *X's are either Y or Z*"; "*Some S's are both P and Q*". Distinguish between a *complex* and a *compound* judgment. Indicate a scheme of assertoric and modal, simple and compound propositions. Is the alternative judgment to be interpreted in the *exclusive*, or the *non-exclusive*, sense? In the former sense, how would you contradict "*Either X or Y*"? Does the alternative proposition give, or can it give, the results of a logical division? Must the alternatives always *differ?* In so far as they differ, do they not mutually *exclude* one another? How is the alternative converted into an "*If*" judgment? Does the former contain any positive element not contained in the latter? Does the justification of the grounds of the judgment "*If S is M it is P*" involve either an endless regress or an appeal to a final alternative judgment? Transvert *If S is M it is P* into alternative form. Are alternative judgments usually modal? Do they, of themselves, yield a square of opposition? Derive all the eductions you can from the propositions: (1) "*Every duty on imports is either protective or a source of revenue;* (2)

Some men are either knaves or fools. Contradict (*a*) the proposition, "*No men are both honest and prosperous in business*"; express the original (*b*) as an alternative proposition; (*c*) as a conditional proposition. Contradict (*b*), and (*c*); and show that the three contradictories are equivalent.

PART III.

CHAP. I.—Distinguish mediate from immediate inference. Explain *major* and *minor extremes*, *premisses*, *conclusion*, *middle term*. Give some definitions of mediate reasoning. Define the *Syllogism*. Distinguish between the *matter* and the *form* of the syllogism; the *remote* and the *proximate* matter. Which is the major premiss of a syllogism? Can you tell this without knowing the conclusion? How is the *formal* element of the syllogism expressed? Does the *formal* validity of a syllogism guarantee the truth of its conclusion *absolutely*, or only *conditionally?* Explain and justify the axiom: "*Ex falso sequitur quodlibet*". Why is it possible to reach a true conclusion through false premisses? Why can we not infer the truth of the premisses from the truth of the conclusion, though we can infer the falsity of (one at least of) the premisses from the falsity of the conclusion? What fallacies arise from not attending to those points? Enumerate the various kinds of syllogisms, and describe each. Does the syllogism presuppose the fundamental laws of thought? Is it an application of them? And of them *alone?* What do you understand by a *Mediate Axiom* of the syllogism? State the mediate axioms of *Identity* and *Diversity*. Are they mathematical? What form of syllogism did Aristotle regard as the *perfect* or *standard* form? Explain: "Syllogistic reasoning is from the general to the particular". State and explain the Aristotelean *Dictum*. Investigate the question whether it refers us to the *extension*, or the *intension*, or *both*, of *P*, *S*, and *M* respectively. How is *M* to be read in the major premiss? Suggest a modified reading of the *Dictum* which will show more clearly the *inferential character* of the syllogistic process? Indicate some mediate axioms based exclusively on the intensive aspect of the terms of the syllogism.

CHAP. II.—Derive the general rules of syllogism from the Aristotelean *Dictum*. Enunciate these rules. What practical use have the rules of structure? What is *Quaternio Terminorum? Undistributed Middle?* Explain why the middle term must be distributed at least once. Illustrate by Euler's circles. Explain *Illicit Major* and *Illicit Minor*. Illustrate each by an example of your own. Why can *illicit major* not occur in affirmative syllogisms? Prove the rule that one premiss must be affirmative. Illustrate it by Euler's circles. Can a conclusion be ever validly inferred from two negative propositions? If so, is the present rule thereby disproved? Give some instances of similar *apparent* violations of the rules of quantity. Prove that a negative premiss necessitates a negative conclusion, and *vice versa;* that two particular premisses yield no conclusion; that "the conclusion follows the weaker premiss"; that a particular major and a negative minor yield no conclusion. Why must the premisses always distribute one term more than the conclusion? Show that either premiss of a valid syllogism, combined with the denial of the conclusion, must prove the denial of the other premiss. Show that *illicit process* in the former syllogism will involve undis-

tributed middle in the latter, and *vice versa*. Show how the violation of the rule (5) forbidding two negative premisses involves the violation of the rule (3) forbidding undistributed middle. Show that the rule against two negatives (5) involves the rule that a negative premiss necessitates a negative conclusion (6, *a*). To how many may the six or eight general rules be reduced? Explain the statement that "undistributed middle involves *indirectly* illicit process". State the rules as given in scholastic treatises on logic, and compare with the more modern statements of these rules.

EXERCISES.—(1) Prove that if the middle term be distributed twice in the premisses of a syllogism, the conclusion must be particular. (2) If the major term is distributed in its premiss and undistributed in the conclusion, determine the syllogism. (3) Prove that if three propositions involving three terms (each of which occurs in two of the propositions) are together incompatible, then (*a*) each term is distributed at least once, and (*b*) one and only one of the propositions is negative; show that these rules are equivalent to the rules of the syllogism. (4) Given the premisses of a valid syllogism, examine (*a*) in what cases it is *possible*, and (*b*) in what cases *impossible*, to determine which is the major and which the minor term. (5) (*a*) Given that the *major* premiss is O, determine the syllogism; (*b*) given that the *minor* premiss is O, determine the syllogism. (6) How many distributed terms may there be in the premisses, *more* than in the conclusion, of a valid syllogism? (7) If the minor premiss of a valid syllogism is negative, can you determine the position of the terms in the major premiss?

CHAP. III.—Define *Figure* and *Mood*. Can you distinguish between the *first* and the *fourth* figure unless you know which premiss is major and which minor? Show how many possible moods of syllogism there are. How can you determine the valid moods? How many of the sixty-four forms do the general rules and corollaries eliminate as invalid? State and prove the *special* rules of each figure. Name the moods which the special rules leave intact in each of the four figures. Why does the special rule of quality precede the special rule of quantity? Why are the rules of the fourth figure hypothetical? How many valid moods are there in each figure? How many valid moods altogether? How many of these are "named" or "original"? What is a *Subaltern Mood?* How many of these are there? Why are there none in the third figure? Repeat the mnemonic lines for the "named" moods. What is a *Strengthened Mood* of syllogism? How many of these are there among the twenty-four? Among the nineteen "named" moods? Are all the "subaltern" moods "strengthened" moods? What is a "fundamental" mood? What are the fundamental moods corresponding to each of the named strengthened moods? Determine directly, *by the general rules of syllogism*, the number of moods in which A, E, I, and O may be proved, respectively. Why can A be proved *only* in the first figure? Why can A or I not be proved in the second? Why can A or E not be proved in the third? What influence have the various suppositions regarding existential import, on the validity of the syllogism?

EXERCISES.—(1) Why do the premisses *E I* yield a conclusion in every figure, and the premisses *I E* in no figure? (2) Besides *E I*, does any other combination yield a conclusion in all the figures? (3) Construct a syllogism in *Cesare*, of which the conclusion will be "No birds are quadrupeds".

(4) Construct a syllogism in *Ferison* to prove that "Some students are not strong". (5) Prove by a syllogism in *Camenes* that "No persecution is justifiable". (6) Why can an *O* proposition not be a premiss in the first figure, a major in the second, or a minor in the third? (7) Why is one special rule common to the first and second figure, and another common to the first and third? (8) Examine these assertions: (*a*) The conclusion of a valid syllogism is simply convertible if in the premisses the extremes have the same quantity; (*b*) the extremes of a valid syllogism have the same quantity in the premisses if the conclusion is simply convertible? (9) "No one will hold that all virtuous men are happy, who remembers how many good men have lost wealth and life for conscience sake." Express this reasoning in *Ferio*, *Festino*, and *Ferison*. (10) Prove that every syllogism which distributes the middle term twice has a strengthened premiss.

CHAP. IV.—What is direct reduction? Is it the only test of the validity of a syllogism outside the first figure? What is its utility? Explain the significance of the consonants in the mnemonic lines. Give examples. Define *Indirect Proof*. On what principles does it depend? How is the principle applied to prove the validity of syllogisms in figures other than the first? Apply it to *Bocardo*. Define *Indirect Reduction*. Is the new syllogism employed in this process the only one involved in the original syllogism? If not, how do you know which of the possible ones to select? May all the moods of the other figures be reduced (1) directly, or (2) at least by the aid of indirect reduction, to any mood of the first figure? Reduce *Celarent* to *Ferio*. Is the first figure the most perfect? Is it the only cogent figure? What are its characteristics? Why is the second called the exclusive figure? What are its characteristics? State its axiom. Justify the passage of thought in the second figure, and compare it with the first. What arguments fall naturally into the third figure? What are its characteristics? What is its axiom? Why is it called the inductive figure? Justify its reasoning. Do any reasonings fall naturally into the fourth figure? Does it exhibit a distinctive type of inference? Are the moods of the fourth figure merely the moods of the first with converted conclusions? Are they the same as the *indirect* moods of the first? Define an indirect mood of the first figure. How many moods of the first figure yield both direct and indirect conclusions? How many direct only? How many indirect only? Does a pair of premisses from which a proposition necessarily follows, always "demonstrate," that proposition or prove it to be true? What is an *antilogism?*

EXERCISES.—(1) Can a purely formal proof be adduced for the statement that a true conclusion may validly follow from premisses that are false? (2) Was Kant right in asserting about the other figures that "the very same conclusion would follow from the same middle term in the first figure by pure and unmixed reasoning"? (3) If the mnemonic of a valid mood ends in "*s*," construct the syllogism. (4) Given I as major premiss, determine the syllogism by reference to the General Canons. (5) Why are all moods with an O premiss excluded from the first and fourth figures, while some such are admitted into the second and third? (6) Prove that if a valid syllogism contains an O premiss its middle term must occupy the same position in both premisses. (7) State which of the following moods are illegitimate or useless, naming the figures in which they are so: AAI, IEO,

IAI, EAO, AEO. (8) Determine by the General Canons of Syllogism the valid moods (*a*) in which the major premiss is I; (*b*) in which the minor premiss is O; (*c*) in which the mnemonic ends in "*s*". (9) Can it be shown that the reasoning in *Bocardo* is founded on the *Dictum de omni et nullo?* (10) Why are *D* and *C* excluded from the mnemonics of the second and third figures respectively?

CHAP. V.—Define the pure hypothetical syllogism. On what axioms is it based? On what does the quantity of each antecedent and of each consequent depend? Do the rules of the categorical syllogism apply here? Construct a pure hypothetical syllogism in *Dimaris*, and reduce it to the first figure. Define the mixed hypothetical syllogism. What is the basis of the reasoning here? What are the valid "moods" and their respective forms? Why is there no valid ground for inference in sublating the antecedent, or in positing the consequent, of the major premiss? Can the mixed hypothetical syllogism be reduced to the categorical? Is the reasoning mediate or immediate? What is a pure disjunctive syllogism? Is the conclusion merely the sum of the premisses? Examine the syllogism: *S is either P or Q; either P or Q is R; therefore S is R.* Define the mixed disjunctive syllogism. Is it "a syllogism one of whose premisses is a disjunctive proposition"? What is the basis of the reasoning here? What is the name of the valid mood? What are its forms? Are they reducible to mixed hypothetical syllogisms? Is the *Modus Ponendo Tollens* always invalid? Why? When the major premiss is *disjunctive* in the strict sense, as distinct from alternative, which mood is valid? Define the *Dilemma.* Explain its structure and its relation to the mixed hypothetical syllogism. How do you arrive at its valid forms? Name, explain, and illustrate these latter. How may the forms be reduced to one another? Is there any *formal* fallacy in the dilemma besides positing consequents or sublating antecedents? What conditions must the premisses fulfil in order that the dilemma be conclusive? In what ways may an inconclusive dilemma be met? What do you understand by "rebutting" a dilemma? May a perfectly valid dilemma be "rebutted"? Why can simple dilemmas and destructive dilemmas not be "rebutted"? State and criticize some alternative views of the dilemma. Give some examples of defective dilemmas. May the simple destructive form be rejected consistently with retaining the simple constructive form?

EXERCISES.—(1) Classify the following argument: "If he managed to escape he must have been either very clever or very rich; but he was both stupid and poor; so he cannot have escaped". (2) "If *X* is true, then either *Y* or *Z* is true: but *Y* is not true". What conclusion can be drawn?

CHAP. VI.—What is an enthymeme? Explain and exemplify the three orders of enthymeme. Can you determine the order, the figure, and the mood, to which a given enthymeme belongs? Supply the missing premiss in the enthymeme "C is D because A is B". What is a polysyllogism? a prosyllogism? an episyllogism? a progressive, and a regressive, polysyllogism? Define the sorites. How many forms are there? Why are they so called? What are the essential points of difference between the two forms? What orders of enthymeme are involved in each? Are pure or mixed hypothetical sorites possible? Illustrate. Determine and prove the special rules for the Aristotelean and the Goclenian forms respectively. Exemplify the sorites in

the second and third figures. In how many moods of each is a sorites possible? Determine, from an analysis of the forms and the special rules of the second and third figures, the special rules for the sorites in these figures. Why is the sorites peculiarly liable to the fallacy of "four terms"? What is the epicheirema? How many kinds are there? Give a concrete example of each kind.

CHAP. VII.—Enumerate four essential features of syllogistic reasoning. What kind of *identity* or *diversity* is involved in the judgments which constitute the categorical syllogism? Is the relation of attribute to subject the only relation that is found directly expressed in all judgments? Indicate some other kinds of relation? Can the *a fortiori* argument be reduced to a syllogism? Is its axiom concerned with the *real*, or with the *conceptual*, order? Is any inference exclusively "formal," or any exclusively "material"? Can we infer *immediately* "from particular to particular"? What does "particular" mean here? Can we do so *mediately* without the intervention of a universal? Is not the argument from analogy an instance of this? And circumstantial evidence? State Mill's view about the function of the syllogism. Distinguish between "logical" and "psychological" inference; between immediate inference and verbal change? State what is known as the "Paradox of Inference". Give your own explanation of the "novelty" and "necessity" which constitute the paradox. Explain these distinctions: The conclusion is "*objectively*," "*subjectively*," "*actually*," "*virtually*" contained in the premisses. How is the fallacy of "*Petitio Principii*" alleged to be committed in the syllogism? Give instances of syllogisms which *do* commit this fallacy; of syllogisms which do *not*. Does the presence of a definition in the premisses involve the fallacy? To what erroneous theory, as to the manner of reaching the universal judgment, may we trace the objection that the syllogism is a *petitio principii?* Is the process by which we reach general truths an *inferential* process? Is it more laborious, or less laborious, than deductive inference? Distinguish between *formal* and *material* generalization. Distinguish between the *logical grounds* and the *ultimate sources* of our knowledge of an inferred conclusion.

END OF VOL. I.

INDEX.

Abscissio infiniti, i., 345.
Absolute terms, i., 70.
Abstract terms, i., 57-63.
Abstraction, i., 4, 205; ii., 14, 24-6.
Accent, fallacy, of, ii., 301, 306-7.
Accident, fallacy of, ii., 297, 301, 310-12.
— predicable, i., 77, 86-8.
Accidental judgments, i., 170-80.
Actio et passio, etc., ii., 81.
— category of, i., 141, 147-8; and cause, ii., 81-2.
Adam, ii., 172.
Adams, ii., 197.
Added determinants, inference by, i., 246.
A Dicto, etc., fallacy of, ii., 301, 309-11.
Adjacent cases, ii., 206-7.
Adversative propositions, i., 271.
Aequipollentia, i., 232, 312.
Affinity, i., 130.
Affirmation and negation, i., 202-3.
Agnosticism, ii., 59-61, 79, 107, 130, 146, 222.
Agreement, method of, ii., 173-5; combination with difference, 179-85.
Albert the Great, ii., 21, 33.
Aldrich, i., 326, 352.
Alternative judgments and propositions, i., 280-91; fallacies of, ii., 314-15.
— subjects, i., 281; predicates, i., 281.
— syllogisms; pure, i., 362; mixed, 363-5.
— terms, i., 198.
Ambiguity in language, i., 44, 189, 192, 307-8.
Ambiguous middle, i., 307.
Amphiboly, fallacy of, ii., 301, 308-9.
Ampliative propositions, i., 170-80.
Analogical use of terms, i., 43-4; predication, 149.
Analogy, i., 393; its function in induction, ii., 135-42, 150, 153-5; force of, 156-8; and enumerative induction, 159.
Analysis, i., 5; in definition, 91; in judgment, 157-8, 174; in method, ii., 9-10; in teaching, 14-16; in induction, 32, 36; experimental, 165 *sqq.*
Analytic method, i., 378; ii., 7; analytico-synthetic, 9.
— chains of reasoning, i., 377, 383-4.
— definition, i., 91-2.
Analytic propositions, i., 170-80, 404-5; ii., 94.
Anselm, St., ii., 233.
Antecedent, *v. consequent*, and *ground*.
Antepraedicamenta, i., 136.
Antilogism, i., 324, 340-1.
Apodeictic judgments, i., 181-5; ii., 215.
Appetitus naturalis, ii., 66.
Apprehension, simple and complex, i., 2, 6.
Approximation, methods of, ii., 204-5; and probability, 279.
Argument, regressive, ii., 45.
Argumentum a fortiori, i., 388; ii., 158.
— *a contrario*, ii., 158.
— *a pari*, ii., 158.
— *a posteriori*, ii., 53, 108, 129, 146, 151, 232-3.
— *a priori*, ii., 53, 232-3.
— *a simultaneo*, ii., 233.
— *ex praescriptione*, ii., 259.
— — *silentio*, ii., 256.
— *ad hominem*, ii., 234, 316-7.
— — *baculum*, ii., 316.
— — *misericordiam*, ii., 316.
— — *ignorantiam*, ii., 316.
Aristotelean enthymeme.
— sorites, i., 379-84.
Aristotle, on moderate realism, i., 9; ii., 231; logical treatises, i., 40; predicables, i., 73; ii., 80; categories, i., 136-42, 147-9; on judgment, 154; on denial, 204; on conversion, 236; ii., 80; on def. of syllogism, i., 292-3; *Dictum de omni et nullo*, 300-4; on indirect reduction, 339; on ἔκθεσις, 349; on indirect moods of the first figure, 350; on hypothetical arguments, 366; on method, ii., 10, 16-17; influence on mediaeval thought, 19; on induction, 24; on inductive syllogism, 28, 31-2, 158; on ascent to the universal, 30-4, 229-32; on scientific induction (ἐμπειρία), 32-4; on causes, 62, 64; on purpose in nature, 67; on παράδειγμα ("example"), 158-9; on probable knowledge, 224; on analogy, 160-1; on truth 210; on enthymeme, 265-8; on demonstration, 223-9; kind of, 232-5; on science, 223-5, 230-2, 237-9, 242; on fallacies, ii., 300-2; analysis of, 303-29.

Art, distinguished from Science, i., 14-16.
Artificial classifications, i., 122-31.
Assent, ii., 210-14.
— and inference, i., 297.
Assertoric propositions, i., 180-5, 255, 257, 284, 289, 290.
Association of ideas, i., 3, 51.
Assumption of axioms and postulates, ii., 13-14, 60, 106-7, 109, 111, 113, 135, 143-5, 147, 243, 255, 298, 322-7.
Atomism, chemical, ii., 140-7.
— mechanical, *v. mechanical view of universe.*
Attributive terms, i., 60.
— view of predication, i., 207-8; reverse of, 208.
Augmentative judgments, i., 170-80.
Augustine, St., i., 142; ii., 10, 19, 327.
Authenticity of manuscripts, ii., 257.
Authority, ii., 249; in science, 251-2, 334; in history, 254.
— divine, ii., 250.
Averages, ii., 279-82; functions of, 285-7; and "law," 289-93.
Averroës, i., 350.
Axioms, of mediate inference, i., 299-301; ii., 50; in philosophy, ii., 10, 13; 117, 142-5, 243.

Bacon, Francis, Lord Verulam, ii., 37-41, 51, 53, 56, 134, 169, 172, 299-300.
Bacon, Roger, ii., 21, 37, 185.
Bain, i., 10, 232; ii., 105, 146.
Balfour, ii., 252-3.
Ballot-box theory of nature, ii., 38, 43.
Balmes, ii., 327.
Barbara, Celarent, etc., history of, i., 326, 352.
Baudot, ii., 281.
Baynes, i., 213.
Begging the Question, *v. Petitio Principii.*
Belief, and science, ii., 97, 250-2; and opinion, 212; and authority, 216-7, 250-3; in uniformity of nature, grounds of, ii., 99-113.
Beliefs, *v. assumption of axioms.*
Bentham, i., 124.
Berkeley, ii., 146.
Bernard, Cl., ii., 148-50.
Bernouilli's theorem, ii., 280-2.
Bertrand, ii., 269, 285, 291.
Bias, ii., 6, 148, 163, 254, 273-4, 319, 324.
Bifid division, *v. dichotomy.*
Borda's experiments, ii., 187.
Bosanquet, i., 11; on judgment, 161; on reciprocal judgment, 276, 288; ii., 45, 106, 242.
Boudon, ii., 269.
Boutroux, ii., 327.
Bowen, i., 214, 232.
Bowne, B. P., ii., 275, 284, 291.
Bradley, i., 11; on judgment, 160-1, 174.
Buckle, ii., 290-1.
Burke, ii., 140.

Cajetan, Cardinal, i., 173.
Calculus of probability, ii., 268-81; applications of, 282-5.
Calvus, fallacy of, i., 379; ii., 306.
Casual and causal connexions in nature, ii., 166, 322.
Categorematic words, i., 34, 36.
Categorical judgment, nature of, i., 154-64.
— propositions, relation to conditionals, i., 269-70; to hypotheticals, 273; to disjunctives, 288-9.
— and hypothetical statement of laws in induction, ii., 58-60, 93-100, 217-23.
Categories, in logic and in metaphysics, i., 135-6; and predicables, 138; Aristotle's enumerated, 141-2; and language, 142-5; and reality, 145-6; Kantian, 150-1; compared with Aristotle's, 152-3.
Cauchie, ii., 254.
Causa essendi, and *causa cognoscendi*, i., 345, 349; ii., 222, 327.
Causal demonstration, ii., 232-3.
— hypotheses, ii., 122-7.
Causality, i., 148; principle of, ii., 61; and uniformity of nature, 71-5; sensist view of, 70-80; and space and time, 80-4; and chance, 268-72.
Cause and condition, ii., 62-3.
— — reason or ground, ii., 61, 85, 227, 231-2.
Causes, definitions and divisions of, ii., 62-6, 180; necessitating and indispensable, 84; non-reciprocating, 85-92; "plurality" of, i., 275; ii., 84-92, 167, 244, 247-8; free or self-determining, 94-5; conjunction of, 195-6; interaction of, 335-7.
Certainty, ii., 211, 263.
Certitude, ii., 211; and prudence, 212; three kinds of, 214-17, 235; in the "human" sciences, 248-59; and probability, 260-3.
Cessante causa, etc., ii., 81.
Chains of reasoning, i., 377-84; ii., 265.
Chance, logic of, ii., 261; concept of, 268-72; elimination of, 280-3; estimation of, 276-8, 282-3; and mechanical view of nature, 291-3.
Change, i., 148; and causation, ii., 81-2.
— of relation, i., 269.
Cicero, i., 379.
Circulus in definiendo, i., 109.
— *in demonstrando*, ii., 320.
Circumstantial evidence, i., 394-5; ii., 262, 265, 267.
Class compartments, i., 117, 217, 250-3, 261.
— inclusion view of judgment, i., 210-16.

Class name, i., 193.
Classification, i., 121-31; and hypothesis, i., 131; ii., 151; of widest concepts, i., 135; and disjunctive judgments, i., 286, 289.
Clarke, on *species infimæ*, i., 80-1; on *prædicamenta*, 137; on singular propositions, 192; on dilemma, 375; on fallacies of method, ii., 12; on method of teaching, 15-16, 211; on "chain" evidence, 265.
Clifford, ii., 138, 239.
Co-division, i., 113.
Coexistences and sequences, ii., 68.
Cognate genus, i., 78.
— species, i., 78.
Collective judgments, i., 189, 300-1, 402-4, 408, 412; ii., 27-32, 48, 320.
— terms, i., 46, 189; ii., 305.
Colligation of facts, ii., 42, 122.
Combination of causes, ii., 195-6.
Common-sense, truths of, ii., 324-5.
"Comparative" sciences, ii., 190.
Compartmental view of judgment, *v. class compartments*.
Complementary terms, 66, *n.* 1.
Complex conception, inference by, i., 247.
— concepts and terms, i., 37, 198, 284-5, 290.
— dilemmas, i., 367-9.
— propositions, i., 169, 197-8, 283-4.
Composition and division, fallacies of, i., 47, 226; ii., 301, 304-6.
— of causes and effects, ii., 85, 195-6.
Compound propositions, i., 169, 197-8; opposition of, 223; 265; and complex, 284-5.
Comprehension of concepts and terms, i., 49, 63, 172.
Comprehensive view of predication, i., 209-10.
Conceivability, sphere of, i., 250.
Concepts, i., 17, 37; divisions of, 42-71; specific, 76; and imagination images in science, ii., 131-2, 149; fallacies incident to, ii., 303-7.
Conceptualism, i., 10.
Conceptualist view of logic, i., 20, 26.
Conceptus objectivus, i., 37 *n.*
Conclusion, its relation to premisses, i., 295-7, 314, 316-7, 401; ii., 225-6, 294-5, 313.
Conclusions, logical grounds and ultimate sources of, i., 410-12.
Concomitant variations, method of, ii., 185-93; and agreement, 190; limitations of, 191, 201; and statistics, 192-3, 282.
Concrete terms, i., 57-63.
Condition and cause, ii., 62-3; totality of conditions, ii., 70-80.
Conditional propositions, distinguished from hypothetical, i., 265-6; and categoricals, 269-70; existential import of, 269-70; modality of, 270.
Conditional syllogisms, pure, i., 356-8; mixed, 358-61.
Condorcet, ii., 285.
Confusion, fallacies of, ii., 299.
Conjunction of causes, ii., 85, 195-6.
Conjunctive propositions, i., 280, 282.
— terms, i., 198.
Connotation, i., 48-52; and denotation, 56; and definition, 93, 104; objectivity of, 175.
Connotative view of predication, i., 208-9.
Consequens, fallacy of, i., 272-3, 279, 297; ii., 312-13, 319.
Consequent, and antecedent in deduction and induction, ii., 53-5.
Consequentia, or consequence in reasoning, i., 295-6, conseqence and sequence, ii., 165.
Consilience of inductions, ii., 42, 142.
Consistency, logic of, i., 20, 22, 253; ii., 118; and truth, i., 19, 22; ii., 118.
Constructive definition, i., 106-7.
— dilemma, i., 367-8.
— hypothetical syllogism, i., 359.
— method, ii., 7.
Content of concepts, i., 48-50.
Contingent judgments and propositions, i., 170-80, 200-2; ii., 99, 218-19.
— being, ii., 61, 221.
— modality, i., 181.
— necessity and law, i., 176; ii., 218-19.
Continuity, of cause and effect, ii., 81-2, 196.
Contradiction, in judgments and propositions, i., 221-4.
— in terms, i., 64-8.
— principle of, i., 24, 222, 224.
Contraposition, i., 241-3; and induction, 243; ii., 152, 170.
Contrary judgments and propositions, i., 224-5.
— terms, i., 68-9.
Contraversion, i., 232.
Conventional element in language, i., 35; in connotation, i., 50-2.
Conventional intension, i., 49.
Converse relation, inference by, i., 247.
Conversion of propositions, i., 232-41; by negation, 241.
Conversio syllogismi, i., 337.
Copernican astronomy, ii., 122, 127, 329.
Copula, logical, i., 154, 165, 202, 249, 386-7, 391.
Copulative propositions, i., 280, 282.
Correlative terms, i., 70.
Criteria of truth, i., 28-9, 161; ii., 4, 107-8, 211-12, 250-2.
Criteriology, i., 28-9, 147; ii., 4.
Crofton, ii., 281.
Crookes, Sir W., ii., 139.
Cross-division, i., 118.

Cumulative evidence, i., 394-5; ii., 141-2, 217, 249, 262.

Darwin, ii., 139, 334.
Davy, ii., 168.
Deductio ad impossibile, v. reductio.
Deduction, and induction, ii., 8, 9, 48-55, 117-19, 243-8 (*cf. inference*, and *method*).
Deductive inference, i., 391-2, 412; definitions of, ii., 51; in mathematics, ii., 25, 243-8.
Definite singular propositions, i., 193.
— numerical propositions, i., 197.
Definition, its functions, i., 89-91; ii., 2; formation of, i., 91-5; fixity of, 95-6; limits of, 96-7; ii., 237; nominal and real, i., 99-106, 134, 252; existential import of, 101-2, 407; private, 106; genetic, 106-7; physical, 107; rules of, 108-11; as involving fallacy in inference, 405-6; not arbitrarily invented in mathematics, ii., 25-6, 230, 237.
Delehaye, ii., 253.
Delorme, ii., 37.
Demonstration, i., 333-4, 345; ii., 52-3, 108, 142; and explanation, 224, 235-9; conditions of, 225-0; kinds of, 232-5; and verification, 245-8.
De Morgan, i., 39, 232, 307, 313, 314, 316, 389; ii., 306, 310, 318, 320, 333.
De Munnynck, ii., 145.
Denial, nature and ground of, i., 203-6.
Denotation, i., 52-64.
De Quincey, i., 103.
Derivative laws, ii., 208.
Descartes, i., 149; ii., 10, 291.
Description, i., 107.
Descriptive hypotheses, ii., 123-5.
Design, *v. purpose.*
Desitive propositions, i., 200.
De Smedt, ii., 253.
Destructive dilemma, i., 367-8.
— hypothetical syllogism, i., 359.
Determination, and negation, i., 204-6.
— of moods of syllogism, i., 320-4, 327-31.
Determinative clauses, i., 157, 198.
Determining cause, ii., 64.
Development, conception of, ii., 10.
De Wulf, i., 93, 318; ii., 10, 17, 19, 128, 138, 144, 231, 252, 334.
Diagnosis, i., 125, 345.
Diagnostic definition, i., 133.
Diagrams, Euler's, i., 211-12, 236, 310.
Dialectic, ii., 52.
Dichotomy, i., 115-7.
Dictum de omni et nullo, i., 209, 300-4; and rules of syllogism, 305-6; and first figure, 344, 386, 389; its real and conceptual import, 390, ii., 27; and uniformity of nature, 115-9.
— *de diverso*, i., 346.
Didactic method, ii., 7, 14-16.
Difference, method of, ii., 175-9; combination with agreement, 179-85.
Differentia, predicable of, i., 77, 81-2.
Dilemma, i., 367-75; formal and material validity of, 370-3.
Diogenes of Laërte, i., 40.
Direct determination of valid moods of syllogism, i., 329-31.
— reduction of syllogisms, i., 335-9.
Discourse, universe of, i., 54, 65, 161, 249-52, 255.
Discovery, and instruction or exposition, ii., 14, 15; and proof, 42-3; deductive and inductive, 243-8.
Discretive propositions, i., 271.
Discursive reasoning, i., 18, 295, 297.
Disjunctive propositions, i., 280-91; in calculus of probability, ii., 275-82.
— syllogisms, pure, i., 362; mixed, 363-5; in induction, ii., 39, 50-2, 142, 172, 182.
Disputationes quodlibetales, ii., 17.
Distinctive explanation, i., 107.
Distribution of terms in propositions, i., 186-8; intensive, 202, 209; of consequent in hypotheticals, i., 270-1, 279, 357.
Distributive use of terms, i., 47; ii., 305.
Divisio, fallacy of, i., 47, 226; ii., 301, 304-6.
Division, logical, functions of, i., 112-13; nature of, 113-4; formal and material, 115-7; rules of, 118-21; and definition, 113; and disjunctive judgments, 286, 289.
Divisions of logic, i., 17-19; ii., 2-7.
Documents, historical, ii., 256-8.
Double method of agreement, ii., 179-81.
Doubt, ii., 213.
— and modality, i., 184-5; in "if" judgments, 267-9.
Dualism, *v. monism, theism.*
Duhem, ii., 129, 132.

Education, and method, ii., 14.
Eductions, i., 229-46; table of, 245; material, 245-7; and existential import, 256-8; from conditionals, 272-3; from hypotheticals, 278-9; from alternatives or disjunctives, 290-1; and rule of quality in syllogism, 311-12.
Effect and cause, ii., 63; as correlative, 80-1; not identical, 82.
Effects, intermixture of, ii., 195-6.
Efficient cause, ii., 62-5; traditional and empirical concepts of, 70-80; kinds of, 71; efficiency not necessity, 73-4; and space relations, 83.
εἰκός, ii., 265.
ἔκθεσις, i., 349, 354.
Elements of syllogism, i., 294-7.
Elimination, in inductive analysis, i., 275; ii., 38, 50, 52, 55, 165-72, 244; in de-

duction, ii., 244-8; of errors in measurement, ii., 203-5; in mediate inference, i., 296-7.
Empiric demonstration, ii., 234.
Empirical generalizations or laws, ii., 205-9, 333-4.
— judgments, i., 170-80.
Empiricism, ii., 59-61, 75 *sqq.*; and uniformity of nature, 102-5.
Ens rationis, i., 31, 72.
— *reale*, i., 30, 31.
Enthymeme, Aristotelean, ii., 265-8.
— modern, i., 376-7.
Enumeration of instances, i., 347-8, 402; ii., 27-32.
Enumerative induction, ii., 27-32, 39, 43, 48, 102-4, 152, 159; and theory of chance, 283-4; and *petitio principii*, 318, 321-2; and generalization, 334.
— judgments, i., 190, 300, 402-4; ii., 27-32.
Epicheirema, i., 378, 383-4.
Epistemology, i., 28-9; ii., 113, 323.
Episyllogism, i., 377.
Episyllogistic chains of reasoning, i., 377-83.
Equational view of judgment, i., 216-18, 234.
Equivalence of propositions, i., 231-2, 312.
Equivocal terms, i., 43-4.
Equivocation, fallacy of, ii., 297, 301, 303-4.
Error, ii., 211, 213; and fallacy, 296-8; sources of, 325-7.
Essence, notion of, i., 75-6; and *proprium*, 83-4, 175-7; ii., 237; nominal and real, i., 76, 84; and substance, 141; and nature, ii., 67.
Essential judgments, i., 170-80.
Ethical truth, ii., 210.
Etymology, and connotation, i., 64.
EUCLID, i., 242; ii., 11, 31, 318.
EULER, i., 211-12, 236, 310.
Evidence, circumstantial, i., 394-5; ii., 262, 267; "chain" evidence, 265; historical, ii., 5-6, 217; nature of, 211-14; sources of, 214-17; extrinsic and intrinsic, 216, 250; mediate and immediate, 226-7, 297-8; external and internal, 257-8; probable, 261; estimation of, ii., 59, 213, 299, 325-7.
Evolution, and classification, i., 130-1; as a standpoint, ii., 10.
Example, argument from, ii., 158-60.
Exceptio probat regulam, ii., 167-8.
Exceptions, significance of, ii., 167; to necessary truths, and to physical laws, 217-23; and rule, ii., 308, 333.
Exceptive propositions, i., 200, 241.
Excluded middle, principle of, i., 24-5, 203, 222, 224, 226, 231-2.
Exclusive figure of syllogism, i., 344; in induction, ii., 39-40.
Exclusive propositions, i., 199-200, 241.
Exemplification, i., 97-9, 103.
Existence, implication of, in definitions, i., 101-2, 252; in categorical judgments, 248-62; in conditionals, 269-70; ii., 96; logical meaning of "exist," i., 251.
Existential formulation of judgment, i., 164, 234, 261.
— propositions, i., 164, 217-8, 221, 251.
Experience and proof, ii., 53, 97, 108-9; and interpretative principles, ii., 142-5, 243.
Experientia (ἐμπειρία = induction), ii., 33-4.
Experiment, and observation, ii., 164-5; "natural," 164; functions of, 165 *sqq.*; practical canon of, 171.
Experimental analysis, difficulties of, ii., 168-72; principles of, 166-7.
— methods, ii., 42, 172-201.
Explanation, distinctive, i., 107.
— scientific, ii., 50, 52, 53, 89, 142; and verification, 207-8, 239-40, 245-8; and demonstration, 224, 235-9; limits of, 239-41; in deduction and in induction, 243-8.
— popular, ii., 238-9.
— ultimate or philosophical, ii., 59-61, 128; and verification, 142, 208-9, 239-43, 245-8; and statistics, 291.
Explanatory hypotheses, ii., 124-5.
Explicative clauses, i., 198.
— propositions, i., 170-80.
Exponible propositions, i., 198-200, 215.
Expositio or ἔκθεσις, i., 349, 354.
Exposition, method of, ii., 14-16; scholastic method of, 16-22.
Extension, of terms, i., 48-57; in propositions, 186-8; in syllogisms, 293, 301-4.
— of hypotheses, *v. hypotheses* and *consilience of inductions*.
— of laws, ii., 206-7.
Extensive view of predication, i., 210-16.
— definition, i., 97-9.
"Extreme case," function of, ii., 333.
Extremes of syllogism, i., 294-5.

Fact, and theory, ii., 60, 149-50; and truth, 235-6; and law, 206, 223, 236-9.
— necessary, i., 180; ii., 208, 222.
Faculties, mental, i., 1-8.
Fallacies and logic, ii., 294-6; and error, 296-8; classifications of, 295, 298-303; formal and material, 302-3.
False analogy, ii., 307, 309, 331-2, 334.
— cause, ii., 327-9.
Feeling, Mill's category of, i., 150.
Figure of speech, ii., 301, 307-8.
Figures of syllogism, i., 319-31; characteristics of each, 343-55.
Final causes, ii., 59, 62, 64, 65-70, 112.

"Finger-post to the unexplained," ii., 194, 197.
First figure of syllogism, special rules of, i., 321; characteristics of, 343-4.
— and second intentions of the mind, i., 32, 72.
Forbes, i., 317 (*v. Palaestra Logica*).
Force of concepts, i., 48.
Form and matter, of thought, i., 20-3, 146, 150, 152, 264-5, 285; ii., 295-6; of judgment, i., 162-4; of syllogism, i., 294-7.
Formal cause, ii., 62, 64, 65, 83.
— contradictories, i., 65-7.
— division, i., 115-7.
— hypotheticals, i., 274.
— logic, i., 19-23, 162, 264-5; ii., 296, 302-3.
— propositions, i., 172-3.
— validity of inference, i., 295-7.
— fallacies, ii., 296-7, 302-3.
"Forms," Bacon on, ii., 38-40.
Formulation of judgment, schemes of, i., 164-6; for "if" judgments, 265-7.
Four terms, fallacy of, i., 307.
Fourfold scheme of propositions, i., 186-8.
Fourth figure of syllogism, special rules of, i., 323-4; critical analysis of, 350-5.
Fowler, i., 232, 375; ii., 156, 160, 187, 189.
Freeman, ii., 253.
Free will and causality, ii., 71, 74.
— and uniformity of nature, ii., 94, 218.
— and science, ii., 94, 217, 249.
— and social statistics, ii., 289-93.
Fundamental syllogisms, i., 326-7.
Fundamentum divisionis, i., 112, 113, 115-6, 119; ii., 311.
— *relationis*, i., 70.

Galen, i., 350-1.
Galenian Figure, i., 319.
General judgments and propositions, two kinds of, i., 190-2 (*cf. collective, enumerative*).
— terms, i., 44-6.
Generalization in language, i., 44, 50, 96.
— formal and material, i., 409; ii., 40, 47, 95-6, 114-19, 216-17; two ideals of, ii., 40, 41; fallacies of, ii., 299, 309, 332-7.
Generalizations, empirical, ii., 206-9.
Generic *differentia*, i., 82.
— judgments, i., 191, 201, 403.
— *propria*, i., 82-3.
Genetic definition, i., 106-7.
Genus, predicable of, i., 77; kinds of, 78.
"Geometrical" induction, ii., 25.
— inferences, *v. mathematical*.
Georgius Scholarius (Gennadius), i., 318.
Gerard, ii., 139.
Gesture "language," i., 34.
Goclenian sorites, i., 379-84.
Grammar, i., 34, 142-5.
Grammatical analysis, i., 35-6, 142, 155.
Green, T. H., i., 11; ii., 106, 220.
Grote, on Aristotle's categories, i., 143-4.
Grounds or reasons, logical, i., 386; and ultimate sources of knowledge i., 410-12; ii., 297; ground and consequent, ii., 54-5; and cause, ii., 61, 85.
— logical, proximate, and ultimate, ii., 99-100, 118-19.
Growth of language, i., 44.

Habit, as quality, i., 141; as category, i., 142, 148.
Habrich, ii., 14.
Hamilton, Sir W., i., 20; on definition, 100-1; on categories, 149; on comprehensive view of predication, 210; on extensive view, and quantification of predicate, 210-16, 332; his postulate, 212; on dilemma, 375.
Hegel and Hegelianism, i., 11, 153; ii., 60-1, 69-70, 83, 105-6, 146, 221-3, 241-3, 324, 334.
Herschel, Sir J., ii., 172, 197.
Heteropathic effects, ii., 195-6.
Hickey, i., 318, 332; ii., 253.
Hird, i., 317 (*v. Palaestra Logica*).
Historical propositions, i., 195, 234.
— evidence, ii., 5-6, 217.
History and Science, ii., 5-6, 253-9.
Hobbes, i., 10, 43.
Hobhouse, on formal science, i., 163.
Holman, i., 307.
Homogeneous effects, ii., 195-6.
Horns of dilemma, i., 371.
Hume, i., 10, 395; ii., 59, 75-6, 146, 194.
Huxley, ii., 21, 147.
Hyperphysical entities in science, ii., 132-5.
Hypothesis, and classification, i., 130-1; ii., 151; and particular judgments, i., 195; and enumerative induction, ii., 32, 152; nature and functions of, 120-2; verification of, 50, 128, 135-42, 146-7, 150, 207, 319, 333; in deduction, 244-8; kinds of, 122-7; conditions for, 148-51; sources of, 151-5; extension of, 142, 155, 157-8, 206.
Hypothetical judgments and propositions, and categorical, i., 263-4; and conditional, 265-6; ii., 94; modality of, i., 273, 277-8; eductions from, 278-9; and disjunctives, 289; fallacies incident to, ii., 312-3.
— necessity of metaphysical laws, ii., 217-23.
— sorites, i., 380-1.
— syllogisms, pure, i., 356-8; mixed, i., 358-61.
Hysteron proteron, ii., 319.

Idealism, *v.* HEGEL and *Phenomenism*.
Identity, and diversity, i., 24, 308; syllogistic axioms of, 229, 386.
— and similarity, ii., 160-1; 218, 220.
— principle of, i., 23-4, 236.
"*Idola*," Lord Bacon's, ii., 37-8, 299-300, 324-5.
"If" judgments and propositions, i., 263-79.
Ignava ratio, i., 374.
Ignorance, ii., 211, 213.
Ignoratio Elenchi, ii., 301, 315-17.
Illative sense, ii., 22.
Illicit process, in inference, i., 309-10; in hypotheticals, 361.
— generalization, fallacies of, ii., 309, 329, 332-7.
Illustrative proof or explanation, ii., 31-2, 238-9.
Immediate inference, i., 229-47; and existential import, 256-8; and mediate, 292-3, 385; and hypothetical syllogism, 361-2; fallacies of, ii., 308-15.
Immutability of truth, i., 161-2; ii., 220.
"Imperfect" figures of syllogism, i., 335.
— induction, ii., 27.
Impersonal judgments, i., 155, 164.
Implication of existence, in propositions, i., 164, 195, 217, 221, 235, 248-62; in definitions, 101-2, 407.
— of terms, i., 48 *sqq*.
Implications and import, of propositions, i., 164-6, 199, 229, 234, 396.
Import, existential, *v. implication of existence*.
Important characters, in classification, i., 127-31; in analogy, ii., 156-8.
Impossible modal judgments, i., 180-5.
Inaccuracy, in measurement, ii., 202-5.
Inceptive propositions, i., 200.
Incompatible attributes, fallacy of, ii., 304; terms, i., 64, 69; judgments, 205-6.
Incomplete induction, *v. imperfect*, and *enumerative*.
Indefinite propositions, i., 194-5; singulars, 193, 200.
— terms, *v. infinite*.
Indesignate propositions, i., 200-2, 270.
Indirect moods of syllogism, i., 323, 350-5.
— proof, i., 339; ii., 233-4, 318, 328.
— reduction, i., 339-43.
Indispensable cause or condition, ii., 64, 84.
Individual substances, i., 75, 139; properties and accidents, 82-8.
Individuation, i., 86, 140.
Induction, aim of, ii., 23, 235; as an inverse process, ii., 54-5.
— and contraposition, i., 243.
— and deduction, ii., 8, 48-55, 117-19, 243-8.
— and inference, i., 408, 411; ii., 24, 48-53, 109, 114.
Induction, and probability, ii., 43, 206-9, 212-14.
— enumerative, ii., 27-32, 39, 40, 43, 48, 102-4, 152, 159; and theory of chance, 283-4.
— fallacies, incident to, ii., 329-37.
— grounds of, ii., 44, 56 *sqq*.
— imperfect and perfect, ii., 27.
— presuppositions of, ii., 55 *sqq*.
— scientific, ii., 32-7, 40, 44.
— steps of, ii., 41-2, 44, 47, 49.
— views of, ii., 24, 32-45.
Inductive inference, i., 408, 411; ii., 24, 48-53.
— method, ii., 8, 13.
— methods, Mill's, ii., 172-200.
— syllogism, i., 347; ii., 27-32; as illustrative "proof," 31-2; 267-8.
Inference, nature of, i., 6, 78, 229, 231, 385-92, 397-401.
— and verbal change, i., 166, 212, 231.
— by added determinants, i., 246.
— by complex conception, i., 247.
— by converse relation, i., 247.
— from particulars, i., 313.
— immediate, i., 229, 292-3, 385.
— inductive and deductive compared, ii., 51-3.
— in "if" judgments, i., 267-9.
— mediate, i., 292-3, 385-92; summary of teaching on, 412-14.
— paradox of, i., 396-401.
Infima species, i., 79-81, 97, 122, 129; ii., 139.
Infinitation, i., 232.
Infinite judgments, i., 202.
— terms, i., 66, 232.
Inseparable *accidens*, i., 87; and *proprium*, 175-7, 200-2.
Integration, i., 214.
Integrity of documents, ii., 258.
Intellect and sense in science, ii., 128-35, 150.
Intension of concepts and terms, i., 48-57; kinds of, 49, 398; and extension, 55-7; in propositions, 207-8; in syllogisms, 293, 301-4; and possibility of inference, 398.
Intentio universalitatis, i., 8, 390.
Intentions, first and second, i., 32, 72, 148, 390.
Interaction, *v. cause*.
Interference of causes, ii., 169, 171.
Intermixture of effects, ii., 195-6.
Interpretation, i., 5, 160; of meaning in judgment, 207 *sqq*.
Intuition of universal truths, i., 407; ii., 24-6, 211, 229, 232, 247-8.
Inventio medii, i., 332; ii., 55, 121, 246.
Inventive method, ii., 7.
Inverse probability, ii., 278-82.
— processes, ii., 54-5.
Inversion, i., 243-5.

Irony, Socratic, ii., 15.
Irrelevant elements, elimination of, i., 276; ii., 165-72, 269.
Irvine, i., 307.
"Isolated" laws, ii., 207.

Janet, ii., 200.
Jevons, i., 39, 232, 244, 375; ii., 37, 43, 55, 137, 168, 169, 194-6, 202, 204, 303.
Joannes a S. Thoma, i., 14, *n.* 2.
John XXI., Pope, i., 317, 352.
Johnson, i., 323.
Joint method of difference and agreement, ii., 181-3.
Joly, M. Henri, ii., 326-7.
Joseph, on form and matter of thought, i., 21, 163, 264-5; ii., 296; on extension and denotation, i., 57; on properties, 85; on principle of individuation, 86, 140; on definition, 92; on basis of division, 119; on the categories, 147; on predication *per accidens*, 156; on verbal and real judgments, 171; on modality, 181, 185; on quantity of judgment, 191; on particular propositions, 194-5; on negation, 205; on immediate inferences, 232, 234-5, 237, 239-40; on existential import of copula, 249; on hypothetical judgment, 264; on disjunctive judgments, 285, 287-8; on definition of syllogism, 293, 366, 391-2; on the *Dictum de omni*, 300, 303-4; on undistributed middle, 309; on figures of syllogism, 344-55; on simple dilemmas, 369, 375; on deductive, inductive, and mathematical inferences, 391-2, 411; ii., 31, 40, 47, 49-53, 117, 142, 172, 231, 247, 248; on subsumption in syllogism, i., 406; on elimination in induction, ii., 38, 50, 52; on mechanical view of reality, 69; on efficient and necessary causality and free will, 73-5; on scope and ideal of science, 75, 108, 224, 229-32, 237-8; on metaphysical assumptions, 134-5, 143-5, 243; on concepts of cause, 80, 88, 90-2, 208; on grounds of belief in uniformity of nature, 102, 107-13, 115-16, 243; on function of hypothesis, 122; on analogy, 158-60; on experimental methods, 173, 175, 186, 191, 194; on inductive explanation, 247; on fallacies; 297, 301, 311, 328.
Joyce, on necessary judgments, i., 176; on traditional rules of syllogism, 317; on mnemonic lines, 352; on logical and real principles in inference, 389-90; on circumstantial evidence, 394-5; on induction not an inference, 408; ii., 114; on inductive syllogism, ii., 27, 31; on scholastic view of induction, 33; on uniformity of nature, 94, 98, 116; on measurement, 202; on explanation, 242-3; of laws, 207; on metaphysical truths, 219; on chance, 274, 280; on fallacies, 328.
Judgment (*v. propositions*), i., 17, 18; Kant's divisions of, 151; process of, 154-8; objective truth of, 158-62; definitions of, 161; problems on import of, 167-8; classifications of, 168-70; necessary and contingent, with synonyms, 170-80; 201-2; ii., 217-23; objectivity of necessary judgments, i., 174-9; modality of, 180-5, 261; quantity of categorical, 188-98; affirmative and negative, 202-6; opposition in categorical judgments, 219 *sqq.*; existential import of, 248-62; hypothetical and conditional, 263-79; disjunctive and alternative, 280-6; inference from "particulars," 386, 392-5; fallacies incident to, ii., 308-15.

Kant, i., 10, 20; on the categories, 146, 150-3; on analytic and synthetic judgments, 177-80; on modality, 183-5; on first figure of syllogism, 334; on realism, ii., 146-7; on fallacies, 300; 334.
Kelvin, Lord, ii., 133, 138.
Kepler, ii., 122.
Key, analytical, i., 125.
Keynes, Dr., on laws of thought, i., 27; ii., 308; on meaning and implication, i., 50, 165-6, 234; on denotation, 55; on negative terms, 66; on definition, 94, 95, 99; on simple, complex and compound judgments and terms, 169, 198-9, 283-4; on verbal and real propositions, 172; on modality, 183; on denial, 203-6; on quantification of predicate, 212-16, 332; on equational reading of propositions, 217-18; on contraposition, 242; on universe of discourse, 55, 249; on existential import, 249-262; on conditional and hypothetical propositions, 267, 270, 274, 276-8; on alternative and disjunctive propositions, 283; on eductions and rules of syllogism, 312; on simplification of rules of syllogism, 315-17; on fourth figure of syllogism, 324; on existential import in syllogism, 331-2; on antilogism, 340-1; on first figure, 344; on second figure, 345; on third figure, 349; on "disjunctive" syllogisms, 365; on dilemmas, 375; on sorites, 382-3; on non-syllogistic inferences, 387; on *petitio principii* in inference, 404-5.
Kinds, natural, i., 129; ii., 139.
Kirsch, ii., 254.

Lambert, i., 344.
Langlois and Seignobos, ii., 253.

Language, ambiguities of, i., 44, 189, 192, 283; ii., 301 *sqq.*
— and categories, i., 142-5.
— and existential import, i., 258-60.
— and thought, i., 35.
— definition of, i., 34.
— functions of, i., 34-7.
— generalization and specialization of, i., 44, 50, 96.
— of animals, i., 34.
— of gesture, i., 34.
Laplace, ii., 134, 138.
Larmor, ii., 133, 134.
Laurie, H., ii., 172.
Law, meanings of, ii., 68-70, 123, 124.
— and cause, 91-2, 240-1; hypotheses of, 123-5.
— and fact, 206, 208, 216, 223, 236-9.
— and property, ii., 237-8.
— and statistical uniformity, 288-93.
— of parsimony, i., 197; ii., 85.
Laws, derivative, ii., 208.
— descriptive and explanatory, ii., 69, 92, 189.
— empirical, ii., 206-9.
— explanation of, ii., 237-43.
— fallacies in establishing, ii., 334-7.
— necessary character of, *v. necessity*, and *truth*.
— of nature, ii., 98, 110, 123, 208; exceptions to, 220.
— of thought, i., 23-28; and syllogism, 298-9; not purely formal, ii., 222.
Lazy argument, i., 374.
Least squares, method of, ii., 205.
Leibniz, i., 27, 178; ii., 21, 100.
Leo XIII., Pope, ii., 21.
Lepidi, ii., 264.
Lesage, ii., 125.
Leverrier, ii., 197.
Lewis, Sir G. C., ii., 336-7.
"Liar," fallacy of, ii., 308.
Liesse, ii., 287.
Limitation, conversion by, i., 236.
Limitative judgments, i., 202.
Limiting clauses, i., 157, 198.
Linea praedicamentalis, i., 78.
Locke, i., 10, 103, 149, 395; ii., 75.
Logica critica, i., 29.
— *dialectica*, i., 29.
— *docens*, i., 16, 38 *n.*
— *utens*, i., 16, 38 *n.*
Logic, and allied sciences, i., 30-41.
— applied, ii., 1, 117-19.
— as science and art, i., 13-16.
— critical, i., 28-9.
— definitions of, i., 38.
— divisions of, i., 17-19.
— formal and material, i., 19-23, 28-9, 162-4, 253; ii., 117-19, 296, 302-3.
— formal object of, i., 14.
— history of, i., 40-7; ii., 2-7.
— natural and artificial, i., 12, 13, 38.
Logic of "chance," or "probability," ii., 261.
— of relatives, i., 247, 391.
— real, i., 28-9.
— scope of, i., 12; ii., 2-7, 294-6.
— sources of, i., 39-41.
— subject-matter of, i., 14, 16-19.
— uses of, i., 38-9; ii., 294.
Logical conceivability, or possibility, i., 249-50, 264.
— equations, i., 216-18, 234.
— truth, ii., 210-11.
— whole and part, i., 114.
Lottin, ii., 293.
Lotze, on negative terms, i., 67; on dilemma, 375; on chance, ii., 283.

Mach, ii., 123, 292.
Magnitude, *v. measurement*, and *quantitative*.
Maher, i., 161, 409; ii., 16, 25, 94, 99, 110, 111.
Major premiss, i., 295, 367.
— term, i., 294, 351-2.
— — illicit process of, i., 309.
Mal-observation, ii., 163, 330-1.
Malus, ii., 153, 163.
Mansel, i., 20; on categories, 142, 152-3, 264, 352, 375.
Mansion, ii., 269.
Manuscripts, value of, ii., 256-8.
Many questions, fallacy of, i., 379; ii., 302, 314-15.
Many-worded terms, i., 37, 197-8, 259, 283-4.
Material and formal, in logic, *v. formal* and *logic*.
— cause, ii., 62, 65, 83.
— division, i., 115.
— fallacies, ii., 302-3.
— obversion, i., 232.
Mathematical axioms, and logical, i., 299, 391; ii., 240.
— calculus of probabilities, ii., 275-85.
— inferences, i., 392; ii., 25, 51, 53, 117, 244-8.
induction, ii., 25.
— science, ii., 25, 54-5, 229-30, 244-8.
— truths, i., 179-80; ii., 220-1.
Matter, of judgment and proposition, i., 262-4.
— of inference and syllogism, i., 294-7.
Mean, *v. averages*.
Meaning in terms, two kinds of, i., 48, 50.
— and implications, of propositions, i., 164-6, 199, 229, 234, 396.
Means, method of, ii., 204-5.
Measurement, ii., 186, 188; nature, units, and limits of, 202-4; methods of, 204-5; and probability, 279; sphere of, 335-6.
Mechanical view of the universe, ii., 6, 59-60, 68-9, 75, 110-13, 130-5, 140-1, 221, 291-3.

Mechanical conceptions in science, ii., 132-3.
Mediate axioms, of syllogism, i., 299-301, 324.
— inference, i., 292-3; and categorical syllogisms, 385-92; and hypothetical syllogisms, 361-2; from particulars, 392-5; remote and proximate materials of, 409-10.
Mellone, Dr. S. H., on denial, i., 204; on collective premisses in syllogism, 402; ii. 9; on geometrical definitions, ii., 25; on scholastic view of induction, 33; on uniformity of nature, 72-3, 97-8, 114, 117; on cause and effect, 81-2; on efficiency and necessity, 73; on plurality of causes, 86-7, 90; on verification of hypothesis, 142; on analogy, 155, 157; on experimental methods, 173-86, 194; on intermixture of effects, 196; on residual phenomena, 197; on fact and law, 208; on demonstration, 227-8; on deduction and induction, 244-5; on enthymeme, 266-7.
Membra dividentia, i., 112.
Memory, and physical certitude, ii., 215.
Metaphor, i., 43-4; ii., 161, 332.
Metaphysical certitude, ii., 106; nature and source of, 214-15; of God's existence, 235.
— necessity, i., 170-80; as abstract and hypothetical, ii., 217-23.
— analysis, i., 93, 114.
— laws or principles, ii., 106, 217-23.
— judgments and propositions, i., 170-80.
Metaphysics, and logic, i., 30-3, 135, 145-7, 252; ii., 106 *sqq.*; 111-13, 130.
Metathesis, i., 336.
Method, i., 19, 378; ii., 1-22; didactic and inventive, 7; analytico-synthetic, 9; of teaching, 14-16; scholastic, 16-22; deductive and inductive, 48-9; in physical science, mediaeval and modern, 128-35; of perceptual inductive analysis, 165-72; fallacies incident to, 315-37; experimental "methods," 172-200; scope of, 197-201; methods of measurement, 204-5.
Methodology, ii., 1, 13-14, 253.
Mercier, Cardinal, i., 91, 101, 145; on the *Dictum de omni*, 301-3, 318; on necessary judgment as premiss of inference, 405; on discovery by inference, 409; on method, ii., 10; 16, 21; examples of inductive process, 45-7, 56-7; on uniformity of nature, 96; on hypothesis, 120-1, 127, 138, 139, 150; on truth, 210; on demonstration, 228; on statistics, 285 *sqq.*
Metrology, ii., 203.
Michael Psellus, i., 318.
Middle Ages, and method, ii., 4, 10; and induction, 35-7; and history, 255.
Middle term, i., 294-5; undistributed, 308-9, 317; finding middle terms, 332-4; ii., 53-4, 244-8; and "cause," 227-8, 247-8.
Mill, J. S., i., 10; on definition, 101; on classification, 127; on categories, 149-50; on connotative reading of judgments, 209; on immediate inference, 234; on existential import, 258; *Nota notae*, 303-4; on inference from particulars, 393-7; on the syllogism as a *petitio principii*, 401-4; ii., 30, 39; on induction, 42-3, 122; on "deductive," method in induction, 45, 198, 247; sensism, 59, 74, 79, 104-5; on "cause," 76-80, 128-31; on plurality of causes, 86; on uniformity of nature, 98-9, 102-5, 110, 116, 321-22; on laws of nature, 146; on analogy, 153, 156; on experimental methods, 169, 172-200; classification of fallacies, 298-9, 306, 307.
Minima sensibilia, ii., 202.
Minor premiss, i., 295, 367.
— term, i., 294, 351-2.
— — illicit process of, i., 309.
Miracles and method, ii., 255.
Mixed demonstration, ii., 233-5.
— syllogisms, i., 297-8; hypothetical, 358-62; not immediate inferences, 361-2; in induction, ii., 49-50; disjunctive, i., 363-6; in induction, ii., 39, 50-2.
Mnemonic lines for moods of syllogism, i., 326, 336; older forms of, 352.
Modal judgments and propositions, i., 180-5, 227, 261, 284, 289, 290.
Modality, objective, i., 180-3; subjective, 183-5.
Modi dicendi, *per se* and *per accidens*, i., 171-4.
Modus, in proposition, i., 180-1; in syllogism, 320, 359-60, 364-5.
Monahan, ii., 141, 143.
Monism, ii., 60, 105-7, 112, 221-3, 242; 323-4.
Monuments, ii., 256.
Moods, of syllogism, i., 320-31; and existential import, 331-2; indirect moods, 351; of hypothetical syllogism, 357-60; of disjunctive syllogism, 364-5; of dilemma, 370.
Moral certitude, ii., 150-1; nature and source of, 215, 217, 221, 248-50, 259, 262.
— laws, ii., 249-50.
— necessity, i., 183, 404-5.
— obligation, necessity of, ii., 78.
— universals, i., 201; ii., 217, 262, 265, 310.
Motio, motus, actio et passio, ii., 81-2.
Müller, Max, on categories and speech, i., 144-5.
Multiple quantification, i., 197.

Name, i., 37; proper, 46, 96.
Named moods of syllogism, i., 325.
Natural classification, i., 122-31.
— experiments, ii., 164.
— judgments and propositions, i., 156, 323, 352.
— kinds, i., 129; ii., 139.
— sciences, ii., 68.
— selection, ii., 139.
Nature, laws of, ii., 98, 110, 123, 208; exceptions to, 220.
— meanings of, i., 75; ii., 66-7.
Naville, Ernst, ii., 151.
Necessary character of truth, i., 161-2, 179-80; ii., 220-3.
— fact, i., 182; ii., 208, 222, 231, 233.
— judgments and propositions, i., 170-80, 200-2, 209, 225; ii., 23-4, 215, 217-23.
— modals, i., 181-3.
— (= necessitating) causes, ii., 64, 71, 84-92.
Necessity, kinds of, i., 176, 183, 404-5, 414; ii., 23, 78, 292; conditional, of physical laws, ii., 110-13, 216, 219, 230, 242-3; of metaphysical laws, 217-23, 229-30.
— and efficiency, ii., 73-4; and unconditionalness, 77-80.
— explanations of, ii., 60, 75-80, 109-10.
Negation, and affirmation, i., 202-3.
Negative definitions, i., 108, 110-11.
— hypotheticals, i., 270, 276-9, 357.
— instances, ii., 167, 170; function of, 171; non-observation of, 330.
— judgments and propositions, i., 202-6.
— premisses, i., 310-12.
— terms, i., 65-7, 202-6, 231-2.
Neo-Platonism, ii., 10.
Newman, Cardinal, i., 39, 163-4, 176, 394; ii., 22, 134, 211.
Newton, ii., 41, 50, 122, 124, 128, 185, 190, 247.
Nomenclature, i., 131-2.
Nominal definitions, and real, i., 99-104, 105-6, 134, 252; ii., 74.
— essences, i., 76, 84.
Nominalism, i., 10.
Nominalist view of logic, i., 20.
Non causa pro causa, fallacy of, ii., 302, 327-9.
Non-connotative terms, i., 62-4.
Non-denotative terms, i., 61.
Non-observation, fallacies of, ii., 163, 165, 329-30.
"*Non per hoc*"; "*non propter hoc*," ii., 328.
"*Non sequitur*," ii., 328.
"Normal mind" and axioms, ii., 325.
Nota notae, i., 209, 303-4.
Notions, transcendental, i., 149.
Numerically definite propositions, i., 197.
Nys, ii., 125, 140.

Objective, and subjective elements in truth, i., 177-80; ii., 220-3; in laws of thought, i., 25-7; ii., 222.
— views of modality, i., 180-5.
— extension, i., 55.
— intension, i., 49.
— reference of judgment, i., 160-2; 203-4, 248-52, 264.
Observation, and experiment, ii., 164-5; repeated observation, 165, 168.
— errors in, ii., 196; and measurement, 202-5; and inference, 263-4, 331.
— fallacies of, ii., 299, 329-31.
Obversion, i., 230-2.
Occam, ii., 144.
Occasionalism, ii., 74.
"Occult" forces and motions, ii., 128-32.
O'Keeffe, ii., 252.
Ollé Laprune, ii., 327.
O'Mahony, ii., 254.
Ontological truth, ii., 58, 210, 212, 235.
Opinion, ii., 213 (*cf. probability*, *belief*).
Opposition of judgments and propositions, i., 219-28; and existential import, 256-7; of conditionals, 270-2; of hypotheticals, 276-8; of disjunctives, 290.
— of terms, i., 64-9.
— of truth and error, ii., 213-4.
— square of, i., 220.
Oral tradition, ii., 256, 258-9.
Organon, Aristotle's, i., 40.
Origin and sources of hypotheses, i., 195; ii., 32, 151-5.
— — — of logic, i., 38-41.
Original moods of syllogism, i., 324-6.
Ostensive reduction, i., 335-9.
Ostwald, ii., 145.

Palaeography, ii., 256.
Palaestra logica, cited, ii., 174, 176, 180-1, 189, 193, 208, 311.
Pantheism, ii., 61, 106-7, 242.
Paradox of inference, i., 396-401.
Paralogism, ii., 297, 300.
Paronyms, ii., 307.
Parsimony, law of, i., 197; ii., 85.
Part, logical, i., 114.
Partial contrapositive, i., 241.
Particular judgments and propositions, i., 194-5; inference from, 313, 386, 392-5.
Partitive conversion, i., 236.
Pascal, ii., 22.
Passio, category of, i., 142, 147-8.
Pasteur, ii., 121, 138, 139, 157, 200.
Pedagogics, ii., 14.
Pejorem sequitur, etc., i., 318.
Per accidens conversion, i., 236-7.
— predication, i., 156, 172.
Percept, i., 3.
Perfect figure of syllogism, i., 300, 319, 335.
— induction, ii., 27.

Periodic changes, ii., 189-90.
Permanent causes, ii., 188.
Permutation, i., 232.
Per se predication, i., 170-4.
" Personal equation," ii., 204.
Petitio Principii, fallacy of, ii., 235, 301, 317-22.
— and syllogism, i., 401-7; and grounds of belief in uniformity of nature, ii., 103-5, 116, 321-2.
PETRUS HISPANUS, i., 317, 318, 352.
Phantasm, i., 3-4.
Phantoms, Bacon's, *v. idola*.
Phenomenism, ii., 59-60, 130, 146, 322-4.
Phenomenon, and cause, ii., 76-80, 128-31.
Philology, on thought and speech, i., 144-5.
Philosophy, i., 19, 135; and method, ii., 10 (*v. sciences*).
— errors in, ii., 325-6.
Physical certitude, ii., 98-100, 106; nature and source of, 215-6, 221.
— causes, ii., 64, 71.
— division, i., 107, 114.
— judgments and propositions, i., 170-80.
— law and necessity, ii., 78, 100-13, 123-4, 221-3.
— sciences, rise of, ii., 4-5; scope of, 26, 62-3, 74, 75, 110-13, 124, 131, 133; and method, 49, 63, 68.
PIAT, ii., 15.
PICTET, ii., 125.
Place, category of, i., 142.
PLATO, i., 9, 10, 11, 92; ii., 19, 20, 336.
Plurality of causes, i., 275; ii., 84-92, 187, 244, 247-8.
Plurative propositions, i., 195-6.
Plures interrogationes, fallacy of, i., 379; ii., 314-5.
POINCARÉ, H., ii., 131, 133, 134.
— L., ii., 201, 202, 203.
POISSON, ii., 285.
Polylemma, i., 367.
Polysyllogism, i., 377-8.
Ponendo tollens, modus, i., 364-5.
Ponens, modus, i., 359.
PORPHYRY, i., 72, 85.
Porphyry's tree, i., 78-9, 116.
Positive instances, function of, ii., 171.
— terms, i., 65-7.
Positivism, ii., 15, 59, 124, 130.
Possible propositions, i., 184.
Possibility, sphere of logical, i., 249-50; and actual reality, 264.
Postpraedicamenta, i., 136.
" *Post hoc ergo propter hoc*," ii., 328-9, 334.
Postulates, ii., 10, 23, 60, 106-7, 109, 111; justification of, 142-5.
Posture, category of, i., 142, 144.
Practical certitude, ii., 262 (*cf. moral certitude*).
Practical science, i., 15.
Praedicabilia, i., 72.
Praedicamenta, i., 7, 136 (*v. categories*).
PRANTL, i., 39, 136, 318.
Predicables, Aristotle's and Porphyry's, i., 72-3; classification of, 73-6; definitions of, 76-7; and categories, 138; as relations, *ibid.*
Predicamental line, i., 78.
Predicate, i., 154-5; distinction from subject, 156-8; distribution of, 187; quantification of, 202-16, 332.
Predication, views of, i., 207 *sqq.*, predicative view of, 207-8, 301, 302-4.
Premisses, i., 292; discovery and proof, of, 375; ii., 1, 2, 23-4; deductive and inductive, 246-7.
Prescription, argument from, ii., 259.
Primae intentiones mentis, i., 32, 73.
Principle, i., 8; logical and real, 135; in syllogism, 295.
Principles, of demonstration, ii., 226; of thought, i., 23-8; and being, 135; ii., 226; "regulative," ii., 13, 143-5, 227; conditional and unconditional, ii., 107-13.
Prius natura, and *prius nobis*, ii., 11, 16, 65, 227-9, 231-2.
Private definition, i., 106.
Privative conception, inference by, i., 232.
— terms, i., 69.
Probability, i. 19; and modality, 183-5; ii., 5, 151; in analogy, 153-61; in induction or generalization, 206-9; 212-14, 263; nature of, 260-3; sources of, 263; calculation of, 262-85; inverse, 278-82.
Probable arguments, ii., 263-8.
Problematic judgments, i., 180-5, 347, 358.
Progressive chains of reasoning, i., 377-83.
Proof, *v. demonstration* and *explanation*.
Proper names, i., 46; and connotation, 63, 96.
Property, i., 77, 82-6; and necessary judgments, i., 171-80; physical, 176; ii., 45-7, 125, 237.
" *Proportio*," ii., 160.
Propositio infinita, i. 202.
Propositions (*v. judgment*), *primi, secundii et tertii adjacentis*, i., 155; structure of, 154-8; natural and inordinate, 156; formulation of, 164-6; simple, complex and compound, 169, 197-200; necessary and contingent, 170-80; *propositio per se nota*, and *per aliud nota*, 170; ii., 24; modal, ii., 180-5, 227; form of, 186; universal and particular, 188-98; collective, 189; singular, 192-3, 227; plurative, 195-6; exponible, 198-200; indesignate, 200-2; views on

import of, 207-18; opposition of categoricals, 219-28, 290; eductions from, 229-46; existential import of, 248-62; hypothetical and conditional, 265-79; disjunctive and alternative, 280-91; inference from "particulars," 386, 392-5.
Proprium (*v. property*), predicable of, i., 77; ii., 152; and essence, i., 83-4; kinds of, 82-3; and necessary judgments, 171-80; and *accidens*, 175-7.
Prosody, fallacy of, ii., 306.
Prosyllogism, i., 377.
πρότερον φύσει, and *π. ἡμῖν*, *v. prius*, etc.
Proxima species, i., 78.
Proximate cause, ii., 63-4.
— matter of syllogism, i., 294.
Proximum genus, i., 78.
PSELLUS, i., 318.
Psychology, and logic, i., 33, 412.
Ptolemaic astronomy, ii., 122, 126, 138, 149, 329.
Pure, empiric, and mixed demonstrations, ii., 234-5.
— categorical syllogisms, i., 297-8; hypothetical, 356-8; disjunctive (alternative), 362.
— propositions, i., 170.
Purely formal division, i., 117.
— logic, i., 20-2, 162, 264-5.
Purpose, in classification, i., 123-31.
— in analogy, ii., 156.
— in nature, ii., 59, 62, 66-70, 112.

"Quadruped" logical, i., 307.
Qualitative analysis, ii., 165 *sqq.*
Quality, category of, i., 141; and relation, ii., 161.
— of categorical propositions, i., 202-6; of conditionals, 270; of hypotheticals, 277.
Quando, category of, i., 142.
Quantification of predicate, i., 212-16; and syllogism, 332.
Quantitative and qualitative methods, ii., 186.
— aspect of facts, ii., 123, 132-5, 141, 160-1, 201, 335.
— determination, ii., 201-5.
Quantity, category of, i., 141.
— of categorical propositions, i., 186 *sqq.*; of conditionals, 270; of hypotheticals, 227.
Quaternio terminorum, i., 307, 317.
Question, in syllogism, i., 293, 295.
QUÉTELET, ii., 145, 288, 289, 290, 292-3.
Quodlibeta, ii., 17.

RABIER, i., 303; ii., 144.
Ramean tree, i., 78.
RAMSAY, ii., 197.
RAMUS, i., 78.
Ratiocinatio, *Ratiocinium*, i., 18, 292.
Ratiocination, fallacies of, ii., 299.
Rational propositions, i., 170-80.
Raw materials of thought, i., 3, 17 n. 2.
RAY, i., 232.
RAYLEIGH, Lord, ii., 197.
Real and logical accidents, i., 86-8, 138.
— definition, i., 99-104.
— essences, i., 76, 84.
— propositions, i., 170-80.
Realism, extreme and moderate, i., 9, 10; Ontologistic and Platonic, 10; truth of realism not self-evident, ii., 107-8; moderate realism and experience, 221-3.
Reality, and thought, i., 42-3, 249-52; ii., 58; possible and actual, i., 264.
— and principle of sufficient reason, ii., 58-64, 113.
— as object of logic and of metaphysics, i., 30-33.
— Empiricist view of, *v. Empiricism.*
— Hegelian-idealist view of, *v. Hegel.*
— Scholastic, Theistic view of, *v. Theism.*
Realm of denotation, i., 52-4, 248-52.
Reason, *v. ground.*
— Principle of sufficient, i., 27; and syllogism, 359; and reality, ii., 58-61, 113.
Reasoning, *v. inference.*
Rebutting dilemmas, i., 371-3.
Reciprocal propositions, i., 237; as the ideal of science, 274-5, 296, 360-1; ii., 85-92, 151-2, 231, 247-8, 312.
— and non-reciprocal causes, ii., 84-92, 167.
Reductio ad absurdum, i., 337, 339-43, 346, 353-4; ii., 233-4, 328.
Reduction of syllogisms, i., 335-44; of hypothetical and disjunctive to categorical, 365-6; of dilemmas, 370-3.
Referential hypotheticals, i., 274.
Refutation, ways of, i., 223, 225, 295.
Regressive chains of reasoning, i., 377, 383-4.
— demonstration, ii., 45, 235.
REINSTADLER, i., 318.
REISCH, ii., 310.
Relation, category of, i., 141, 148; and ratio, ii., 160-1.
Relative terms, i., 70.
Relatives, logic of, i., 247, 391.
Remote matter of syllogism, i., 294.
Remotive propositions, i., 280, 282.
Renaissance, influence on logical method, ii., 4, 10, 36, 128-9.
Repugnant propositions, i., 205-6; terms, i., 69.
Resemblance, in classification, i., 128-31; in analogy, ii., 156-8.
Residual phenomena, ii., 193, 197.
Residues, method of, ii., 193-7.
Rhetoric, i., 34.
Rhetorical syllogism, ii., 268.
RICHARD, P., ii., 20, 22.
RICKABY, ii., 106, 147, 208, 253, 255.

Roberts, W. J., ii., 64.
Roger Bacon, ii., 21, 37, 185.
Rother, ii., 249-50.
Rule, and exception, ii., 167, 217-23.

Satolli, i., 101.
Scepticism and dogmatism, ii., 113.
Schemes of judgments and propositions, i., 164-6.
Scholastic debates, ii., 16-19.
— metaphysics, ii., 128-31, 228.
— method, ii., 4; appraised, 19-22.
— teaching methods, ii., 16-19.
— terminology, ii., 17-19, 317.
— philosophy, and method, ii., 10, 19, 111, 128-35.
— view of science, *v. science.*
— — of causes, ii., 62, 64, 78.
— — of demonstration, ii., 225 *sqq.*
Scholastics, on categories, i., 146; on necessary judgments, 170-83; on existential import, 252-4, 256; on rules of syllogism, 312, 318; on "*inventio medii*," 332-4; on method, ii., 4, 16-19, 128-35; on induction, 33-7; on truth, 58-9; on purpose in nature, 67-70; on basis of necessary causation, 78; on plurality of causes, 86; on belief in uniformity of nature, 100-2; on demonstration, 233; on authority in science, 252.
Science, and art, i., 15.
— and belief, ii., 97, 250-2.
— free will, and necessary causation, ii., 72, 94, 217.
Sciences, special, and philosophical, ii., 3-7, 101, 111, 124, 131, 133-4, 142, 168, 208, 226-7, 240-1.
— speculative and practical, i., 15.
— deductive and inductive, ii., 4-5, 7-9, 235, 243-8.
— historical, ii., 253-9.
— human, ii., 6, 248-9.
Scientific knowledge, ideal of, ii., 23, 108, 134, 208, 223-32.
— nomenclature, i., 131-2.
— terminology, i., 132-4.
Scope of concepts, i., 48.
— of logic, i., 19-23.
Scottish philosophers, on uniformity of nature, ii., 101.
Scotus, Duns, ii., 21, 33, 35-7, 93.
Second figure of syllogism, rules of, i., 322; characteristics of, 344-6.
Secundae intentiones mentis, i., 32, 72, 390.
Secundum quid, fallacy of, ii., 309-11.
Selection, in sense perception, ii., 162-3.
Self-contradictory concepts, ii., 304; judgments, 308.
σημεῖον, ii., 266-7.
Sensation and thought, i., 2-4.
Sensism, ii., 59, 79, 104-5, 130, 323-4.
Sensus communis, i., 3.
Sentroul, ii., 210.
Separable accidents, i., 86-8, 201-2.
Sequence and consequence, ii., 165, 328-9, 334.
— causality, ii., 80-4.
Sextus Empiricus, i., 303, 401.
Shyreswood, W. of, i., 352.
Sign, i., 35.
Sigwart, i., 66, 185, 206, 350; ii., 45, 290.
Silence, argument from, ii., 256.
Simple, complex, and compound judgments, i., 169, 197-200, 283-4.
— and complex terms, i., 198, 290.
— apprehension, i., 2, 17.
— inspection, fallacies of, ii., 299.
— contraposition, i., 241.
— conversion, i., 231-40.
— dilemmas, i., 367-8.
"*Simplex indicium veri*," ii., 144-5.
Singular concepts, i., 63, 394.
— judgments and propositions, i., 192-3, 227, 239.
— terms, i., 45-6.
Situs, category of, i., 142, 144.
Smith's "Nut to crack," ii., 318.
Socrates, i., 92; ii., 15, 331.
Sophisms, ii., 300-1.
Sorites, i., 378-84; ii., 265, 304.
— fallacy of, i., 379; ii., 306.
Space and causality, ii., 83.
Spalding, i., 232.
Specialization in language, i., 44, 50, 96.
Species, predicable of, i., 76-7.
— *fixed infimæ*, i., 79-81, 97, 130; ii., 139.
— and *genus*, in logic and in biology, i., 80, 130.
Specific and individual essences, i., 75-6.
— *differentia*, i., 79-82.
— *proprium*, i., 82-4.
Specification, of terms in judgment, i., 213-16.
Speculative science, i., 15.
Speech, parts of, and logical terms, i., 35; and categories, 143.
Spencer, H., ii., 14, 146, 320-1.
Sphere of application in concepts, i., 52-4, 248-52.
— of reference in judgments, i., 160-2, 203-4, 248-52.
— of inductive analysis, limitation of, ii., 168, 182-3.
Spinoza, i., 149; ii., 10.
Square of opposition, i., 220.
Standards of measurement, ii., 202.
Statistics, ii., 5; and concomitant variations, 192-3, 282; uses of, 285-7; and induction, 287; interpretation of, 287-93.
Stock, St. G., i., 226, 232, 375.
Stoics, on categories, i., 149; on sorites, 379.
Strength of analogies, ii., 156-8.

Strengthened syllogisms, i., 326-7.
Subaltern moods of syllogism, i., 325.
— opposition, i., 220-1.
Subcontrary opposition, i., 226-7.
Subdivision, i., 113.
Subject, grammatical, logical, and ultimate, of propositions, i., 155, 161, 248-9, 264.
Subjective intension, i., 49, 63.
— view of modality, i., 183-5.
Substance, category of, i., 139, 141, 143; and attribute, 366, 386-7; in induction, ii., 83, 129.
Substances, first and second, i., 75, 139.
Substantial terms, i., 46-7.
Sufficient reason, *v. reason.*
— (= necessitating) cause, ii., 64, 71, 79, 168, 244.
Suggestion and implication, i., 51, 63, 258; and inference, 394; in teaching, ii., 15.
— of hypotheses, ii., 151-5.
Summa Theologica, ii., 17.
Summum genus, i., 78, 96, 127.
Suppositio, i., 48.
— *materialis*, i., 37.
Suprasensible not unknowable, ii., 59-60.
Suprema genera rerum, i., 137.
Suspicion, ii., 213.
Syllogism, syllogisms, etymology and definitions of, i., 293, 385-92; matter and form of, 294-7; formal validity expressed as a hypothetical proposition, 296; pure and mixed, 297-8; and laws of thought, 298-9; mediate axioms of, 299-304; general canons of, 305-18; figures and moods of, 319-331; and existential import, 331-2; and quantification of predicate, 332; reduction of, 335-44; figures compared, 344-55; mixed, 356-75; and mediate inference, 385-92; characteristics of syllogistic reasoning, 385-6; and deductive inference, 391-2, 412; ii., 51-3; and fallacy of *petitio principii*, i., 401-7; and uniformity of nature, ii., 115-19; probable syllogism, ii., 159, 263-8; rhetorical, ii., 268; demonstrative, ii., 225-9.
Syllogistic reasoning, *v. syllogism*, and *inference.*
Symbolic logic, i., 117.
Symbols, use of, i., 23; for experimental methods, ii., 199-200.
Syncategorematic words, i., 34, 36.
Synonyms, i., 172.
Synthesis, in definition, i., 91; in judgment, i., 157-8, 171; in method, ii., 9-10; and analysis in teaching, ii., 14-16.
Synthetic method, i., 378; ii., 7-10.
— chains of reasoning, i., 377-83.
— judgments and propositions, i., 170-80; ii., 94; synthetic *a priori*, i., 179.
Systematic conceptions, ii., 127, 137-42, 145, 319.

Tabulæ, Lord Bacon's, ii., 172.
Tautology, i., 272, 276.
Teaching, methods of, ii., 14-22.
τεκμήριον, ii., 232, 266.
Teleology and mechanicism in philosophy of nature, *v. mechanical*, and *purpose.*
Tendency, defined, ii., 196.
Terminology, scientific, i., 132-4.
Terms, definition of, i., 37; divisions of, 42-71.
— ambiguous, i., 44, 189.
— distribution of, in proposition, i., 186-8.
— relation to concepts and to things, i., 42-3, 100.
— simple and complex, single-worded and many-worded, i., 37, 197-8, 259, 283-5, 290.
Testimony, qualities of, ii., 250; not ultimate, 251; criteria of, 254-6; and probability, 264; and calculus of probability, 284-5.
Tests of truth, *v. criteria.*
Tetralemma, i., 367.
Theism, philosophy of, ii., 61, 78; and uniformity of nature, 100-2, 106-7, 109-10, 112-13, 143; as a verifiable hypothesis, 145-7; 230-2, 234-6, 240-3.
Theophrastus, i., 351.
Theory and fact, ii., 60, 149-50.
— and hypothesis, ii., 124.
— of knowledge, i., 28-9.
Thesis, in syllogism, i., 292.
Third figure of syllogism, special rules of, i., 322-3; characteristics of, 346-50.
Thomas Aquinas, St., i., 13, 15, 32, 43, 141, 148, 173, 394; ii. 3, 12, 17, 21, 29, 33-4, 47, 67, 82, 127, 138, 147, 210, 228, 252, 326.
Thomson, Archbishop, i., 20, 212, 375.
Thought and language, i., 35.
— and sensation, i., 2-4.
— and things, i., 42-3, 249-52.
— form and matter of, i., 20-3, 146, 150, 152.
— laws of, *v. Laws of Thought.*
Time, category of, i., 142-4, 150; and causation, ii., 80-4, 228.
— *of* predication and *in* predication, i., 161-2.
Tollendo ponens, *modus*, i., 364; in induction, ii., 39.
Tollens, *modus*, i., 359.
Toohey, ii., 214-5.
Total cause, ii., 63-5.
Totum divisum, i., 112.
— *logicum*, i., 114.
— *metaphysicum*, i., 114
Traditional logic, i., 29; and existential import, 252-4, 256; 272.
— scheme of propositions, i., 186-8.

Transcendental notions, i., 149.
Transversion, i., 269.
Tree of Porphyry, i., 78-9.
Tres modi sciendi, i., 90; ii., 2.
Truth, and judgment, i., 158-62; objective character of, i., 160-2, 179-80; ii., 220-3; in hypothetical judgments, i., 263-4; ii., 217-23; in alternative judgments, i., 290, 371; definition and kinds of truth, ii., 210, 213, 235; truth indivisible, 214, 260-1; attainable, 325-7; necessity of, *v. necessity*.
"*Tu quoque*," ii., 316.
TYNDALL, ii., 140.
Type, definition by, i., 97-9.
Types, organic and inorganic, i., 129.

Ubi, category of, 142.
UEBERWEG, on nominal definition, i., 100-1; on dilemma, 375; on regressive reasoning, ii., 9; on induction, 34; on application of syllogism, 117, 227.
Ultimate sources of inferred conclusions, i., 410-12, 414.
Unconditional principles in science and philosophy, ii., 107-13.
Undistributed middle, i., 308.
Undue assumption of axioms (*cf. assumption*), ii., 255, 317, 322-7, 334.
— rejection of axioms, ii., 323.
Uniformity of nature, principle of, as categorical, ii., 96-9, 109; as hypothetical, 94-6, 109.
— grounds of belief in, 99-113, 116, 320-22.
— recognized by Scholastics, ii., 35-7, 67, 93.
— relation to deduction and syllogism, ii., 116-19; to induction, 114-16.
— and causality, ii., 71-2.
— and possibility of science, ii., 73-4, 93-4.
— sphere of application of, ii., 94.
Unitary terms, i., 46.
Units of measurement, ii., 202.
Unity of nature, ii., 105-6.
Universal concepts, formation of, i., 4-8, 63; direct and reflex, 138.
— grammar, i., 34.
— judgment in reasoning, immediate, i., 231-2, 236; syllogistic, 313-5; mediate, 392-5; errors on its function, 395 *sqq.*, 403; apprehension and application of, 407-12; ii., 216-17, 262-3.
— judgments and propositions, categorical, i., 188-98.
Universality, grades of, in judgment, i., 188-92, 201-2; 404-6; ii., 23, 215-17.
Universals, controversies on significance of, i., 8-11; ii., 107-8, 221-3.
Universe, ultimate views of, *v. Hegel, mechanical, scholastic, theism*,
— of discourse, i., 54, 65, 161, 249-52, 255.
Univocal terms, i., 43-4; predication, 149.

Validity of inference, formal and material, i., 294-7, 314-17, 401; in dilemmas, 370-3.
— of thought, i., 19-23.
VEITCH, i., 39, 174; ii., 242.
VENN, Dr. J., on denotation, i., 55; on definition, 90, 102; on classification, 125, 129, 130; on existential import, 259; on "if" judgments, 267-70; on alternative judgments, 281; on discovery by inference, 400, 408; on induction and inference, 411-12; on enumerative treatment of induction, ii., 43; on inverse processes, 55, 248; on causes, 62; on plurality of causes, 87-9; on uniformity of nature, 93, 100, 105, 117-19; on experimental analysis, 169; on symbolic notation of experimental methods, 200; on chance, 269, 271, 273, 283-4.
Vera causa, ii., 128.
Verbal disputes, i., 103-5.
— division, i., 114.
— propositions, i., 170-80.
— transformations, and inference, i., 166, 212, 231, 239, 396.
Verification, *v. hypothesis*.
Vicious circle, i., 401; ii., 113, 235, 320.
"*Vires occultae*," *v. occult*.
Vis cogitativa, i., 394.
VREGILLE, Père de, ii., 132.

WALKER, L. J., ii., 147.
WARD, ii., 130, 131, 133, 134, 135.
WATT, ii., 122.
Weakened conclusions, i., 325.
Weaker premiss, i., 317-8; ii., 264-5.
WEISMANN, ii., 129, 131, 139, 140.
WELTON, Prof. J., on principle of sufficient reason, and reality, i., 27; ii., 59-60; on *proprium*, i., 83; on definition, i., 90, 93, 98-9, 102, 110; on division, i., 117, 127-8; on categories, 137-8, 144; on subject and predicate, 156-7; on grounds of negation, 205-6; on quantification of predicate, 213-16; on material inferences, 247; on conditional judgments, 271; on disjunctive judgments, 286, 288-9; on direct determination of valid moods of syllogism, 329-31; on second figure, 345; on pure hypothetical syllogisms, 357-8; on pure disjunctive syllogisms, 362; on dilemmas, 375; on non-syllogistic mediate inferences, 391; on regressive reasoning, ii., 8-9; on method of teaching, 15, 16; on inductive syllogism, 29; on scholastic view of induction, 33; on induction as an inverse process, 55; on mechanical view of reality, 69, 133, 141; on efficient

causality, 73, 83; on cause and effect as identical, 82; on plurality of causes, 86-7; on "uniformity" and "unity" of nature, 103, 105-6; on hypothesis, 123, 141; on analogy, 154, 156; on experimental analysis, 168-72, 198; on methods of measurement, 204-5; on necessary truth, 217-23; on explanation, 239, 241-3; on chance, 271-2, 274-5, 277-85; on averages, 289-93; on fallacies, 303, 311, 317, 320-1, 329-37.
WHATELY, Archbishop, i., 20, 375; ii., 302-3, 310.
"When," category of, i., 142.
"Where," category of, i., 142.
WHEWELL, on terminology, i., 133-4; on induction, ii., 42, 122; on experimental methods, 198-9.
Whole, logical and metaphysical, i., 114.
WILLIAM OF SHYRESWOOD, i., 352.
WILLMANN, ii., 14.
WINDELBAND, ii., 229-31.
WINDLE, ii., 129, 131, 139, 140.
Words, classification of, in grammar and in logic, i., 34.
Working hypothesis, ii., 126-7.

ZENO, i., 369, 371, 375.
ZIGLIARA, i., 81, 318, 332, 342; ii., 2, 19, 86, 212, 253, 260.

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